Technological Progress, Organizational Change and the Size of the Human Resources Department*

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Abstract

Innovative workplace practices based on multi-tasking and ICT that have been diffusing in most OECD countries since the 1990s have strong consequences on working conditions. Available data show together with the emergence of new organizational forms like multi-tasking, the increase in the proportion of workers employed in managerial occupation and the increase in skill requirements. This paper proposes a theoretical model to analyze the optimal number of tasks per worker when switching to multi-tasking raises coordination costs between workers and between tasks. Firms can reduce coordination costs by assigning more workers to human resources management. Human capital is endogenously accumulated by workers. The model reproduces pretty well the regularities observed in the data. In particular, exogenous technological accelerations tend to increase both the number of tasks performed and the skill requirements, and to raise the fraction of workers devoted to management.

Keywords: Information Technology, Organizational Change, Human Capital, Multi-Tasking.

Journal of Economic Literature: J22, J24, L23, O33, C62.

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1 Introduction

Admittedly the recent Information and Communication Technologies (ICT) revolution has favoured the development of numerous innovative workplace practices in most OECD countries (see for example Osterman, 2004, for the US case, and Boucekkine and Crifo, 2007, for more recent empirical evidence for OECD countries). A view emerges as the productivity gains from investing in ICT cannot be significant without an appropriate evolution in the workplace organization towards more flexibility, which in turn is likely to rise the demand for skilled labour. This view is advocated and tested for example in the well-known paper of Bresnahan *et al.* (2002).

Table 1 illustrates this rising organizational flexibility following the ICT revolution. Work organization inside firms evolved from specialization to multi-tasking, and flexible forms of workplace organization have largely diffused in most OECD economies. For example, the two thirds of American firms and 57% of German establishments now rely on multi-tasking, and the proportion of workers involved in organizational changes increased continuously (and even doubled) in Great Britain and France.

Table 1: Tasks and computer use in the EU (% of workers), 1991-2000

	1991	1996	2000
Job involving complex tasks		57	56
Job involving repetitive tasks	23.3	16	15
Working with computers	13.9	38	41

Source: European surveys on working conditions, European foundation for the improvement of living and working conditions.

Parallel to this trend, we can also observe an increasing employment share of skilled workers in major OECD countries during the 1990s along with the dissemination of ICT, as shown in Table 2 (additional evidence confirming this trend in other OECD countries can be found in Boucekkine and Crifo, 2007).

Table 2: High-skilled (HS) ICT workers in the European Union and the United States, Average annual employment growth (1995-01) (* = in 2001)

	HS workers	HS ICT-related workers	Share of HS ICT workers in total occupations*
United States	2.79	5.29	2.63
France	1.67	7.11	2.05
Italy	5.99	8.58	1.30
Belgium	2.13	8.91	2.01
Germany	1.66	9.41	1.90
Denmark	3.08	9.49	2.58
EU	2.79	10.11	2.01
United Kingdom	1.37	12.63	2.60

Source: OECD (July 2004) based on the Eurostat Labour Force Survey and the US Current Population Survey, May 2003.

A definitely much less stressed aspect of organizational change, which is central in this paper, is the impact on human resources departments. In general, one would expect that the role of such department will be significantly altered in a situation where flexible forms of work organization are so massively adopted. Indeed, a quick look at the data confirms this intuition. In particular, the management ratio increased in many OECD countries during the twentieth century, but started decreasing in the early 2000s. In France for instance, the percentage of workers employed in managerial and professional specialty occupations rose from 7.1% in 1982 to 11.1% in 2000 and decreased to 4.8% in 2004. Similarly, this ratio ranges from 10.5% in 1970 to 14.6% in 2000 and 13.9% in 2004 in the United States. Additional evidence for other OECD countries is reproduced in Table 3. With reference to other countries, the management ratio (managers as percentage of the workforce) in 2004 amounted to 9.2% in Australia, 10.1% in Sweden, 10.7% in Hong-Kong, 12.5% in India and 13.2% in the Netherlands (Future Foundation and SHL, 2004).

Table 3: Employment in managerial occupations in selected countries over the twentieth century (in percentage of total employment)

US, 1910-2004	1910	1960	1970	1980	1990	2000	2004
	6.4	11.1	10.5	11.2	12.6	14.6	13.9
UK, 1911-2004	1911	1961	1971	1981	1991	2000	2004
	3.4	2.7	3.7	5.3	14.3	16.3	13.6
Canada 1950-2001		1950	1970	1980	1992	1999	2001
		8.4	10.0	7.7	13.0	11.7	10.4
Norway, 1981-2004	1981	1985	1991	1995	1997	2000	2004
	5.3	6.5	6.6	7.0	7.8	8.0	7.2
France, 1982-2004				1982	1989	2000	2004
				7.1	7.5	11.1	4.8
Denmark, 1981-2004	1981	1985	1991	1995	1997	2000	2004
	4.0	3.4	4.4	6.5	7.0	7.1	4.6
Germany, 1976-2004			1976	1980	1989	1999	2004
			3.8	3.2	4.1	4.9	3.3
Japan, 1960-2007		1960	1970	1980	1990	1999	2007
		2.1	2.6	4.0	3.8	3.3	2.7

Sources: UK, US, Canada, France, Germany, Japan data for the period 1950-2000: Aoyama and Castells (2002) and for year 2004: OECD (2002-2004). US data for year 1910: Wyatt and Hecker (2006). UK data for year 1911: Gallie (2001). Canada data for year 2001:

Statistics Canada. Japan data for year 2007: Japan Labour Force Survey. Denmark, Norway data for the period 1981-2004: ILO Labour Statistics Database and for year 2004: OECD (2002-2004).

We see from Table 3 that two groups of countries may be distinguished. In the first group, managerial occupations represent between 10 to 14% of total employement: US, UK, Canada are representative countries of this group[1]. Such countries have a relatively large proportion of managers in percentage of the workforce and can be considered as more managerial or "bureaucratic" than the second group of countries (Norway, France, Denmark, Germany, Japan), for which managers represent a much lower fraction of the workforce (between 2 to 7% of total employment).

Interestingly, the management ratio has started to decrease in both groups of countries in the beginning of 2000s. Since this period corresponds to the wide-scale diffusion of computer usage and multi-tasking, the restructuration process accompanying organizational changes and multi-tasking might be affecting human resources departments as well. This observation is consistent with the recent literature emphasizing that firms tend to be delayering and hierarchies are becoming flatter. In particular, advances in information technology may expand the effective span of control of top managers (Rajan and Wulf, 2006) and recent organizational changes tend to rely on the decentralization of authority, delayering of managerial functions and increased multi-tasking (Caroli and Van Reenen, 2001).

In the light of all these data, the evolution of skills, job content and work organization observed in many OECD countries over the past decades can be summarized in the following three main characteristics:

- increase in the proportion of workers employed in managerial occupations, together with a recent decrease since the ICT revolution
- diffusion of innovative workplace practices based on multi-tasking and computer use
- increase in skills requirements

The fact that the development of multi-tasking seems to be accompanied by an increase and then a decrease in the management ratios could suggest that coordinating workers and tasks should become more complex in the new economy. Becker and Murphy (1992) already argued that unlike Adam Smith's argument, specialization and the optimal division of labour would be determined more by the cost of combining specialized workers - i.e. coordination costs - than by the extent of the market. In particular, two types of coordination costs matter: the costs incurred by firms when coordinating workers' effort across various tasks and the costs of versatility incurred by workers subject to increased pressure and work rhythms. This contribution is only concerned with the former type of costs.

[1]This group also comprises Australia, Sweden, Hong-Kong, India and the Netherlands.

This paper studies the determination of the optimal number of tasks performed per worker in an economy where individuals devote time to production and human capital accumulation, and where multi-tasking both increases production and gives rise to coordination costs. Coordination costs could take several forms, which are surveyed in detail in the next section. In particular, we shall distinguish between horizontal coordination costs, which involve the costs of coordinating the tasks accomplished by each production worker, and vertical coordination costs, which reflect coordinating different workers. In our model, the first can reduce the coordination costs by assigning more workers to pure coordination non-productive tasks. We then examine how the economy reacts to permanent exogenous technological accelerations. The model is able to reproduce the three stylized facts outlined above. Importantly enough, the model delivers a permanent trend towards multi-tasking and human capital accumulation following permanent technological accelerations, while the size of the human resources department, that is the fraction of workers devoted to reduce coordination costs, is also significantly raised and then reduced.

The paper produces two important contributions to the literature. In first place, it brings out a novel modelling of coordination costs and the associated human resources management workers. The framework is additionally found to replicate the recent evidence on the evolution of the size of human resources departments in the OECD countries following the ICT revolution. Second, it generalizes the findings of Boucekkine and Crifo (2007) in many respects. These authors have produced a simple OLG model with a fixed given number of tasks with human capital accumulation but without coordination costs. In this paper, the number of tasks is endogenous in the presence of well-motivated coordination costs. In this sense, it has a much broader scope than Boucekkine and Crifo's 2007 model.

The paper is organized as follows: Section 2 defines precisely coordination costs. Section 3 presents the setup of the general equilibrium model and derives the optimality and equilibrium conditions. Section 4 derives the steady-state values and the associated comparative statics. Section 5 reports some simulations exercises to analyze the impact of permanent exogenous technological accelerations on the economy. Section 6 concludes, while the main computations are reported in the Appendix.

2 Defining and accounting for coordination costs

As we analyze within-firm work organization, the evidence we report here is focused on "internal" coordination costs, i.e. intra (or within) firm coordination costs[2].

From a theoretical perspective, the literature on coordination problems mostly focuses on the division of labour and the returns to workers' specialization (Yang

[2] External coordination costs correspond for instance to the costs of finding suppliers, negotiating contracts and paying bills to them and such issues are outside the scope of our model. External coordination costs are examined for instance in the transaction costs literature or in the industrial organization literature.

and Borland, 1991, Tamura, 1992). Some papers examine the impact of ICT on coordination (Brynjolfsson *et al.*, 1994), but very few papers analyze the role of coordination costs per se (see e.g. Becker and Murphy, 1992, or Hobijn, 1999). As coordination costs are multi-faceted, the theoretical literature has focused on various types of costs borne by the firm and the employees, among which:

– Switching costs between activities in terms of time, response delay or reduced goods quality (Lien *et al.* 2003, 2006).

– Communication costs and knowledge acquisition costs between workers (see Dessein and Santos, 2003, Bolton and Dewatripont, 1994, Radner, 1993, Garicano, 2000, Borghans and Ter Weel, 2006). In Garicano's model, for instance, higher layers of problem solvers in the firm increases the utilization rate of knowledge, thus economizing on knowledge acquisition, at the cost of increasing the communication required.

– Administrative coordination costs (assigning tasks, allocating resources and integrating outputs) and expertise coordination costs (managing knowledge and skill dependencies for complex, nonroutine intellectual tasks) (see Faraj and Sproull, 2000).

– Contracts incompleteness (see Acemoglu, Antras and Helpman, 2005). The basic idea here is that a firm decides a range of tasks to be performed and the division of labour among its workers. Coordination costs come from the fact that only a fraction of the activities that workers have to realize are contractible, the others are nonverifiable and noncontractible (and lead to under-investment).

To account for such a variety of coordination costs, we propose here a typology that distinguishes two categories of internal coordination costs: horizontal and vertical coordination costs.

- **Horizontal coordination costs** are the costs (at the firm level) of coordinating the tasks realized by each production worker. They embed switch costs between tasks, inter-tasks learning and information sharing costs, but also the strain associated with versatility and stressful working conditions. Such costs mainly concern production workers.

- **Vertical coordination costs** are the costs of coordinating workers, that is the "costs of managers and others who decide when, where and how to produce" (Brynjolfsson *et al.*, 1994). Coordinating workers is a costly activity which induces expenses in addition to the direct wage costs of human resources managers and specialists. Vertical coordination costs mainly concern workers employed in human resources services.

From an empirical perspective, an important and complex problem lies in measuring the costs resulting from work organization and occupational conditions and changes. Coordination costs may result in poor working conditions, excessive and unmanageable pressures leading to absenteeism, sick pay, turnover, injuries, compensation and litigation costs, damage to equipment and production or reduced performance. Yet, the costs of coordinating activities and workers are not limited to safety and health conditions in the workplace. Precise empirical evidence on the extent of coordination costs is hard to find in the economics literature (unlike psychology, sociology, management and computer science where coordination costs are subject to a wider attention), with a notable exception in the literature on transaction costs[3], but several estimates may help evaluating the extent of horizontal and vertical coordination costs.

As regards horizontal coordination costs, task-switching induce behavioural (neural or cognitive) switch costs. When people switch back and forth between tasks, there are both mental and physical costs, due for instance to 'inter-tasks learning'. Such costs persist even when people know in advance the identity of the new task and have ample time to prepare for the switch. Empirically, task-switching costs are measured by psychologists and neuroscientists in terms of response time and acuracy, that is slower response and lower percentage of correct response[4]. Recent evidence shows that the behavioural costs of task switching may lead to around 3% of errors on average[5].

Overall, if we restrict our attention to an average error rate of 3% due to switch costs, horizontal coordination costs might represent a minimal loss of 3% of firm output, which would be the least costly scenario.

As regards vertical coordination costs, there are financial losses evaluated in terms of human resources management when tasks assignment becomes more complex. More precisely, the time wasted by managers and directors in managing errors in tasks coordination and underperformance has been documented in a recent study by the Future Foundation. In 2004, US managers reported a waste of 34 days per year and senior executives 7 weeks a year - an hour per day - in managing employees underperformance. Overall, the reported time spent redoing or correcting mistakes of others amounts to 21% in Hong Kong, 16% in India, 13% in the US, 12% in Australia, 9% in the UK, 11% in the Netherlands and 7% in Sweden[6] (Future Foundation and SHL, 2004). Financial losses due to vertical coordination costs may also include the loss of wages or income and the additional expenditures (health care and

[3]For Wallis and North (1986) transactions costs can be decomposed into motivation costs (agency costs and conflict of interests among managers, owners and debt holders as well as costs of cheating and opportunistic behaviours) and coordination costs (costs of obtaining information, coordinating input in production and measurement costs). Both marketed and non-marketed transaction costs (e.g. resources spent in waiting, getting permits to do business, cutting through red tapes etc.) are concerned. In developed economies, the transaction sector would represent 60% of GNP and non-marketed transaction costs 11.3% of GDP per capita (Wang, 2003).

[4]For Lien *et al.* (2006), when a typical response to a single stimulus takes 300 milliseconds, adding a second task increases the response to about 800 milliseconds. Extending the difference to a car driving 60 miles an hour, the response rate more than doubles.

[5]This figure is a gross average of percentage of errors observed in some recent experiments in psychology. See e.g. Schneider and Logan (2005)

[6]This survey was conducted in 2004 over 700 managers across sectors over these 7 countries.

medical treatment) associated with failures of tasks coordination[7]. More generally, the annual (vertical) cost of coordinating employees poorly able to deal with the tasks they face ranged from 0.6% of GDP in Sweden, 1.1% in Australia, 1.5% in the UK and in the US, 1.8% in the Netherlands, 2.3% in India to 3% in Hong Kong (Future Foundation and SHL, 2004).

In principle, vertical coordination may increase or decrease with the management ratio since a higher proportion of managers both facilitates efficient coordination of workers and tasks but also raises bureaucratic management costs. We shall therefore consider both scenarios (more efficient or more bureaucratic vertical coordination costs).

Overall, if we restrict our attention to the time wasted managing underperformers, vertical coordination costs might be approximated by a minimal shortfall around 2% of firm output, which would correspond to the least costly scenario.

3 The model

The model proposed in this paper considers an economy in discrete time (from 0 to ∞) with an active population of size L[8]. The firm occupational structure is composed of two types of jobs: human resources jobs (in fraction ρ of the workforce employed) and production jobs (in fraction $1-\rho$ of the workforce employed). Workers devote time to production (either in the human resources service or in the production service) and to human capital accumulation.

3.1 Technology and coordination costs

The economy is characterized by a representative firm that produces a homogeneous (numeraire) good according to the following technology:

$$y_t = A_t \cdot \int_0^{n_t} \left[(1-\rho_t)\cdot h_t \cdot x_t(i) \cdot L_t\right]^{1-\alpha} di \qquad 0<\alpha<1 \tag{1}$$

[7] In the US, health care costs and individuals' health insurance have been rising by approximately 50% over the past two decades and part of this rise could be imputed to organizational changes (Cartwright and Cooper, 1997). In the 2000 European working condition survey, work-related stress was the second most common work-related health problem across European economies. Stress results in greater sickness absenteeism, impaired performance and productivity higher turnover rates and injuries. In the British industry for instance, almost 40% of all absenteeism could be attributed to stress at the workplace at a cost of £4.2 billion in 2000 (Hoel *et al.*, 2001). Workers involved in innovative work organizational practices in fact tend to be subject to greater psychological discomfort and to face more mental strain than their non innovative counterparts (Askenazy *et al.*, 2002).

[8] Note that when labour is divisible L measures either the number of workers for a fixed working time, or the volume of hours worked when working time can vary (given a fixed upper bound). Here we consider the second interpretation.

where A_t is a productivity parameter, L_t is the volume of hours worked with human capital h_t, $x_t(i)$ is the time devoted to task i, n_t is the number of tasks performed per worker and ρ_t is the fraction of the workforce in the personnel (human resources) service.

The worker's productive time is equal to T_t, hence we also have the constraint:

$$\int_0^{n_t} x_t(i)di = T_t \tag{2}$$

Tasks are symmetric, i.e. $x_t(i) = x_t$, and from (2) we then get:

$$x_t = \frac{T_t}{n_t}$$

and substituting this expression in (1) we obtain the production function:

$$y_t = A_t \cdot [(1 - \rho_t) \cdot h_t \cdot T_t \cdot L_t]^{1-\alpha} n_t^\alpha \tag{3}$$

Producing the good implies two types of costs: production costs and coordination costs (a similar element can be found in Brynjolfsson *et al.*, 1994). Since production requires physical resources and knowledge about how to combine them, production costs correspond to traditional costs of transforming inputs into output (physical - productive - resources expenses) whereas coordination costs correspond to the costs of combining and managing interactions and dependencies between resources (tasks and/or workers). In our model, labour is the sole input, therefore production costs equal the total wage bill and coordination costs depend on the number of tasks realized per worker (n) and on the fraction of workers in the human resources service (ρ).

The firm's profits (given that output is the numeraire) then write:

$$\pi_t = A_t \cdot [(1 - \rho_t) \cdot h_t \cdot T_t \cdot L_t]^{1-\alpha} n_t^\alpha - C(n, \rho) - w_t \cdot h_t \cdot T_t \cdot L_t$$

where w_t is the wage rate per efficiency unit of labour and $C(n, \rho)$ represents coordination costs measured as pure output loss.

The coordination costs function depends on horizontal and vertical coordination costs as follows[9]:

$$C(n, \rho) = \frac{h(n, \rho) \cdot v(\rho)}{d}$$

where $h(n, \rho)$ denotes horizontal coordination costs and $v(\rho)$ denotes vertical coordination costs, while d reflects the extent of coordination costs (a higher d reduces the importance or magnitude of coordination costs).

[9] In a different (hierarchical) context, Garicano and Rossi-Hansberg (2006) formalize the cost of knowledge acquisition by workers as the cost of solving problems of a given difficulty level multiplied by the knowledge cost required to solve a given proportion of problems. Our approach is very different since we focus on horizontal and vertical costs of coordinating workers and tasks whereas Garicano and Rossi-Hansberg focus on the costs of knowledge acquisition and poblem-solving in knowledge hierarchies. However, they also assume a multiplicative interaction between both types of costs.

Horizontal coordination costs are the firm-level costs of coordinating the tasks realized by each production worker. They embed switch costs between tasks, inter-tasks learning and information sharing costs, but also the strain associated with versatility and stressful working conditions. The higher the number of tasks per worker, the higher the switch costs, the higher the inter-tasks learning and information sharing costs and the higher the stress costs. Since production workers are in fraction $1-\rho$, we assume that the costs of coordinating n tasks for each worker writes:

$$h(n,\rho) = n^{\xi} \cdot (1-\rho)^{\theta}$$

where $\xi, \theta > 0$ (we do not impose $\xi \neq \theta$).

In other words, the costs of horizontal tasks coordination are based upon the idea that allocating workers' attention over various activities is likely to raise the occurrence of mistakes on the job. This increasing risk of production failure reduces the value of output and profits by an amount that depends both on the number of tasks per capita n and on the size of the production service $(1-\rho)$, given elasticities parameters ξ and θ that will be discussed below.

Vertical coordination costs are the costs of coordinating workers. Consistently with Section 2, they reflect the administrative cost of running a human resources department in a firm. In our set-up, such a cost involves of course a direct labour cost, which is already accounted for by the last term of the profit expression given above, but also an extra-cost depending on the size of the human resources department to be operated. To this end, we assume that vertical coordination costs is a function of the share of workers in the human resources service, that is:

$$v(\rho) = \rho^{\eta}$$

where η can be positive or negative. In particular, when $\eta > 0$ (that will be the case considered in the simulations) vertical coordination costs increase with the increase in the share of workers in the human resources service (and this corresponds to a more bureaucratic situation). On the other hand, when $\eta < 0$ coordination costs decrease with the increase in the proportion of human resources employees in the economy (and this can be interpreted as a less bureaucratic scenario).

The fact that vertical coordination costs are linked to the size of human resources service and add to the wage bill can be related to the literature on vertical integration. Indeed, since Williamson's theory of transaction costs, it is well known that a major limit to the growth of firms lies in the costs of internal organization. As explained by Joskow (2003), the volume of auditing information that must be processed by management grows non-linearly with the size and scope of the firm and becomes more difficult to use to control costs and quality effectively and to adapt to changing market conditions. Moreover, monitoring becomes also more difficult in large organizations and there are therefore potential shirking problems resulting from low power internal compensation incentives. In sum, the decision whether or not to vertically integrate comes from a tradeoff between the costs of market-based arrangements and the costs of internal organization described by the relatively inferior adaptive properties of bureaucratic hierarchies to rapidly changing outside opportunities over the longer term and the difficulty of designing compensation mechanisms to give managers and employees appropriate incentives to control costs and product quality[10]. Here, we rely on this debate on the "make or buy" decision by accounting for the fact that firms relying on large human resources departments are characterized by specific management costs (bureaucratic and incentives costs) labelled as vertical coordination costs.

We now make clear two working assumptions for the optimization problem of the firm to make sense. We first need to ensure that function $C(n, \rho)$ is increasing in n and decreasing in ρ. We need the latter condition on ρ to have an interior solution to the optimization problem tackled. Indeed, an increase in ρ decreases production labour, and therefore production. To balance this negative impact on profits, we need the increase in ρ lowers the coordination costs. With analytical forms postulated, we need the decrease in horizontal coordination costs due to an increment in ρ more than compensates the induced rising vertical coordination costs (when $\eta > 0$). This property will be put in more formal terms in Proposition 2.

The convexity of the cost function can be easily investigated. We have the following result:

Proposition 1 *The cost function* $C(n_t, \rho_t) = \frac{n_t^{\xi} \cdot (1-\rho_t)^{\theta} \cdot \rho_t^{\eta}}{d_t}$ *is convex when* $\xi > 1$, $\theta < 1$ *and* η *is small enough (either positive or negative).*

Proof. The first-order partial derivatives of the cost function are given by:

$$\frac{\partial C}{\partial n_t} = \frac{\xi n_t^{\xi-1} (1-\rho_t)^{\theta} \rho_t^{\eta}}{d_t} \qquad \frac{\partial C}{\partial \rho_t} = \frac{n_t^{\xi}}{d_t} (1-\rho_t)^{\theta-1} \rho_t^{\eta-1} [\eta - (\eta+\theta) \rho_t]$$

while the second-order partial derivatives are:

$$\begin{aligned}
\frac{\partial^2 C}{\partial n_t^2} &= \frac{\xi(\xi-1) n_t^{\xi-2} (1-\rho_t)^{\theta} \rho_t^{\eta}}{d_t} \\
\frac{\partial^2 C}{\partial \rho_t \partial n_t} &= \frac{\partial^2 C}{\partial n_t \partial \rho_t} = \frac{\xi n_t^{\xi-1}}{d_t} (1-\rho_t)^{\theta-1} \rho_t^{\eta-1} [\eta - (\eta+\theta) \rho_t] \\
\frac{\partial^2 C}{\partial \rho_t^2} &= \frac{n_t^{\xi}}{d_t} (1-\rho_t)^{\theta-2} \rho_t^{\eta-2} [\eta(\eta-1)(1-\rho_t) - \rho_t \eta (\theta-1) + \\
&\quad -\rho_t \eta (\eta+\theta)(1-\rho_t) - \rho_t^2 (\eta+\theta)(\theta-1)]
\end{aligned}$$

and the hessian matrix is:

$$H = \begin{bmatrix} \frac{\partial^2 C}{\partial n_t^2} & \frac{\partial^2 C}{\partial \rho_t \partial n_t} \\ \\ \frac{\partial^2 C}{\partial n_t \partial \rho_t} & \frac{\partial^2 C}{\partial \rho_t^2} \end{bmatrix}$$

[10] However, as highlighted by Joskow (2003), this literature has focused much more on the inefficiencies of market transactions than it has on the strengths and weaknesses of internal organization. Our focus on vertical and horizontal coordination costs can therefore provide an interesting complementary contribution to this debate.

and for the cost function to be convex this matrix must be positive definite, i.e. we must have:

$$H_1 > 0 \qquad H_2 > 0$$

With reference to the first condition we have:

$$H_1 > 0 \Rightarrow \frac{\partial^2 C}{\partial n_t^2} > 0 \Rightarrow \frac{\xi(\xi-1)\, n_t^{\xi-2}(1-\rho_t)^{\theta}\rho_t^{\eta}}{d_t} > 0 \Rightarrow \xi > 1$$

i.e. it is satisfied if $\xi > 1$, while with reference to the second condition we have:

$$H_2 > 0 \Rightarrow \det H > 0 \Rightarrow \frac{\partial^2 C}{\partial n_t^2}\cdot\frac{\partial^2 C}{\partial \rho_t^2} - \left(\frac{\partial^2 C}{\partial \rho_t \partial n_t}\right)^2 > 0$$

that is:

$$\frac{\xi(\xi-1)\, n_t^{\xi-2}(1-\rho_t)^{\theta}\rho_t^{\eta}}{d_t}\cdot\frac{n_t^{\xi}}{d_t}(1-\rho_t)^{\theta-2}\rho_t^{\eta-2}\left[\eta(\eta-1)(1-\rho_t) - \rho_t\eta(\theta-1) + \right.$$
$$\left. -\rho_t\eta(\eta+\theta)(1-\rho_t) - \rho_t^2(\eta+\theta)(\theta-1)\right] - \left[\frac{\xi n_t^{\xi-1}}{d_t}(1-\rho_t)^{\theta-1}\rho_t^{\eta-1}\left[\eta-(\eta+\theta)\rho_t\right]\right]^2 > 0$$

that leads to:

$$\frac{\xi n_t^{2\xi-2}(1-\rho_t)^{2\theta-2}\rho_t^{2\eta-2}}{d_t^2}\left[(\xi-1)\eta(\eta-1)(1-\rho_t) - \rho_t\eta(\xi-1)(\theta-1) + \right.$$
$$-\rho_t\eta(\xi-1)(\eta+\theta)(1-\rho_t) - \rho_t^2(\xi-1)(\eta+\theta)(\theta-1) +$$
$$\left. -\xi\eta^2 + 2\xi\eta\rho_t(\eta+\theta) + \xi\left(\eta^2+2\eta\theta+\theta^2\right)\rho_t^2\right] > 0$$

The fraction outside the square bracket is positive, while considering the expression inside the square bracket and letting η tend to 0 (both in the case of η positive and in the case of η negative) we get:

$$-\rho_t^2(\xi-1)\theta(\theta-1) + \xi\theta^2\rho_t^2 > 0,$$

which holds when $\xi > 1$ and $\theta < 1$, and hence the cost function is convex under the conditions of the proposition. ■

At this point the profit function writes:

$$\pi_t = A_t \cdot \left[(1-\rho_t)\cdot h_t \cdot T_t^d \cdot L_t\right]^{1-\alpha} n_t^{\alpha} - \frac{n_t^{\xi}\cdot(1-\rho_t)^{\theta}\cdot\rho_t^{\eta}}{d_t} - w_t \cdot h_t \cdot T_t^d \cdot L_t$$

where $d_t > 0$ and where T_t^d denotes now the working time demanded by the firm. In the decentralized economy the firm's optimization program is then given by:

$$\max_{n_t, T_t^d, \rho_t} \quad A_t\left[(1-\rho_t)\, h_t T_t^d L_t\right]^{1-\alpha} n_t^{\alpha} - \frac{n_t^{\xi}(1-\rho_t)^{\theta}\rho_t^{\eta}}{d_t} - w_t h_t T_t^d L_t$$

The first-order conditions of this program are:

$$
\begin{aligned}
\frac{\partial \pi_t}{\partial n_t} &= \alpha A_t \left[\left(1-\rho_t\right) h_t T_t^d L_t\right]^{1-\alpha} n_t^{\alpha-1} - \frac{\xi n_t^{\xi-1} \left(1-\rho_t\right)^{\theta} \rho_t^{\eta}}{d_t} = 0 \\
\frac{\partial \pi_t}{\partial T_t^d} &= \left(1-\alpha\right) A_t \left[\left(1-\rho_t\right) h_t T_t^d L_t\right]^{-\alpha} \left(1-\rho_t\right) h_t L_t n_t^{\alpha} - w_t h_t L_t = 0 \\
\frac{\partial \pi_t}{\partial \rho_t} &= -\left(1-\alpha\right) A_t \left[\left(1-\rho_t\right) h_t T_t^d L_t\right]^{-\alpha} h_t T_t^d L_t n_t^{\alpha} - \frac{n_t^{\xi}}{d_t} \left[\eta \left(1-\rho_t\right)^{\theta} \rho_t^{\eta-1} - \theta \left(1-\rho_t\right)^{\theta-1} \rho_t^{\eta}\right] = 0
\end{aligned}
$$

from which we obtain:

$$
\alpha A_t \left(1-\rho_t\right)^{1-\alpha} \left(h_t T_t^d L_t\right)^{1-\alpha} n_t^{\alpha} = \frac{\xi n_t^{\xi} \left(1-\rho_t\right)^{\theta} \rho_t^{\eta}}{d_t} \tag{4}
$$

$$
\left(1-\alpha\right) A_t \left(1-\rho_t\right)^{1-\alpha} \left(h_t T_t^d L_t\right)^{-\alpha} n_t^{\alpha} = w_t \tag{5}
$$

$$
\left(1-\alpha\right) A_t \left(1-\rho_t\right)^{-\alpha} \left(h_t T_t^d L_t\right)^{1-\alpha} n_t^{\alpha} = \frac{n_t^{\xi}}{d_t} \left[\theta \left(1-\rho_t\right)^{\theta-1} \rho_t^{\eta} - \eta \left(1-\rho_t\right)^{\theta} \rho_t^{\eta-1}\right] \tag{6}
$$

The second-order conditions of the problem of the firm, that guarantee the presence of a maximum, are checked in Appendix 7.1.

Equation (4) gives the optimality condition for the number of tasks by equalizing the marginal productivity of a task (that is, the increase in output due to an additional task) with the marginal cost of a task (that is, the marginal increase in horizontal coordination costs). Similarly, the optimal demand for productive time T determined in equation (5) equalizes the marginal product of productive time with its marginal remuneration.

Equation (6) is the most important condition of the firm's block, as it provides the optimality condition for the fraction of labour devoted to coordination tasks. The left-hand side clearly reflects the loss in production induced by diverting workers from production. The right-hand side reflects the marginal impact of a larger share of human resources specialists on the costs of internal coordination. By definition, this impact is twofold. On the one hand, more workers in the human resources department implies less workers in production, and less productive workers means a lower exposure by the firm to the risk of productive mistakes, thereby a lower level of horizontal costs of tasks coordination. On the other hand however, more workers in the human resources department drives the vertical component of coordination costs in the opposite direction by construction. More human resources specialists means indeed a higher bureaucratic and incentives burden, thereby driving vertical coordination costs upward (in the case of $\eta > 0$). For condition (6) to make sense, it is of course necessary to ensure that the right-hand side is positive at least locally, which is done in Proposition 2. Note that this needed property amounts to guarantee that the coordination costs function $C(n, \rho)$ is a decreasing function of ρ, a point made before.

3.2 Household and human capital accumulation

The household in the economy has a utility function given by:

$$u\left(c_t\right) = \frac{c_t^{1-\tau} - 1}{1-\tau} \qquad \tau > 0$$

where c_t is consumption, and to simplify the analysis we then assume $\tau = 1$, that is:

$$u\left(c_t\right) = \ln c_t \tag{7}$$

The household is then endowed with one unit of time supplied each period, that is spent on working (the fraction T_t^s) or on human capital accumulation (the fraction $1-T_t^s$), and the accumulation of human capital is described by the following equation:

$$h_{t+1} = E_t \cdot h_t^\delta \cdot \left(1 - T_t^s\right)^{1-\delta} \qquad 0 < \delta < 1 \tag{8}$$

where E_t is an efficiency parameter.

The household's intertemporal optimization program in the decentralized economy is then given by[11]:

$$\max_{\{c_t, T_t^s, a_{t+1}, h_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \ln c_t$$

$$s.t. \quad \begin{aligned} a_{t+1} &= \left(1 + r_t\right) a_t + w_t h_t T_t^s - c_t \\ h_{t+1} &= E_t h_t^\delta \left(1 - T_t^s\right)^{1-\delta} \end{aligned}$$

where β is the discount factor (with $0 < \beta < 1$) and a_t represents the assets held at time t.

The Lagrangian for this problem is:

$$\begin{aligned} \mathcal{L} \;=\; & \sum_{t=0}^{\infty} \left\{ \beta^t \ln c_t + \beta^t \mu_t \left[\left(1 + r_t\right) a_t + w_t h_t T_t^s - c_t - a_{t+1}\right] + \right. \\ & \left. + \beta^t \lambda_t \left[E_t h_t^\delta \left(1 - T_t^s\right)^{1-\delta} - h_{t+1}\right] \right\} \end{aligned}$$

[11] plus the standard transversality conditions.

and the first-order conditions are given by:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_t} &= \frac{\beta^t}{c_t} - \beta^t \mu_t = 0 \\
\frac{\partial \mathcal{L}}{\partial T_t^s} &= \beta^t \mu_t w_t h_t - \beta^t \lambda_t (1-\delta) E_t h_t^{\delta} (1 - T_t^s)^{-\delta} = 0 \\
\frac{\partial \mathcal{L}}{\partial a_{t+1}} &= \beta^{t+1} \mu_{t+1} (1 + r_{t+1}) - \beta^t \mu_t = 0 \\
\frac{\partial \mathcal{L}}{\partial h_{t+1}} &= \beta^{t+1} \mu_{t+1} w_{t+1} T_{t+1}^s + \beta^{t+1} \lambda_{t+1} \delta E_{t+1} h_{t+1}^{\delta-1} \left(1 - T_{t+1}^s\right)^{1-\delta} - \beta^t \lambda_t = 0 \\
\frac{\partial \mathcal{L}}{\partial \mu_t} &= \beta^t \left[(1 + r_t) a_t + w_t h_t T_t^s - c_t - a_{t+1}\right] = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda_t} &= \beta^t \left[E_t h_t^{\delta} (1 - T_t^s)^{1-\delta} - h_{t+1}\right] = 0
\end{aligned}$$

from which, rearranging and substituting, we get the following relevant equations:

$$\frac{1}{c_t} = \frac{\beta}{c_{t+1}} (1 + r_{t+1}) \tag{9}$$

$$\frac{w_t h_t}{c_t (1-\delta) E_t h_t^{\delta} (1 - T_t^s)^{-\delta}} = \frac{\beta \delta w_{t+1} \left(1 - T_{t+1}^s\right)}{c_{t+1} (1 - \delta)} + \frac{\beta w_{t+1} T_{t+1}^s}{c_{t+1}} \tag{10}$$

$$a_{t+1} = (1 + r_t) a_t + w_t h_t T_t^s - c_t \tag{11}$$

$$h_{t+1} = E_t h_t^{\delta} (1 - T_t^s)^{1-\delta} \tag{12}$$

Equation (9) is the typical Euler equation for optimal consumption over time. Equation (10) is the optimality condition for human capital accumulation after substitution of the Lagrange multipliers λ_t and μ_t using the first-order conditions with respect to consumption and production time supply notably. The left-hand side measures the marginal cost of human capital accumulation, which is simply reflected in the wage forgone in period t due to education. The right-hand side is the sum of two marginal benefit terms: the increase in the marginal productivity of education time in $t+1$ and the increasing labour remuneration in the same period. Equations (11) and (12) are just the law of evolution of consumer's wealth and the education technology respectively.

3.3 Market equilibrium conditions

Together with the solution of the problem of the firm and of the household, the stationary equilibrium of the decentralized economy is characterized also by the market equilibrium condition:

$$y_t = c_t + \frac{n_t^{\xi} (1 - \rho_t)^{\theta} \rho_t^{\eta}}{d_t}$$

from which:

$$A_t \left[(1-\rho_t) h_t T_t^s L_t\right]^{1-\alpha} n_t^{\alpha} = c_t + \frac{n_t^{\xi} (1-\rho_t)^{\theta} \rho_t^{\eta}}{d_t} \tag{13}$$

and by the labour market equilibrium condition:

$$T_t^d = T_t^s = T_t \tag{14}$$

We are now able to set the following definition of equilibrium for the economy under investigation:

Definition *Given the initial condition h_0, an equilibrium is a path:*

$$\{r_t, T_t, h_t, \rho_t, n_t, w_t, c_t, a_t\}_{t \geq 0}$$

that satisfies the equations (4)-(6) and (9)-(13) derived above and the corresponding standard transversality conditions.

4 Steady-state and comparative statics of the model

Given the decentralized economy considered in the previous Section, it is now possible to obtain the steady-state values of the different variables (for the computations see Appendix 7.2), that are given by:

$$r = \frac{1-\beta}{\beta} \tag{15}$$

$$T = \frac{1-\beta\delta}{1+\beta-2\beta\delta} \tag{16}$$

$$h = E^{\frac{1}{1-\delta}} \frac{\beta (1-\delta)}{1+\beta-2\beta\delta} \tag{17}$$

$$\rho = \frac{\alpha\eta}{\alpha(\xi+\eta+\theta)-\xi} \tag{18}$$

$$\begin{aligned} n \;=\; & \left(\frac{\alpha A d}{\xi}\right)^{\frac{1}{\xi-\alpha}} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\alpha}{\xi-\alpha}} \left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}} \cdot \\ & \cdot \left(\alpha(\xi+\theta)-\xi\right)^{\frac{1-\alpha-\theta}{\xi-\alpha}} (\alpha\eta)^{\frac{\eta}{\alpha-\xi}} \left(\alpha(\xi+\eta+\theta)-\xi\right)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}} \end{aligned} \tag{19}$$

$$\begin{aligned} w \;=\; & (1-\alpha)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\ & \cdot \left(\alpha(\xi+\theta)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} (\alpha\eta)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha(\xi+\eta+\theta)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} \end{aligned} \tag{20}$$

$$c = \frac{\xi-\alpha}{\xi}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \cdot \tag{21}$$
$$\cdot\left(\alpha(\xi+\theta)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}} (\alpha\eta)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha(\xi+\eta+\theta)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}$$

$$a = \frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \cdot \tag{22}$$
$$\cdot\left(\alpha(\xi+\theta)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} (\alpha\eta)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha(\xi+\eta+\theta)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}$$

The following result can then be stated:

Proposition 2 *Provided the following restrictions on the parameters hold:*

$$\delta < \frac{1+\beta}{2\beta} \qquad \textit{and} \qquad \begin{cases} \dfrac{\xi}{\xi+\theta} < \alpha < \xi & \textit{if } \eta > 0 \\[2ex] \alpha < \dfrac{\xi}{\xi+\theta} < \xi & \textit{if } \eta < 0 \end{cases}$$

there exists a unique steady state of the model with $0 < T < 1$ *and* $0 < \rho < 1$ *where the values of the different variables are given by the expressions (15)-(22).*

Proof. The steady-state values of the variables are obtained in Appendix 7.2. Concerning the restrictions on the parameters, given the expression obtained for T:

$$T = \frac{1-\beta\delta}{1+\beta-2\beta\delta}$$

the fact that $0 < T < 1$ implies $0 < \frac{1-\beta\delta}{1+\beta-2\beta\delta} < 1$, and since the numerator is positive (because $0 < \beta < 1$ and $0 < \delta < 1$) we must have (for the first inequality to hold):

$$1+\beta-2\beta\delta > 0 \Rightarrow \delta < \frac{1+\beta}{2\beta}$$

while the second inequality is always verified (being $\delta < 1$). Given the expression obtained for ρ:

$$\rho = \frac{\alpha\eta}{\alpha(\xi+\eta+\theta)-\xi}$$

then, the fact that $0 < \rho < 1$ implies $0 < \frac{\alpha\eta}{\alpha(\xi+\eta+\theta)-\xi} < 1$. At this point it is necessary to distinguish the case $\eta > 0$ and the case $\eta < 0$. If $\eta > 0$ the restriction $0 < \rho < 1$ requires:

$$\alpha(\xi+\eta+\theta)-\xi > 0 \Rightarrow \alpha > \frac{\xi}{\xi+\eta+\theta}$$

and also:

$$\alpha\eta < \alpha(\xi+\eta+\theta)-\xi \Rightarrow \alpha > \frac{\xi}{\xi+\theta}$$

Since we also have $\alpha < \xi$ (because the latter must be larger than 1 for the cost function to be convex), this implies the following restrictions:

$$\frac{\xi}{\xi + \theta} < \alpha < \xi$$

If $\eta < 0$, on the other hand, the restriction $0 < \rho < 1$ requires:

$$\alpha\left(\xi + \eta + \theta\right) - \xi < 0 \Rightarrow \alpha < \frac{\xi}{\xi + \eta + \theta}$$

and also (since both the numerator and the denominator of the fraction that defines ρ are negative numbers, so that the sign of the inequality must be reversed):

$$\alpha\eta > \alpha\left(\xi + \eta + \theta\right) - \xi \Rightarrow \alpha < \frac{\xi}{\xi + \theta}$$

so that we have the following restrictions:

$$\alpha < \frac{\xi}{\xi + \theta} < \xi$$

that guarantee that in the steady-state the values of n, w, c, a are all positive. ■

The steady-state values obtained above can then be used for the comparative statics analysis of the decentralized equilibrium. The results (see computations in Appendix 7.3) can be summarized in the following Table (for the missing elements, it is not possible to derive analytically the sign of the relationship between the variable and the parameter that affects it, and it is necessary to resort to simulations to have this indication):

Table 4: Comparative statics of the model

	r	T	h	ρ	n	w	c	a
α	0	0	0	$-$				
β	$-$	$-$	$+$	0				
δ	0	$+$	$+$	0				
ξ	0	0	0	$+$				
θ	0	0	0	$-$				
η	0	0	0	$+$				
A	0	0	0	0	$+$	$+$	$+$	$+$
E	0	0	$+$	0	$+$	$-$	$+$	$+$
L	0	0	0	0	$+$	$-$	$+$	$-$
d	0	0	0	0	$+$	$+$	$+$	$+$

These results can be interpreted as follows (they are relative to the case $\eta > 0$, while in the case of $\eta < 0$ the only changes concern the elements of the column of ρ, that change all their sign).

The interest rate r depends negatively on the discount factor (β), and the same factor (that represents also the degree of impatience) has a negative impact on the time spent working T and a positive impact on human capital h. A higher productivity of human capital (through the elasticity parameter δ) has a positive influence on the time spent working T and (either through the efficiency parameter E or through the elasticity parameter δ - in this latter case if E is high enough -) on human capital h. In other words, when households are less impatient or when human capital is more productive, the level of human capital improves.

The fraction of the workforce in the human resources service ρ depends positively on the weight (elasticity) of tasks in horizontal coordination costs (ξ) and on the weight of human resources in vertical coordination costs (η) and it depends negatively on the weight of production workers in horizontal coordination costs (θ) and on the weight of tasks in total output (α). In other words, more workers are allocated to human resources management when coordination costs are more resources intensive or consuming, or when the production failure risk (due to switch costs between tasks) is higher. On the contrary, more workers are allocated to production activities when task specialization is more productive or when bureaucratic coordination is more costly.

The intensity of multi-tasking n (the number of tasks per individual) as well as the level of consumption c increase with the efficiency of human capital E, the size of the workforce L and the technological efficiency of production A; and they decrease with the extent of coordination costs (recall that a higher d means a lower importance of coordination costs).

The wage rate w decreases with the efficiency of human capital E, the size of the workforce L and the extent of coordination costs, and increases with the technological efficiency of production A.

The level of assets a decreases with the size of the workforce L and the extent of coordination costs and increases with the efficiency of human capital E and the technological efficiency of production A.

5 Short-term dynamics of the model

The model considered can then be simulated in order to study the effects of different types of technological shocks that can hit the economy. Hereafter we present some representative simulation experiments on a calibrated model. Since the model is in many aspects highly stylized (no physical capital for example), calibrating it on a real economy is a daunting task. Below we consider a benchmark calibration and the corresponding short-term dynamics. The equilibrium paths generated by this calibration have been checked to be qualitatively robust to numerous parameters' changes. While the quantitative results displayed below are only indicative and should not be taken too seriously, the qualitative dynamics are found to be strongly robust.

5.1 Calibration

As usual, the simulation of the model requires a calibration, and the values chosen for the different parameters are reported in the following table:

Table 5: Calibration of the model, benchmark case

Parameter	*Symbol*	*Value*
Parameter α in the production function	α	0.65
Psychological discount factor	β	0.99
Productivity of human capital	δ	0.9
Parameter ξ in the coordination costs function	ξ	1.2
Parameter θ in the coordination costs function	θ	0.7
Parameter η in the coordination costs function	η	0.01
Productivity of output	A	1
Efficiency of human capital	E	2
Size of the workforce	L	1
Parameter d in the coordination costs function	d	0.5

In particular, on the one hand these parameters are such that they satisfy the restrictions obtained above (in order to have convexity of the cost function and existence of the steady-state). On the other hand, the same parameters allow to obtain steady-state values of some relevant quantities that are reasonable on the basis on the empirical evidence that is available. For instance, with this parameterization in the steady state the fraction of time devoted to production is slightly more than 0.5, the ratio consumption/output is close to 0.95 and the fraction of the workforce in the human resources services is close to 0.15 (this is consistent with the numbers for UK and US in recent years, see Table 3).

It is important to observe that this calibration considers a value of the parameter η positive and small, that corresponds to a "high bureaucratic" scenario, in which both coordination costs and fraction of the workforce in the human resources services are quite high. Considering a value of η negative would correspond to a "less bureaucratic" situation, in which coordination costs decrease as the proportion of human resources employees increases in the economy, but in any case the qualitative results would not be altered, and there would be no major difference in terms of organizational change.

5.2 Simulation

Starting from the values chosen, different types of shocks are considered, and the consequences on the economy are analyzed. In particular, it is possible to consider a shock on the productivity of output A, a shock on the efficiency of human capital E and a shock on the coordination costs through the parameter d. All these shocks are permanent.

The first situation considered is an increase in A (the productivity of output). From the analytical results derived previously we know that this increase has a long run effect on the number of tasks performed per worker and on output (that both increase). The simulation allows studying the behaviour of all quantities also in the short run (Figures $1.1 - 1.4$).

In particular, the simulations show that the number of tasks per worker n increases also in the short run, immediately after the increase in A, and remains at this level in the long run. The output level behaves in the same way, since it increases immediately after the increase in A, and then remains at this value in the long run. Also the fraction of the workforce in the human resources service ρ increases immediately after the increase in A, but then it returns to its initial level, since the long run value of this variable is not affected by A. The same holds for the level of human capital, that is characterized by an increase (of small amount) followed by a decrease, and for the allocation of time between production and human capital accumulation (with an initial increase of the time devoted to productive activity immediately after the shock on A, that in the end returns to its initial level).

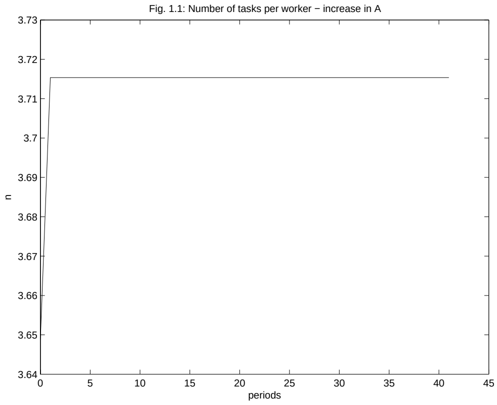


Fig. 1.1: Number of tasks per worker − increase in A

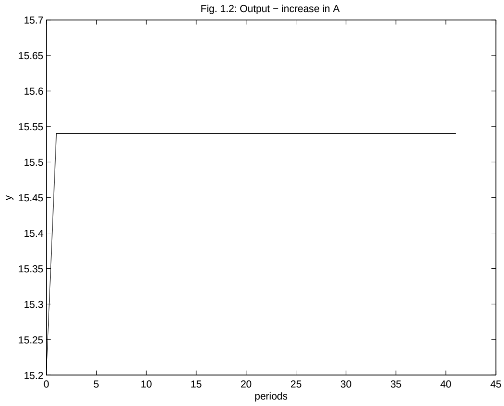


Fig. 1.2: Output − increase in A

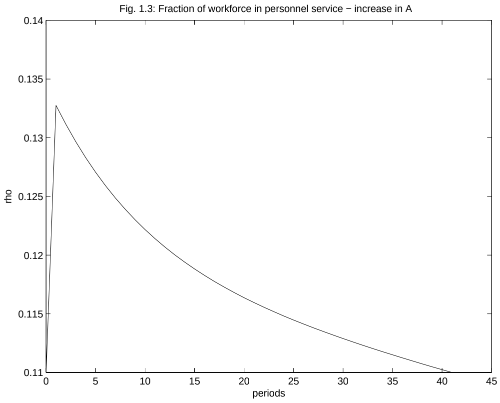


Fig. 1.3: Fraction of workforce in personnel service − increase in A

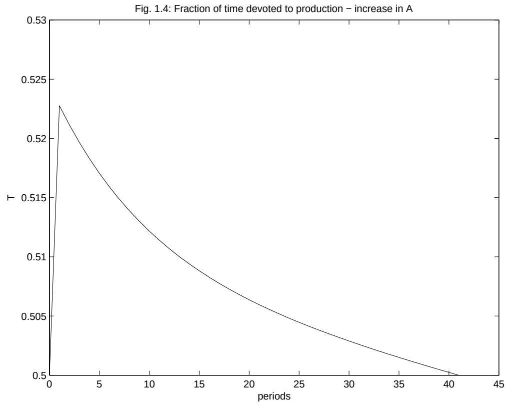
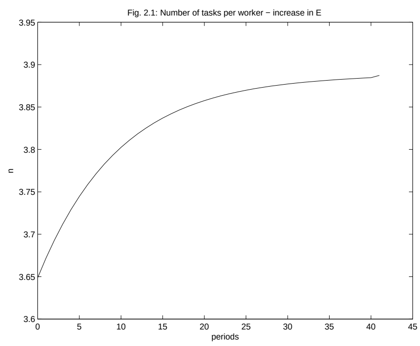


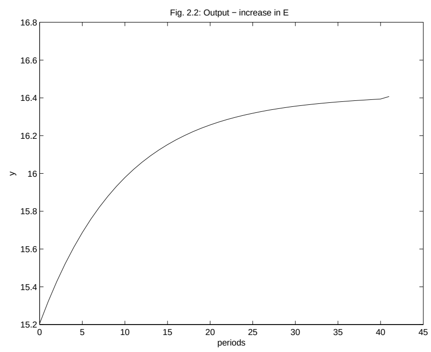
Fig. 1.4: Fraction of time devoted to production − increase in A

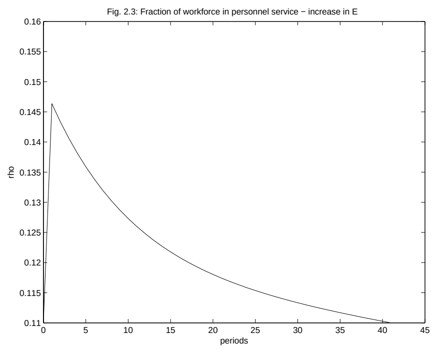
If we consider an increase in the parameter d that characterizes the extent of coordination costs (a rise in d reduces the importance of coordination costs), we get the same results as for the increase in A.

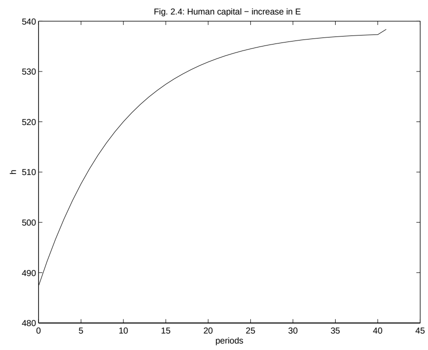
More precisely, the increase in d increases the intensity of multi-tasking and the output level immediately after the shock, and this effect persists also in the long run. The fraction of the workforce in the human resources service, the level of human capital and the allocation of time in favour of production, on the other hand, increase immediately after the shock on d, but then return to their initial levels since this shock does not affect their long run values. Intuitively, an increase in d corresponds to a reduction in coordination costs and represents therefore the same kind of qualitative productivity shock as a rise in the technological parameter A.

The last situation analyzed is an increase in the efficiency of human capital E. In this case, we have shown analytically that a rise in E has a long run effect on the number of tasks performed per worker, on the level of output and on the level of human capital (that all increase).









In the short run (Figures 2.1 − 2.4), the timing of the effects of the shock on the different variables is different from the previous cases. In effect, the number of tasks per worker n gradually increases after the rise in E and reaches, in the long run, a level higher than the initial one (while in the other cases examined there is immediately the jump in the variable, that then remains at the new value). Following the rise in E, output also increases gradually reaching the new long run level, and the same happens for the level of human capital. On the contrary, the fraction of the workforce in the human resources service and the allocation of time in favour of productive activity initially increase but then return to their initial levels, since their long run values are not affected by the shock on E.

It is important to observe that the results of these simulations (in particular, as outlined above, from the qualitative point of view) are consistent with the available data, concerning for instance the intensity of multi-tasking and other flexible organizational forms, and the behaviour of the workforce employed in the human resources service (see Tables 2 and 3) following the ICT Revolution of the 90s. This fraction of the workforce has increased in major economies and started to decrease in the early 2000s. This is the kind of behaviour that also emerges from the simulation exercises of the present model, which therefore behaves extremely well in replicating the organizational features of the ICT revolution. Interestingly enough, our model predicts a transitory increase in the size of the human resources department in response to all the performed permanent technological shocks, which certainly needs to be corroborated on updated data.

6 Conclusion

This paper develops a model to analyze the intensity of multi-tasking under various exogenous technological accelerations. The model has two original characteristics: it includes endogenous coordination costs, and it introduces the size of the human resources department as a key variable for the firms to control their coordination costs. In our modelling, and building on recent economic and management literature, we distinguish between vertical and horizontal coordination costs, which proves crucial in the equilibrium properties of the model. The model also includes endogenous human capital accumulation, and therefore bring together enough ingredients to study some highly relevant stylized facts identified in the introduction section in OECD data. Although technological progress is exogenous in our set-up, and no technology adoption decision is to be taken, we believe that the model offers a useful shortcut to analyze the consequences of technological accelerations on workplace organization. The fact that all performed numerical simulations corroborate the ability of the model to replicate the observed stylized facts is a good indication of that.

Two extensions are currently in our agenda. One concerns the incorporation of an explicit technology adoption decision where the firms have to decide whether they should buy a more efficient technology also involving costly organizational restructuring. This sensitive issue is left in the dark in our framework. A second issue concerns a component of coordination costs not treated in this paper, but already explicitly mentioned in the Introduction, that's the negative impact of flexible organizational forms (like multi-tasking) on the health of workers by inducing more stress, pressure and so on, as documented for example by Askenazy *et al.* (2002). We are currently studying the normative implications of such a situation (Boucekkine *et al.*, 2008).

References

[1] Acemoglu, D., P. Antras and E. Helpman (2005), "Contracts and the Division of Labor", NBER Working Paper 11356.

[2] Aoki, M. (1986), "Horizontal vs. Vertical Information Structure of the Firm", American Economic Review 76 (5), 971-983.

[3] Askenazy, P. (2001), "Innovative Workplace Practices and Occupational Injuries and Illnesses in the United States", Economic and Industrial Democracy 22 (4), 485-516.

[4] Askenazy, P. and E. Caroli (2006), "Innovative Work Practices, Information Technologies and Working Conditions: Evidence for France", IZA Working Paper 2321.

[5] Askenazy, P., E. Caroli and V. Marcus (2002), "New Organizational Practices and Working Conditions: Evidence from France in the 1990's", Recherches Economiques de Louvain 68 (1-2), 91-110.

[6] Aoyama, Y., and M. Castells (2002), "An empirical assessment of the information society: employment and occupational structures of G-7 countries, 1920-2000 ", International Labour Review 141, (1-2), 123-159.

[7] Barro, R. and X. Sala-i-Martin (1996), Economic Growth, Mc Graw Hill.

[8] Becker, G. S. and K. M. Murphy (1992), "The Division of Labor, Coordination Costs, and Knowledge", The Quarterly Journal of Economics 107 (4), 1137-1160.

[9] Bolton, P. and M. Dewatripont (1994), "The Firm as a Communication Network", The Quarterly Journal of Economics 109 (4), 809-839.

[10] Borghans, L. and B. ter Weel (2006), "The Division of Labour, Worker Organisation and Technological Change", Economic Journal 116 (509), F45-F72.

[11] Boucekkine, R. and P. Crifo (2007), "Human Capital Accumulation and the Transition from Specialization to Multi-tasking", Macroeconomic Dynamics forthcoming.

[12] Boucekkine, R., P. Crifo and C. Mattalia (2008), "Is there too much multi-tasking following the ICT revolution? ", working paper in progress.

[13] Brenner, M., D. Fairris and J. Ruser (2004), "Flexible Work Practices and Occupational Safety and Health: Exploring the Relationship between Cumulative Trauma Disorders and Workplace Transformation", Industrial Relations 43 (1), 242-266.

[14] Bresnahan, T., E. Brynjolfsson and L. Hitt (2002), "Information Technology, Workplace Organization and the Demand for Skilled Labor: Firm-Level Evidence", Quarterly Journal of Economics 117, 339-376.

[15] Brynjolfsson, E., T. Malone, V. Gurbaxani and A. Kambil (1994), "Does Information Technology Lead to Smaller Firms?", Management Science 40 (12), 1628-1644.

[16] Caroli, E. and J. Van Reenen (2001). "Skilled-Biased organizational change: evidence from a panel of british and french establishments.", Quarterly Journal of Economics 116 (4), 1449-1492.

[17] Cartwright, S. and C. Cooper (1997), Managing Workplace Stress. London: Sage Publications, Inc.

[18] Crowston, K. (1997), "A Coordination Theory Approach to Organizational Process Design", Organization Science 8 (2), 157-175.

[19] Dessein, W. and T. Santos (2003), "The Demand for Coordination", NBER Working Paper N. W10056.

[20] Economic and Social Research Council (2008), Manamgement in the UK, ESRC Fact Sheet.

[21] Fairris, D. and M. Brenner (2001), "Workplace Transformation and the Rise in Cumulative Trauma Disorder", Journal of Labor Research 22 (1), 15-28.

[22] Faraj, S. and L. Sproull (2000), "Coordinating Expertise in Software Development Teams", Management Science 46 (12), 1554-1568.

[23] Future Foundation, and SHL (2004), "Getting the Edge in the New People Economy", Joint research project report.

[24] D. Gallie (2001), " Skill Change and the Labour Market: Gender, Class and Unemployment", Paper given at the National Institute for Economic and Social Research conference Disadvantage in the Labour Market: Diversity and Commonality in Causes, Consequences and Redress.

[25] Garicano, L. and E. Rossi-Hansberg (2006), "Organization and Inequality in a Knowledge Economy ", Quarterly Journal of Economics 121 (4), 1383-1435.

[26] Garicano, L. (2000), "Hierarchies and the Organization of Knowledge in Production", Journal of Political Economy 108 (5), 874-904.

[27] Gordon, F. and D. Risley (1999), The Costs to Britain of Workplace Accidents and Work-Related Ill Health in 1995/6. Second Edition, HSE. London: HSE Books.

[28] Gordon, D. (1994), "Bosses of different stripes: a cross- national perspective on monitoring and supervision", American Economic Review 84 (2), 375-379.

[29] Hobijn, B. (1999),“The Division of Labor in an Increasing Complex World”, New York University Working Paper.

[30] Hoel, H., K. Sparks and C. Cooper (2001), “The Cost of Violence/Stress at Work and the Benefits of a Violence/Stress-Free Working Environment”, Report commissioned by the International Labour Organisation, conducted by the University of Manchester Institute of Science and Technology.

[31] Joskow, P.. (2003). Vertical Integration. Handbook of New Institutional Economics, Kluwer.

[32] Labour Force Survey UK (2006), Office National Statistics, Historical Quarterly Supplement - Table 16: All in employment by occupation (seasonally adjusted).

[33] Lien, M-C., E. Ruthruff and D. Kuhns (2006), “On the Difficulty in Task Switching: Assessing the Role of Task-Set Inhibition”, Psychonomic Bulletin and Review, 13 (3), 530-535.

[34] Lien, M-C., R. Schweickert and R. Proctor (2003), “Task Switching and Response Correspondence in the Psychological Refractory Period Paradigm”, Journal of Experimental Psychology 29 (3), 692-712.

[35] OECD (2002-2004), Babies and Bosses – Reconciling Work and Family Life, series, OECD, Paris

[36] Osterman, P. (2004), “How Common is Workplace Transformation and Who Adopts It?”, Industrial and Labor Relations Review 47, 173-189.

[37] Radner, R. (1993), “The Organisation of Decentralized Information Processing”, Econometrica 61 (5), 1109-1146.

[38] Rajan, R. and J. Wulf (2006), “The flattening firm: evidence from panel data on the changing nature of corporate hierarchies”, Review of Economics and Statistics, November 2006, 88(4): 759–773

[39] Ramaciotti, D. and J. Perriard (1999), “Certification Qualité Selon ISO 9000 et Fréquence des Accidents du Travail dans un Groupe d’Entreprises Suisses”, Actes du XXXIVème Congrès de la SELF-CAEN, 653-660.

[40] Schneider, D. and Logan G. (2005). “Modeling Task Switching Without Switching Tasks: A Short-Term Priming Account of Explicitly Cued Performance”. Journal of Experimental Psychology: General, Vol. 134, No. 3, 343–367

[41] Tamura, R. (1992), “Efficient Equilibrium Convergence: Heterogeneity and Growth”, Journal of Economic Theory 58, 355-376.

[42] Wallis, J. and D. North (1986), “Measuring the Transactions Sector in the American Economy, 1870-1970”, in Long Term Factors in American Economic Growth, S. Engerman and R. Gallman eds., University of Chicago Press.

[43] Wang, N. (2003), "Measuring Transaction Costs: an Incomplete Survey", Ronald Coase Institute Working Paper 2.

[44] Wyatt, I., and D. Hecker (2006), "MOccupational changes during the twentieth century", Monthly Labor Review, March 2006, 35-57.

[45] Yang, X. and J. Borland (1991), "A Microeconomic Mechanism for Economic Growth", Journal of Political Economy 99, 460-482.

7 Appendix

7.1 Second-order conditions at the steady-state in the decentralized economy

To check, at the steady state, the second-order conditions of the problem of the firm that guarantee the presence of a maximum it is possible to observe that we have (at the steady-state):

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial n^2} &= \alpha(\alpha-1) A\left[(1-\rho) h T^d L\right]^{1-\alpha} n^{\alpha-2} - \frac{\xi(\xi-1)(1-\rho)^\theta \rho^\eta}{d} n^{\xi-2} \\
\frac{\partial^2 \pi}{\partial (T^d)^2} &= -\alpha(1-\alpha) A\left[(1-\rho) h T^d L\right]^{-\alpha-1} (1-\rho)^2 h^2 L^2 n^\alpha \\
\frac{\partial^2 \pi}{\partial \rho^2} &= -\alpha(1-\alpha) A\left[(1-\rho) h T^d L\right]^{-\alpha-1} h^2 \left(T^d\right)^2 L^2 n^\alpha + \\
& -\frac{n^\xi}{d}\left[\eta\left[(\eta-1)(1-\rho)^\theta \rho^{\eta-2} - \theta(1-\rho)^{\theta-1}\rho^{\eta-1}\right] + \right. \\
& \left. -\theta\left[\eta(1-\rho)^{\theta-1}\rho^{\eta-1} - (\theta-1)(1-\rho)^{\theta-2}\rho^\eta\right]\right] \\
\frac{\partial^2 \pi}{\partial T^d \partial n} &= \frac{\partial^2 \pi}{\partial n \partial T^d} = \alpha(1-\alpha) A\left[(1-\rho) h T^d L\right]^{-\alpha}(1-\rho) h L n^{\alpha-1} \\
\frac{\partial^2 \pi}{\partial \rho \partial n} &= \frac{\partial^2 \pi}{\partial n \partial \rho} = -\alpha(1-\alpha) A\left[(1-\rho) h T^d L\right]^{-\alpha} h T^d L n^{\alpha-1} + \\
& -\frac{\xi n^{\xi-1}}{d}\left[\eta(1-\rho)^\theta \rho^{\eta-1} - \theta(1-\rho)^{\theta-1}\rho^\eta\right] \\
\frac{\partial^2 \pi}{\partial \rho \partial T^d} &= \frac{\partial^2 \pi}{\partial T^d \partial \rho} = -(1-\alpha)^2 A\left[(1-\rho) h L\right]^{-\alpha} h \left(T^d\right)^{-\alpha} L n^\alpha
\end{aligned}$$

The hessian matrix is then:

$$H = \begin{bmatrix} \frac{\partial^2 \pi}{\partial n^2} & \frac{\partial^2 \pi}{\partial T^d \partial n} & \frac{\partial^2 \pi}{\partial \rho \partial n} \\ \frac{\partial^2 \pi}{\partial n \partial T^d} & \frac{\partial^2 \pi}{\partial (T^d)^2} & \frac{\partial^2 \pi}{\partial \rho \partial T^d} \\ \frac{\partial^2 \pi}{\partial n \partial \rho} & \frac{\partial^2 \pi}{\partial T^d \partial \rho} & \frac{\partial^2 \pi}{\partial \rho^2} \end{bmatrix}$$

and in order to have a maximum for the problem of the firm the sequence of the signs of the north-west principal minors of this matrix must be:

$$H_1 < 0 \qquad H_2 > 0 \qquad H_3 < 0$$

With reference to this aspect we have:

$$H_1 < 0 \Rightarrow \frac{\partial^2 \pi}{\partial n^2} < 0 \Rightarrow \alpha(\alpha-1) A\left[(1-\rho) h T^d L\right]^{1-\alpha} n^{\alpha-2} - \frac{\xi(\xi-1)(1-\rho)^\theta \rho^\eta}{d} n^{\xi-2} < 0$$

that is always true (since it must be $\xi > 1$). We then have:

$$H_2 > 0 \Rightarrow \frac{\partial^2 \pi}{\partial n^2} \cdot \frac{\partial^2 \pi}{\partial \left(T^d\right)^2} - \left(\frac{\partial^2 \pi}{\partial T^d \partial n}\right)^2 > 0$$

that implies:

$$\left[\alpha\left(\alpha-1\right) A\left(1-\rho\right)^{1-\alpha}\left(hL\right)^{1-\alpha}\left(T^d\right)^{1-\alpha} n^{\alpha-2} - \frac{\xi\left(\xi-1\right)\left(1-\rho\right)^{\theta}\rho^{\eta}}{d} n^{\xi-2}\right] \cdot$$
$$\cdot \left[-\alpha\left(1-\alpha\right) A\left(1-\rho\right)^{1-\alpha}\left(hL\right)^{1-\alpha}\left(T^d\right)^{-\alpha-1} n^{\alpha}\right] +$$
$$-\left[\alpha\left(1-\alpha\right) A\left(1-\rho\right)^{1-\alpha}\left(hL\right)^{1-\alpha}\left(T^d\right)^{-\alpha} n^{\alpha-1}\right]^2 > 0$$

that leads to:

$$\frac{\alpha}{d}\left(1-\alpha\right)\xi\left(\xi-1\right) A\left(1-\rho\right)^{1-\alpha+\theta}\rho^{\eta}\left(hL\right)^{1-\alpha}\left(T^d\right)^{-\alpha-1} n^{\alpha+\xi-2} > 0$$

that is always true (since $\xi > 1$). We finally have:

$$H_3 < 0 \Rightarrow \det H < 0$$

that can be checked during the simulations (because it is not possible to obtain an analytic solution for this inequality). In this case the second-order conditions of the problem of the firm are satisfied and the value found is effectively a maximum.

7.2 Steady-state in the decentralized economy

The steady-state of the decentralized economy is obtained considering all the quantities constant in the first-order conditions of the problem of the firm and of the household and in the market equilibrium conditions, that is:

$$\alpha A\left(hT^dL\right)^{1-\alpha}\left(1-\rho\right)^{1-\alpha} n^{\alpha} = \frac{\xi}{d} n^{\xi}\left(1-\rho\right)^{\theta}\rho^{\eta} \tag{23}$$

$$\left(1-\alpha\right) A\left(hT^dL\right)^{-\alpha}\left(1-\rho\right)^{1-\alpha} n^{\alpha} = w \tag{24}$$

$$\left(1-\alpha\right) A\left(hT^dL\right)^{1-\alpha}\left(1-\rho\right)^{1-\alpha} n^{\alpha} = \frac{n^{\xi}}{d}\left(1-\rho\right)^{\theta}\rho^{\eta}\left[\theta - \frac{\eta\left(1-\rho\right)}{\rho}\right] \tag{25}$$

$$\frac{1}{c} = \frac{\beta}{c}\left(1+r\right) \tag{26}$$

$$\frac{h}{\left(1-\delta\right) E h^{\delta}\left(1-T^s\right)^{-\delta}} = \frac{\beta\delta\left(1-T^s\right)}{1-\delta} + \beta T^s \tag{27}$$

$$c = whT^s + ra \tag{28}$$

$$h = Eh^{\delta}\left(1-T^s\right)^{1-\delta} \tag{29}$$

$$A\left(hT^{s}L\right)^{1-\alpha}(1-\rho)^{1-\alpha}n^{\alpha}=c+\frac{n^{\xi}(1-\rho)^{\theta}\rho^{\eta}}{d} \tag{30}$$

$$T^{d}=T^{s}=T \tag{31}$$

From these equations it is possible to get the steady-state values of the different variables. In particular, from (26) we have:

$$\beta(1+r)=1\Rightarrow 1+r=\frac{1}{\beta}\Rightarrow r=\frac{1-\beta}{\beta}$$

Using from now on (31), so that we only write T, from (27) and (29) we obtain:

$$\begin{aligned}
\frac{Eh^{\delta}(1-T)^{1-\delta}}{(1-\delta)Eh^{\delta}(1-T)^{-\delta}} &= \frac{\beta\delta(1-T)}{1-\delta}+\beta T\Rightarrow\frac{1-T}{1-\delta}=\frac{\beta\delta(1-T)}{1-\delta}+\beta T\\
&\Rightarrow\ 1-T=\beta\delta(1-T)+\beta T(1-\delta)\\
&\Rightarrow\ 1-\beta\delta=T(1+\beta-2\beta\delta)\\
&\Rightarrow\ T=\frac{1-\beta\delta}{1+\beta-2\beta\delta}
\end{aligned}$$

From (29) (using the expression just found for T) we then get:

$$\begin{aligned}
h &= Eh^{\delta}\left(1-\frac{1-\beta\delta}{1+\beta-2\beta\delta}\right)^{1-\delta}\Rightarrow h^{1-\delta}=E\left(\frac{\beta(1-\delta)}{1+\beta-2\beta\delta}\right)^{1-\delta}\\
&\Rightarrow\ h=E^{\frac{1}{1-\delta}}\frac{\beta(1-\delta)}{1+\beta-2\beta\delta}
\end{aligned}$$

From (23) and (25) we then have:

$$\begin{aligned}
\frac{1-\alpha}{\alpha} &= \frac{1}{\xi}\left[\theta-\frac{\eta(1-\rho)}{\rho}\right]\\
&\Rightarrow\ \frac{1-\alpha}{\alpha}\xi=\frac{\theta\rho-\eta+\eta\rho}{\rho}\Rightarrow\frac{1-\alpha}{\alpha}\xi\rho=(\eta+\theta)\rho-\eta\\
&\Rightarrow\ \left(\eta+\theta-\frac{1-\alpha}{\alpha}\xi\right)\rho=\eta\Rightarrow\frac{\alpha\eta+\alpha\theta-\xi+\alpha\xi}{\alpha}\rho=\eta\\
&\Rightarrow\ \rho=\frac{\alpha\eta}{\alpha(\xi+\eta+\theta)-\xi}
\end{aligned}$$

From (23) (using the expressions found above for the different variables) we obtain:

$$\begin{aligned}
\alpha Ad(hTL)^{1-\alpha}(1-\rho)^{1-\alpha} &= \xi n^{\xi-\alpha}(1-\rho)^{\theta}\rho^{\eta}\\
&\Rightarrow\ n^{\xi-\alpha}=\frac{\alpha Ad(hTL)^{1-\alpha}(1-\rho)^{1-\alpha}}{\xi(1-\rho)^{\theta}\rho^{\eta}}\\
&\Rightarrow\ n=\left(\frac{\alpha Ad}{\xi}\right)^{\frac{1}{\xi-\alpha}}(hTL)^{\frac{1-\alpha}{\xi-\alpha}}(1-\rho)^{\frac{1-\alpha-\theta}{\xi-\alpha}}\rho^{\frac{\eta}{\alpha-\xi}}
\end{aligned}$$

$$\Rightarrow \quad n=\left(\frac{\alpha Ad}{\xi}\right)^{\frac{1}{\xi-\alpha}}\left(E^{\frac{1}{1-\delta}}\cdot\frac{\beta\left(1-\delta\right)}{1+\beta-2\beta\delta}\cdot\frac{1-\beta\delta}{1+\beta-2\beta\delta}\cdot L\right)^{\frac{1-\alpha}{\xi-\alpha}}\cdot$$
$$\cdot\left(1-\frac{\alpha\eta}{\alpha\left(\xi+\eta+\theta\right)-\xi}\right)^{\frac{1-\alpha-\theta}{\xi-\alpha}}\left(\frac{\alpha\eta}{\alpha\left(\xi+\eta+\theta\right)-\xi}\right)^{\frac{\eta}{\alpha-\xi}}$$
$$\Rightarrow \quad n=\left(\frac{\alpha Ad}{\xi}\right)^{\frac{1}{\xi-\alpha}}E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}}L^{\frac{1-\alpha}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{1-\alpha}{\xi-\alpha}}\cdot$$
$$\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{1-\alpha-\theta}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}}$$

From (24) we then have:

$$w \quad = \quad \left(1-\alpha\right)A\left(hTL\right)^{-\alpha}\left(1-\rho\right)^{1-\alpha}n^{\alpha}$$
$$\Rightarrow \quad w=\left(1-\alpha\right)A\left(E^{\frac{1}{1-\delta}}\cdot\frac{\beta\left(1-\delta\right)}{1+\beta-2\beta\delta}\cdot\frac{1-\beta\delta}{1+\beta-2\beta\delta}\cdot L\right)^{-\alpha}\left(1-\frac{\alpha\eta}{\alpha\left(\xi+\eta+\theta\right)-\xi}\right)^{1-\alpha}\cdot$$
$$\cdot\left(\frac{\alpha Ad}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}E^{\frac{\alpha(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\alpha)}{\xi-\alpha}}\cdot$$
$$\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(1-\alpha-\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(1-\alpha-\theta-\eta)}{\alpha-\xi}}$$
$$\Rightarrow \quad w=\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\cdot$$
$$\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}$$

From (30) we have:

$$c \quad = \quad A\left(hTL\right)^{1-\alpha}\left(1-\rho\right)^{1-\alpha}n^{\alpha}-\frac{n^{\xi}\left(1-\rho\right)^{\theta}\rho^{\eta}}{d}$$
$$\Rightarrow \quad c=A\left(E^{\frac{1}{1-\delta}}\cdot\frac{\beta\left(1-\delta\right)}{1+\beta-2\beta\delta}\cdot\frac{1-\beta\delta}{1+\beta-2\beta\delta}\cdot L\right)^{1-\alpha}\left(1-\frac{\alpha\eta}{\alpha\left(\xi+\eta+\theta\right)-\xi}\right)^{1-\alpha}\cdot$$
$$\cdot\left(\frac{\alpha Ad}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}E^{\frac{\alpha(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\alpha)}{\xi-\alpha}}\cdot$$
$$\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(1-\alpha-\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(1-\alpha-\theta-\eta)}{\alpha-\xi}}+$$
$$-\frac{1}{d}\left(\frac{\alpha Ad}{\xi}\right)^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot$$
$$\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi(1-\alpha-\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\xi\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\xi(1-\alpha-\theta-\eta)}{\alpha-\xi}}\cdot$$
$$\cdot\left(1-\frac{\alpha\eta}{\alpha\left(\xi+\eta+\theta\right)-\xi}\right)^{\theta}\left(\frac{\alpha\eta}{\alpha\left(\xi+\eta+\theta\right)-\xi}\right)^{\eta}$$

$$\Rightarrow \quad c = \frac{\xi-\alpha}{\xi}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \cdot$$
$$\cdot (\alpha(\xi+\theta)-\xi)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}} (\alpha\eta)^{\frac{\alpha\eta}{\alpha-\xi}} (\alpha(\xi+\eta+\theta)-\xi)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}$$

Finally from (28) we obtain:

$$\begin{aligned}
a &= \frac{1}{r}(c-whT) \\
&\Rightarrow a = \frac{\beta}{1-\beta}\Bigg[\frac{\xi-\alpha}{\xi}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \cdot \\
&\quad \cdot (\alpha(\xi+\theta)-\xi)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}} (\alpha\eta)^{\frac{\alpha\eta}{\alpha-\xi}} (\alpha(\xi+\eta+\theta)-\xi)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} + \\
&\quad -(1-\alpha)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot (\alpha(\xi+\theta)-\xi)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} (\alpha\eta)^{\frac{\alpha\eta}{\alpha-\xi}} (\alpha(\xi+\eta+\theta)-\xi)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} \cdot \\
&\quad \cdot E^{\frac{1}{1-\delta}} \cdot \frac{\beta(1-\delta)}{1+\beta-2\beta\delta} \cdot \frac{1-\beta\delta}{1+\beta-2\beta\delta}\Bigg] \\
&\Rightarrow a = \frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \cdot \\
&\quad \cdot (\alpha(\xi+\theta)-\xi)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} (\alpha\eta)^{\frac{\alpha\eta}{\alpha-\xi}} (\alpha(\xi+\eta+\theta)-\xi)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

7.3 Comparative statics in the decentralized economy

The comparative statics of the model in the decentralized economy can be obtained from equations (15)-(22). In particular, from (15) we have:

$$\frac{dr}{d\beta} = \frac{-\beta-1+\beta}{\beta^2} = -\frac{1}{\beta^2}$$

From (16) we then have:

$$\begin{aligned}
\frac{\partial T}{\partial \beta} &= \frac{-\delta(1+\beta-2\beta\delta)-(1-\beta\delta)(1-2\delta)}{(1+\beta-2\beta\delta)^2} = \frac{-\delta-\beta\delta+2\beta\delta^2-1+2\delta+\beta\delta-2\beta\delta^2}{(1+\beta-2\beta\delta)^2} = \\
&= \frac{\delta-1}{(1+\beta-2\beta\delta)^2}
\end{aligned}$$

and also:

$$\begin{aligned}
\frac{\partial T}{\partial \delta} &= \frac{-\beta(1+\beta-2\beta\delta)+2\beta(1-\beta\delta)}{(1+\beta-2\beta\delta)^2} = \frac{-\beta-\beta^2+2\beta^2\delta+2\beta-2\beta^2\delta}{(1+\beta-2\beta\delta)^2} = \\
&= \frac{\beta-\beta^2}{(1+\beta-2\beta\delta)^2} = \frac{\beta(1-\beta)}{(1+\beta-2\beta\delta)^2}
\end{aligned}$$

From (17) we get:

$$\frac{\partial h}{\partial E} = \frac{1}{1-\delta} E^{\frac{1}{1-\delta}-1} \frac{\beta(1-\delta)}{1+\beta-2\beta\delta} = E^{\frac{\delta}{1-\delta}} \frac{\beta}{1+\beta-2\beta\delta}$$

and also:

$$\begin{aligned}
\frac{\partial h}{\partial \beta} &= E^{\frac{1}{1-\delta}} \frac{(1-\delta)(1+\beta-2\beta\delta) - \beta(1-\delta)(1-2\delta)}{(1+\beta-2\beta\delta)^2} = \\
&= E^{\frac{1}{1-\delta}} \frac{1+\beta-2\beta\delta-\delta-\beta\delta+2\beta\delta^2-\beta+2\beta\delta+\beta\delta-2\beta\delta^2}{(1+\beta-2\beta\delta)^2} = \\
&= E^{\frac{1}{1-\delta}} \frac{1-\delta}{(1+\beta-2\beta\delta)^2}
\end{aligned}$$

and then:

$$\begin{aligned}
\frac{\partial h}{\partial \delta} &= E^{\frac{1}{1-\delta}} \frac{-\beta(1+\beta-2\beta\delta) + 2\beta^2(1-\delta)}{(1+\beta-2\beta\delta)^2} + \frac{\beta(1-\delta)}{1+\beta-2\beta\delta} E^{\frac{1}{1-\delta}} \frac{1}{(1-\delta)^2} \log E = \\
&= E^{\frac{1}{1-\delta}} \frac{-\beta-\beta^2+2\beta^2\delta+2\beta^2-2\beta^2\delta}{(1+\beta-2\beta\delta)^2} + E^{\frac{1}{1-\delta}} \frac{\beta}{(1-\delta)(1+\beta-2\beta\delta)} \log E = \\
&= E^{\frac{1}{1-\delta}} \frac{\beta^2-\beta}{(1+\beta-2\beta\delta)^2} + E^{\frac{1}{1-\delta}} \frac{\beta}{(1-\delta)(1+\beta-2\beta\delta)} \log E = \\
&= \frac{E^{\frac{1}{1-\delta}}}{1+\beta-2\beta\delta} \left[\frac{\beta(\beta-1)}{1+\beta-2\beta\delta} + \frac{\beta}{1-\delta} \log E \right]
\end{aligned}$$

From (18) we have:

$$\begin{aligned}
\frac{\partial \rho}{\partial \alpha} &= \frac{(\alpha(\xi+\eta+\theta)-\xi)\eta - \alpha\eta(\xi+\eta+\theta)}{(\alpha(\xi+\eta+\theta)-\xi)^2} = \\
&= \frac{\alpha\xi\eta+\alpha\eta^2+\alpha\theta\eta-\xi\eta-\alpha\xi\eta-\alpha\eta^2-\alpha\theta\eta}{(\alpha(\xi+\eta+\theta)-\xi)^2} = -\frac{\xi\eta}{(\alpha(\xi+\eta+\theta)-\xi)^2}
\end{aligned}$$

and then:

$$\frac{\partial \rho}{\partial \theta} = \frac{-\alpha\eta\cdot\alpha}{(\alpha(\xi+\eta+\theta)-\xi)^2} = -\frac{\alpha^2\eta}{(\alpha(\xi+\eta+\theta)-\xi)^2}$$

and also:

$$\frac{\partial \rho}{\partial \xi} = \frac{-\alpha\eta(\alpha-1)}{(\alpha(\xi+\eta+\theta)-\xi)^2} = \frac{\alpha\eta(1-\alpha)}{(\alpha(\xi+\eta+\theta)-\xi)^2}$$

and finally:

$$\begin{aligned}
\frac{\partial \rho}{\partial \eta} &= \frac{(\alpha(\xi+\eta+\theta)-\xi)\alpha - \alpha\eta\cdot\alpha}{(\alpha(\xi+\eta+\theta)-\xi)^2} = \\
&= \frac{\alpha^2\xi+\alpha^2\eta+\alpha^2\theta-\alpha\xi-\alpha^2\eta}{(\alpha(\xi+\eta+\theta)-\xi)^2} = \frac{\alpha[\alpha(\xi+\theta)-\xi]}{(\alpha(\xi+\eta+\theta)-\xi)^2}
\end{aligned}$$

From (19) we then have:

$$\begin{aligned}
\frac{\partial n}{\partial E} &= \frac{1-\alpha}{(1-\delta)(\xi-\alpha)}\left(\frac{\alpha Ad}{\xi}\right)^{\frac{1}{\xi-\alpha}} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}-1} L^{\frac{1-\alpha}{\xi-\alpha}}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}}\cdot \\
&\cdot(\alpha(\xi+\theta)-\xi)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}(\alpha(\xi+\eta+\theta)-\xi)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}} = \\
&= \frac{1-\alpha}{(1-\delta)(\xi-\alpha)}\left(\frac{\alpha Ad}{\xi}\right)^{\frac{1}{\xi-\alpha}} E^{\frac{1-\xi(1-\delta)-\alpha\delta}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\alpha}{\xi-\alpha}}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}}\cdot \\
&\cdot(\alpha(\xi+\theta)-\xi)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}(\alpha(\xi+\eta+\theta)-\xi)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}}
\end{aligned}$$

and then:

$$\begin{aligned}
\frac{\partial n}{\partial L} &= \frac{1-\alpha}{\xi-\alpha}\left(\frac{\alpha Ad}{\xi}\right)^{\frac{1}{\xi-\alpha}} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\alpha}{\xi-\alpha}-1}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}}\cdot \\
&\cdot(\alpha(\xi+\theta)-\xi)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}(\alpha(\xi+\eta+\theta)-\xi)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}} = \\
&= \frac{1-\alpha}{\xi-\alpha}\left(\frac{\alpha Ad}{\xi}\right)^{\frac{1}{\xi-\alpha}} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\xi}{\xi-\alpha}}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}}\cdot \\
&\cdot(\alpha(\xi+\theta)-\xi)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}(\alpha(\xi+\eta+\theta)-\xi)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}}
\end{aligned}$$

and also:

$$\begin{aligned}
\frac{\partial n}{\partial A} &= \frac{1}{\xi-\alpha}\left(\frac{\alpha Ad}{\xi}\right)^{\frac{1}{\xi-\alpha}-1}\frac{\alpha d}{\xi} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\alpha}{\xi-\alpha}}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}}\cdot \\
&\cdot(\alpha(\xi+\theta)-\xi)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}(\alpha(\xi+\eta+\theta)-\xi)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}} = \\
&= \frac{1}{\xi-\alpha}\left(\frac{\alpha Ad}{\xi}\right)^{\frac{1-\xi+\alpha}{\xi-\alpha}}\frac{\alpha d}{\xi} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\alpha}{\xi-\alpha}}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}}\cdot \\
&\cdot(\alpha(\xi+\theta)-\xi)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}(\alpha(\xi+\eta+\theta)-\xi)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}}
\end{aligned}$$

and finally:

$$\begin{aligned}
\frac{\partial n}{\partial d} &= \frac{1}{\xi-\alpha}\left(\frac{\alpha Ad}{\xi}\right)^{\frac{1}{\xi-\alpha}-1}\frac{\alpha A}{\xi} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\alpha}{\xi-\alpha}}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}}\cdot \\
&\cdot(\alpha(\xi+\theta)-\xi)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}(\alpha(\xi+\eta+\theta)-\xi)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}} = \\
&= \frac{1}{\xi-\alpha}\left(\frac{\alpha Ad}{\xi}\right)^{\frac{1-\xi+\alpha}{\xi-\alpha}}\frac{\alpha A}{\xi} E^{\frac{1-\alpha}{(1-\delta)(\xi-\alpha)}} L^{\frac{1-\alpha}{\xi-\alpha}}\left(\frac{\beta(1-\delta)(1-\beta\delta)}{(1+\beta-2\beta\delta)^2}\right)^{\frac{1-\alpha}{\xi-\alpha}}\cdot \\
&\cdot(\alpha(\xi+\theta)-\xi)^{\frac{1-\alpha-\theta}{\xi-\alpha}}(\alpha\eta)^{\frac{\eta}{\alpha-\xi}}(\alpha(\xi+\eta+\theta)-\xi)^{\frac{1-\alpha-\theta-\eta}{\alpha-\xi}}
\end{aligned}$$

From (20) we get:

$$\begin{aligned}
\frac{\partial w}{\partial E} &= \frac{\alpha\left(1-\xi\right)}{\left(1-\delta\right)\left(\xi-\alpha\right)}\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}-1} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} = \\
&= \frac{\alpha\left(1-\xi\right)}{\left(1-\delta\right)\left(\xi-\alpha\right)}\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(2-\xi-\delta)-\xi(1-\delta)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

and then:

$$\begin{aligned}
\frac{\partial w}{\partial L} &= \frac{\alpha\left(1-\xi\right)}{\xi-\alpha}\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}-1} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} = \\
&= \frac{\alpha\left(1-\xi\right)}{\xi-\alpha}\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(2-\xi)-\xi}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

and also:

$$\begin{aligned}
\frac{\partial w}{\partial A} &= \frac{\xi}{\xi-\alpha}\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}-1} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} = \\
&= \frac{\xi}{\xi-\alpha}\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}} A^{\frac{\alpha}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

and finally:

$$\begin{aligned}
\frac{\partial w}{\partial d} &= \frac{\alpha}{\xi-\alpha}\left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}-1} \frac{\alpha}{\xi} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}} = \\
&= \frac{1-\alpha}{\xi\left(\xi-\alpha\right)}\alpha^{\frac{\xi}{\xi-\alpha}}\left(\frac{d}{\xi}\right)^{\frac{2\alpha-\xi}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\alpha(1-\xi)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
&\quad \cdot \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

From (21) we have:

$$\begin{aligned}
\frac{\partial c}{\partial E} &= \frac{\xi\left(1-\alpha\right)}{\left(1-\delta\right)\left(\xi-\alpha\right)}\cdot\frac{\xi-\alpha}{\xi}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}-1}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\
&\quad\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}=\\
&= \frac{1-\alpha}{1-\delta}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\alpha(1-\xi)+\delta(\xi-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\
&\quad\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

and then:

$$\begin{aligned}
\frac{\partial c}{\partial L} &= \frac{\xi\left(1-\alpha\right)}{\xi-\alpha}\cdot\frac{\xi-\alpha}{\xi}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}-1}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\
&\quad\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}=\\
&= \left(1-\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\
&\quad\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

and also:

$$\begin{aligned}
\frac{\partial c}{\partial A} &= \frac{\xi}{\xi-\alpha}\cdot\frac{\xi-\alpha}{\xi}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}-1}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\
&\quad\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}=\\
&= \left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\alpha}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\
&\quad\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

and finally:

$$\begin{aligned}
\frac{\partial c}{\partial d} &= \frac{\alpha}{\xi-\alpha}\cdot\frac{\xi-\alpha}{\xi}\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}-1}\frac{\alpha}{\xi}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\
&\quad\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}=\\
&= \frac{\alpha^{2}}{\xi^{2}}\left(\frac{\alpha d}{\xi}\right)^{\frac{2\alpha-\xi}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^{2}}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\cdot\\
&\quad\cdot\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\theta)-\xi}{\alpha-\xi}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

Finally, from (22) we get:

$$\begin{aligned}
\frac{\partial a}{\partial E} &= \frac{\xi\left(1-\alpha\right)}{\left(1-\delta\right)\left(\xi-\alpha\right)}\cdot\frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}-1}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\cdot\\
&\cdot\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}=\\
&= \frac{\xi\left(1-\alpha\right)}{\left(1-\delta\right)\left(\xi-\alpha\right)}\cdot\frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\alpha(1-\xi)+\delta(\xi-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\cdot\\
&\cdot\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

and then:

$$\begin{aligned}
\frac{\partial a}{\partial L} &= \frac{\alpha\left(1-\xi\right)}{\xi-\alpha}\cdot\frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}-1}\cdot\\
&\cdot\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}=\\
&= \frac{\alpha\left(1-\xi\right)}{\xi-\alpha}\cdot\frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{2\alpha-\xi(1+\alpha)}{\xi-\alpha}}\cdot\\
&\cdot\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

and also:

$$\begin{aligned}
\frac{\partial a}{\partial A} &= \frac{\xi}{\xi-\alpha}\cdot\frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\xi}{\xi-\alpha}-1}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\cdot\\
&\cdot\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}=\\
&= \frac{\xi}{\xi-\alpha}\cdot\frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}}A^{\frac{\alpha}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\cdot\\
&\cdot\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}$$

and finally:

$$\begin{aligned}
\frac{\partial a}{\partial d} &= \frac{\alpha}{\xi-\alpha}\cdot\frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{\alpha}{\xi-\alpha}-1}\frac{\alpha}{\xi}A^{\frac{\xi}{\xi-\alpha}}E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}}L^{\frac{\alpha(1-\xi)}{\xi-\alpha}}\cdot\\
&\cdot\left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}}\left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}}\left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}}\left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}=
\end{aligned}$$

$$
\begin{aligned}
= \quad & \frac{\alpha^2}{\xi\left(\xi-\alpha\right)} \cdot \frac{\beta}{1-\beta}\left(\frac{\xi-\alpha}{\xi}L-1+\alpha\right)\left(\frac{\alpha d}{\xi}\right)^{\frac{2\alpha-\xi}{\xi-\alpha}} A^{\frac{\xi}{\xi-\alpha}} E^{\frac{\xi(1-\alpha)}{(1-\delta)(\xi-\alpha)}} L^{\frac{\alpha(1-\xi)}{\xi-\alpha}} \cdot \\
& \cdot \left(\frac{\beta\left(1-\delta\right)\left(1-\beta\delta\right)}{\left(1+\beta-2\beta\delta\right)^2}\right)^{\frac{\xi(1-\alpha)}{\xi-\alpha}} \left(\alpha\left(\xi+\theta\right)-\xi\right)^{\frac{\xi-\alpha(\xi+\theta)}{\xi-\alpha}} \left(\alpha\eta\right)^{\frac{\alpha\eta}{\alpha-\xi}} \left(\alpha\left(\xi+\eta+\theta\right)-\xi\right)^{\frac{\alpha(\xi+\eta+\theta)-\xi}{\xi-\alpha}}
\end{aligned}
$$