

# Dynamic Option Adjusted Spread and the Value of Mortgage Backed Securities

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## Abstract

We extend a reduced form model for pricing pass-through mortgage backed securities (MBS) and provide a novel hedging tool for investors in this market. To calculate the price of an MBS, traders use what is known as option-adjusted spread (OAS). The resulting OAS value represents the required basis points adjustment to reference curve discounting rates needed to match an observed market price. The OAS suffers from some drawbacks. For example, it remains constant until the maturity of the bond (thirty years in mortgage-backed securities), and does not incorporate interest rate volatility. We suggest instead what we call dynamic option adjusted spread (DOAS). The latter allows investors in the mortgage market to account for both prepayment risk and changes of the yield curve.

*Keywords:* Asset pricing, Mortgage Backed Securities, Term Structure.

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## 1. Introduction

Mortgage Backed Securities (MBS) are securities collateralised by residential mortgage loans. The MBS market has grown to become the largest fixed income market in the United States. The reason of this enormous growth is probably due to the higher return and lower risk profile compared to other fixed income securities.

However, although the market is growing very quickly, nevertheless there are still quite a few issues concerning the pricing and risk management of these securities. Because of the borrowers' prepayment option in the underlying mortgage loans, mortgage-backed securities have characteristics similar to those of callable bonds. Unlike callable bonds, however, for which the issuers' refinancing strategies are assumed to be close to optimal, mortgage borrowers may be slow to refinance when it would be financially favourable and sometimes prepay when it is financially unfavourable.

Investors in mortgage-backed securities hold long positions in noncallable bonds and short positions in call (prepayment) options. The noncallable bond is effectively a portfolio of zero coupon bonds, and the call option gives the borrower the right to prepay the mortgage at any time prior to the maturity of the loan. Therefore, the value of the MBS is the difference between the value of the noncallable bond and the value of the call (prepayment) option. In the market place, dealers generally price the mortgage by pricing these two components separately.

To evaluate the call option, the Option-Adjusted Spread methodology uses option pricing techniques. When the option component is quantified and taken away from the total yield spread, the yield to maturity of a non-benchmark bond can be compared to a risk-free rate of a benchmark security<sup>1</sup>.

Any model employed to value a MBS should be able to value the noncallable component of a mortgage and the call option component. *Ceteris paribus*, given that interest rate and prepayment risks have been accounted for, and incorporated in the theoretical model, one would expect the theoretical price of an MBS to be equal to its market price. If these values are not equal, then market participants demand compensation for the unmodeled risks.

The difference in values might be due to unmodeled risks which are attributable to the structure and liquidity of the bond. One of these unmodeled risks is

the forecast error associated with the prepayment model. For example, the actual prepayment may be faster or slower than what the model predicts. In this case, the OAS is the market price for the unmodeled risks. Because there is no agreement on how to model prepayments among mortgage holders, and many different interest rate models exist, option-adjusted spread calculation suffers from the lack of a standard term.

The academic literature in this area has mainly focused on modelling OAS dynamics such that the embedded mortgage call option price can be estimated and consequently the mortgage priced (see for example, Dunn and Spatt (1986), Liu and Xu (1998), Schwartz and Torous (1992) amongst others). However, these models although helping to clarify a number of issues concerning the pricing of MBS, are not used in practice. On the other hand, many researchers working in financial institutions, and amongst them top academics, have instead opted for econometric models to estimate the parameters of interest to calibrate reduced form models and price MBS (see for example Chen (2004)). Therefore, from a practitioner's point of view reduced form models seem to be the ideal way of pricing MBS. However, since most of these models are proprietary models their functional form is not known in the market.

This paper is organised as follows: we discuss the MBS model used in this study in Section 2, Section 3 discusses the interest rate model and its calibration, Section 4 presents a numerical example, Section 5 the dynamic option adjusted spread, Section 6 presents the empirical results finally Section 7 concludes.

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<sup>1</sup> See our application of option adjusted spread in this paper.

## 2. The Mortgage Backed Security Model

Consider the following probability space  $(\Omega, F, P)$ , and suppose the process  $\psi(t, D, C, z)$ , representing the price process of a mortgage backed security, is adapted to the filtration  $F$ . The price process depends on the risk neutral vector of discount bond price  $D_i$   $0 < i < N$ , with  $Q$  being the risk neutral probability measure, and the state variable  $z$ . Also denote with  $C_t$  the cash-flow paid by the mortgage at  $t$ .

Define the price process for a mortgage at time  $T$  when  $z = 0$  as the expected value of the discounted future cash-flows:

$$E(\psi) = E^Q \left[ \sum_{t_i=0}^T C_i D_i \right] \quad (1)$$

The main problem when determining the price of this security is that it is not simply determined by discounting  $C_i$ , since the borrower can at each time consider a prepayment action. In the introduction we have already mentioned different ways of modelling the prepayment option when pricing MBS. In this paper we shall follow Chen (2004) and implement a reduced form model<sup>2</sup>. In general, when pricing MBS one has to, first, generate the mortgage cash flows  $\tilde{C}(D, z)$  using, for example, a reduced form model. Once cash-flows have been generated, the value of the mortgage can be obtained by discounting the simulated cash flows between  $1 < i < N$ :

$$E(\tilde{\psi}) = E^Q \left[ \sum_{t_i=0}^T \tilde{C}_i D_i \right] \quad (2)$$

If we use Monte Carlo to generate  $m$  paths for  $\tilde{C}_t | m$ , we have that

$$E^Q(\tilde{\psi}) = \frac{1}{m} \sum_{t_i=0}^T \left[ \sum_{m=1}^M \tilde{C}_i D_i \right], \lim_{m \rightarrow \infty} \tilde{C}_i | m \rightarrow C$$

and the solution of (2) gives the value of the mortgage. Using Equation (2) one can also estimate the option adjusted

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<sup>2</sup> Refer to the Appendix for a description of the model.

spread  $z$  in the following way. Define with  $P$  the observed market price of the mortgage. We can compute  $z$  using a root finding method to solve (3) below:

$$\tilde{\psi}(t_0, \tilde{C}, D, z) = P \quad (3)$$

### 3. The Term Structure Model

To solve Equation (2) one has to simulate the term structure of interest rates out of the maturity of the mortgage. We extend the above model by using a two factor Heath, Jarrow, and Morton (1992) model (HJM). The HJM model is a class of models, and therefore one needs to specify the initial forward rates and volatilities to specify the model itself. Below we explain the way we have dealt with this problem.

The HJM model attempts to construct a model of the term structure of interest rates that is consistent with the observed term structure. The state variable in this model is the forward rate in time  $t$  for instantaneous borrowing at a later time  $T$ ,  $F(t, T)$ . In differential form the model can be written as:

$$dF(t, T) = m(t, T)dt + \sum_{k=1}^N \sigma_k(t, T)dW_k(t) \quad \text{for } 0 \leq t \leq T \quad (4)$$

Or also in integral form

$$F(t, T) = F(0, T) + \int_0^t m(v, T)dv + \int_0^t \sum_{k=1}^N \sigma_k(v, T)dW_k(v) \quad (5)$$

Here  $F(0, T)$  is the fixed initial forward rate curve,  $m(t, T)$  is the instantaneous forward rate drift,  $\sigma(t, T)$  is the instantaneous volatility process of the forward rate curve, and  $W$  is a standard Brownian motion process. The model above is very general and encompasses all the short rate models such as, for example, the Hull and White (1993) model.

The drift process is specified as:

$$m(t, T) = \sum_{k=1}^N \sigma_k(t, T) \int_t^T \sigma(t, s) ds \quad (6)$$

The hardest problem when using the HJM model to simulate  $F(t, T)$  is that the model is specified in terms of instantaneous forward rates and the latter are not observable in the market. To overcome the problem we use the following deterministic specification for the volatilities, and the Musiela parameterization:

$$\sigma_k(t, T) = \bar{\sigma}_k(t, T - t)$$

That means that our model belongs to the Gaussian class of models and maturity is specified as time to maturity. Therefore, if we set  $\tau = T - t$  it follows that:

$$d\bar{F}(t, \tau) = \bar{m}(t, \tau)dt + \bar{\sigma}(t, \tau)dW(t) \quad (7)$$

With the drift specified as:

$$\bar{m}(t, \tau) = \bar{\sigma}(t, \tau) \int_0^\tau \bar{\sigma}(t, s) ds + \frac{\partial}{\partial \tau} \bar{F}(t, \tau) \quad (8)$$

We use the above parameterisation when simulating the forward rates. The spot rate  $z(t)$  used to discount the cash flows can be determined from (7) as follows:

$$z(t) \equiv \lim_{\tau \rightarrow t} \bar{df}(t, \tau)$$

To use the two factor model above, one has to specify the initial forward rates and volatilities. In this application we have used Bloomberg to obtain the forward rates necessary to initiate the process. Also, we have used Bloomberg to obtain implied volatilities on interest rate caps necessary for the calibration of our model. Two volatilities are used. The first is set fixed for all the maturities and equal to the implied volatility of a thirty year interest rate cap option. The second refers to implied volatilities of interest rates cap with maturities 1 to 30 years. An Euler discretization scheme, with 360 time steps and 5000 simulations, is used.

#### 4. Numerical Example

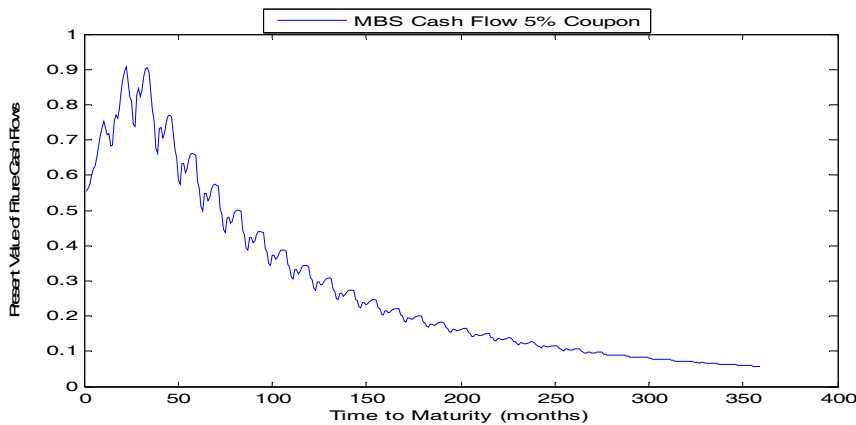
Table 1 shows a sample of simulated prices for the mortgage backed security using the model described above with their standard errors.

**Table 1: Mortgage Backed Security Valuation with 5% coupon rate**

$\tilde{\psi}_0$ %	102.1375	102.1236	<b>102.1786</b>	102.1993	102.1504	102.1547
SE	0.070569	0.069554	<b>0.063124</b>	0.06940	0.073031	0.063799

Consider, for example, the mortgage with value equal to 102.1786%.

Suppose the size of the underlying mortgage pool is \$1,000,000.00, the price of a mortgage-backed security issued from the underlying pool will be \$1,021,786.00. The observed market price is assumed to be 100% of the par value. One can therefore compute, using a root finding method, the option adjusted spread that in this example is 46 basis points.



**Figure 1.**

Figure 1 above shows simulated paths of the monthly cash flows of the mortgage. As the bond approaches maturity the value of the prepayment option decreases and consequently the mortgage cash flow becomes less uncertain.

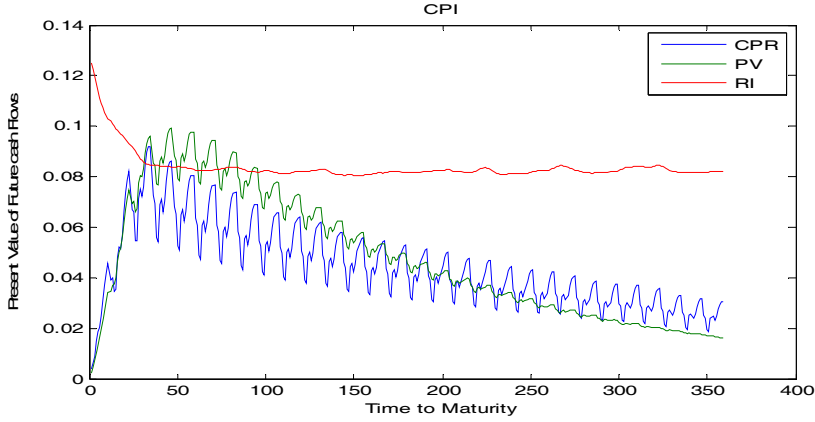
## **5. Dynamic Option Adjusted Spread**

The option-adjusted spread (OAS) above can be viewed as a measure of the yield spread. It is constant over the benchmark curve chosen for the valuation process. The reason why this spread is referred to as option-adjusted is because the cash flows of the underlying security are adjusted to reflect the embedded option. Most market participants find it more convenient to think about yield spread than price differences. One issue with the option spread is that it assumes the yield spread to stay unchanged over the maturity of the bond. Therefore, if future interest rates become volatile, the OAS remains unchanged. This implies that traders will have to compute it and recalibrate their models frequently. In this section we propose a modification of the OAS that we call Dynamic Option Adjusted Spread (DOAS). The DOAS allows one to capture prepayment risk as well as changes in the yield curve. A potential investor holding a mortgage can use the DOAS as a hedging tool.

Figure 2 below shows the conditional prepayment rate (CPR) function, the refinancing incentive (RI) and the portfolio value (PV). At the beginning of the mortgage there is a positive spread (i.e. the difference between the value of the portfolio and the cash flow of the mortgage). The difference would compensate the investor if the option is exercised by the borrower. The spread is particularly relevant in the first one hundred months which, in general, corresponds to the time when the



prepayment risk is higher. As the prepayment risk becomes less accentuate, the spread decreases.



**Figure 2.**

From an investor point of view the DOAS can be viewed as an investment<sup>3</sup>. The value of this portfolio can be positive or negative depending on the spread adjustment. A bond having a positive OAS has a positive portfolio value. On the other hand, a bond with a negative OAS will have a negative portfolio value.<sup>4</sup>

To compute the dynamic option adjusted spread, we used the following procedure. Use simulations to simulate the cash-flows, at each  $t$ , over the lifetime of the mortgage. Compute the option adjusted spread (i.e  $z$ ) at  $t_0$  and use it to adjust the cash-flows of the bond at each  $t$ . You have computed the adjusted cash-flows. The difference, at each  $t$ , between the plain vanilla bond cash flows and the mortgage cash flows, is the dynamic option adjusted spread in  $t$ . The summation of these up to  $t_0$  is the portfolio value

$$PV_0 = E^Q \sum_{i=0}^n [(\psi) - E(\tilde{\psi})]_i \quad (9)$$

Equation (9) describes the way we computed the portfolio value. Therefore the portfolio value is just the difference between a non-callable bond and a callable bond.

<sup>3</sup> We call this investment a portfolio value (PV).

<sup>4</sup> OAS can be negative when the mortgage coupon is low but interest rate volatility is relatively high. In this case investors in this market might not be very concerned with the MBS optionality, at least not in the short run.

It might be worth noticing that, by buying a MBS and investing in the above portfolio, the investor has indeed created a synthetic non-callable bond but with the difference that he is also hedging against interest rate risk.

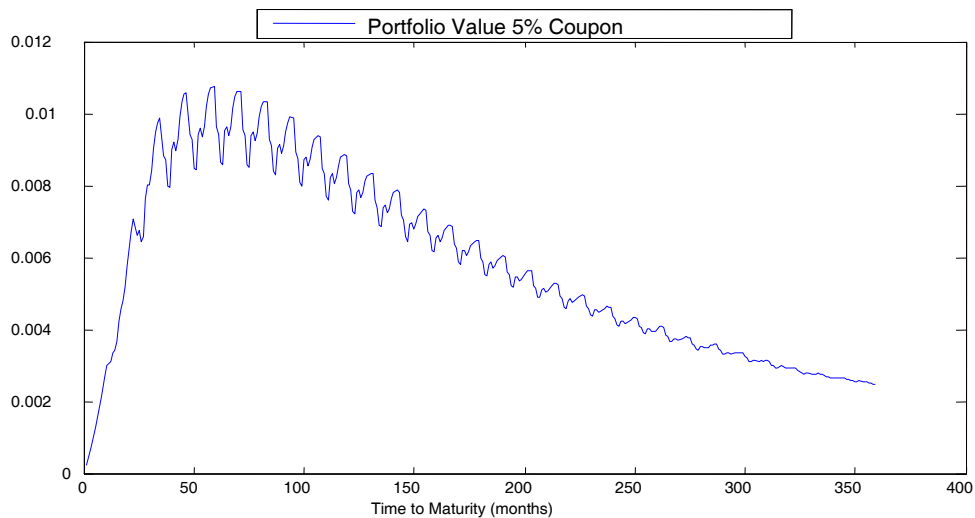
## 5.1 Numerical Example

Table 2 shows estimated portfolio values using Monte Carlo simulations. We also report standard errors.

**Table 2: Portfolio Values (5% Coupon) and Their Standard Errors.**

PV %	2.071118	2.07150	<b>2.07006</b>	2.06917	2.07241	2.07217
SE	0.00344	0.003308	<b>0.00289</b>	0.00324	0.003322	0.002969

The DOAS we use in our example is 2.07006% par value. If we assume that the pool size of the mortgage is \$1,000,000.00, the portfolio value will be \$ 20,700.60. The investor can buy this option to hedge interest rate risk. In the next section, we show this with an example.



**Figure 3.**

## 5.2 Numerical Example

The investor can use the portfolio described above as a hedging instrument against prepayment risk in general and changes of the yield curve. The examples below show exactly this.

Example1: 5% Coupon rate:

Investor *A* buys at time  $t_0$  a 30-year mortgage-backed security with the price of the MBS being 100% of the face value. The investor receives Treasury rate plus 46 basis point (OAS). We assume the pool size to be \$1,000,000.

Another investor, say Investor *B*, buys at time  $t_0$  the same mortgage and also buys a DOAS option. The DOAS option is 2.07006% of the par value. Therefore the value of this investment will be 102.07%.

Suppose at time  $t_1$  the interest rate volatility increases from 13bp to 26bp. What is the impact of this increase on the MBS price, and the investor's portfolio?

At time  $t_1$ , the price of the mortgage drops to 99.8534 % or \$ 998,534.00. Therefore that implies a \$1,466 loss on the mortgage for Investor *A*.

On the other hand, the value of the investment for the Investor *B*, is given by:

$$\begin{aligned} \text{Pay-off} &= \text{bond value at time } t_1 - \text{bond value at time } t_0 \\ &+ (\text{portfolio value at time } t_1 - \text{portfolio value at time } t_0) \end{aligned}$$

$$\text{Pay-off} = 99.8534 - 100 + (2.08289 - 2.07006) = - 0.1337 \text{ or } \$1,337$$

Example2: 6% coupon rate:

We report below another example choosing a coupon rate that is above the initial interest rate used in the simulation. Investor *A* buys at time  $t_0$  the mortgage and receives interests plus 227.70 basis points.<sup>5</sup> Investor *B* buys the same mortgage but also invests into a DOAS option whose price is 9.9080% for a total of 109.908%.

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<sup>5</sup> OAS has been calculated as in (5).

Suppose that at time  $t_1$  the interest rates volatility increases, as before, from 0.00132 to 0.00264. What is the impact of this increase on the bond price, and the investor's portfolio? At time  $t_1$  the price of the mortgage drops to 99.9825 % or \$ 999,825.00. The loss for the Investor A is therefore \$ 175.00. As a consequence of the increase in interest rate volatility the value of the DOAS option increases to 9.9275%. The pay-off for the Investor B is therefore given by:

$$\begin{aligned} \text{Pay-off} &= \text{bond value at time } t_1 - \text{bond value at time } t_0 \\ &+ (\text{portfolio value at time } t_1 - \text{portfolio value at time } t_0) \\ \text{Pay-off} &= 99.9825 - 100 + (9.9275 - 9.9080) = 0.0020 \% \text{ or } \$20.00 \end{aligned}$$

## 6. Empirical Results

Table 3 shows MBS prices with different coupons and also the option adjusted spread. We note that the price of the mortgage increases as the coupon rate increases.

**Table 3: Mortgage-Backed Security Values and Dynamic Option Adjusted Spreads**

<b>Coupon Rate %</b>	<b>5.00</b>	<b>5.50</b>	<b>6.00</b>	<b>6.50</b>	<b>7.00</b>
<b>MBS Price</b>	<b>102.17</b>	<b>106.28</b>	<b>110.26</b>	<b>114.21</b>	<b>117.58</b>
<b>SE</b>	<b>0.06312</b>	<b>0.06204</b>	<b>0.07211</b>	<b>0.06606</b>	<b>0.05265</b>
<b>OAS bp</b>	<b>46.18</b>	<b>135.66</b>	<b>227.70</b>	<b>321.05</b>	<b>412.33</b>
<b>DOAS %</b>	<b>2.0700</b>	<b>6.0034</b>	<b>9.9080</b>	<b>13.7460</b>	<b>17.0849</b>
<b>SE</b>	<b>0.00289</b>	<b>0.00837</b>	<b>0.01223</b>	<b>0.01690</b>	<b>0.02427</b>

The highest price is reached when the coupon is 7% and it is 117.58. Such a high premium clearly cannot be explained just by **par** plus a number of refinancing points. These high prices are consistent with what generally is observed in the market where mortgage prices can easily reach these levels (see also Longstaff, 2004, for a discussion on this issue).

Conditionally on the interest rate level used in our simulation, we note that higher coupon rates will increase the incentive for the borrower to repay the mortgage and this clearly will affect the spread that an eventual investor would require as a compensation for the prepayment option. In fact our model suggests a spread on the Treasury curve of more than 400bp when a 7% coupon is considered. We have also computed standard errors from the simulation by using 100 independent trials of the model in section 2.

At the bottom of Table 3, we report the simulated dynamic options adjusted values. As we see, given the interest rate level used in the simulation, the value of the option increases as the coupon increases. This is consistent with a higher prepayment risk implicit with higher coupons. As we showed above an investor might decide to buy this option, and pay a higher price for the mortgage, if he wishes to be hedge against prepayment risk and changes in the slope of the yield curve.

## Conclusions

The Mortgage Backed Securities market is the largest fixed income market in the United States. These assets are collateralised by a pool of mortgages and allow investors to gain higher interest rates with a relatively lower risk compared to other fixed income instruments. Given the importance of these securities, in the last decade, there has been a proliferation of models trying to explain the optimal prepayment behaviour of the borrower. The main problem with most of these models is that they cannot always explain, within a rational model, how borrowers decide to refinance their loans. Therefore, some of these models have tried to model the prepayment action as an endogenous problem (see Stanton and Wallace, 1998 amongst the others), but MBS prices obtained by using these models cannot generally match market prices.

If on one hand various different models have been proposed in the literature to price MBS. On the other hand there has been very little done in terms of the hedging and risk management of these securities. In this paper we have tried to fill this gap.

We extend a reduced form model to price MBS and propose a novel approach to managing interest rates risk. We show that an investor in this market, by taking a long position on an option (DOAS), can hedge out interest rate risk. The DOAS is simply the difference between the cash flows of a non-callable bond and a callable bond over the maturity of the mortgage. The concept of DOAS can be easily extended to other fixed income securities such as callable bonds and a variety of exotic swaps.

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## Appendix 1

The model assumes that four factors (i.e. refinancing incentive, burnout, seasoning, and seasonality) explain 95% of the variation in prepayment rates. These factors are then combined into one model to project prepayments:

$$CPR_t = RI_t \times AGE_t \times MM_t \times BM_t$$

where,  $RI_t$  represents the refinancing incentive;  $AGE_t$  represents the seasoning multiplier;  $MM_t$  represents the monthly multiplier;  $BM_t$  represents the burnout multiplier.

Therefore, the prepayment model is:

$$CPR_t = RI_t \times AGE_t \times MM_t \times BM_t$$

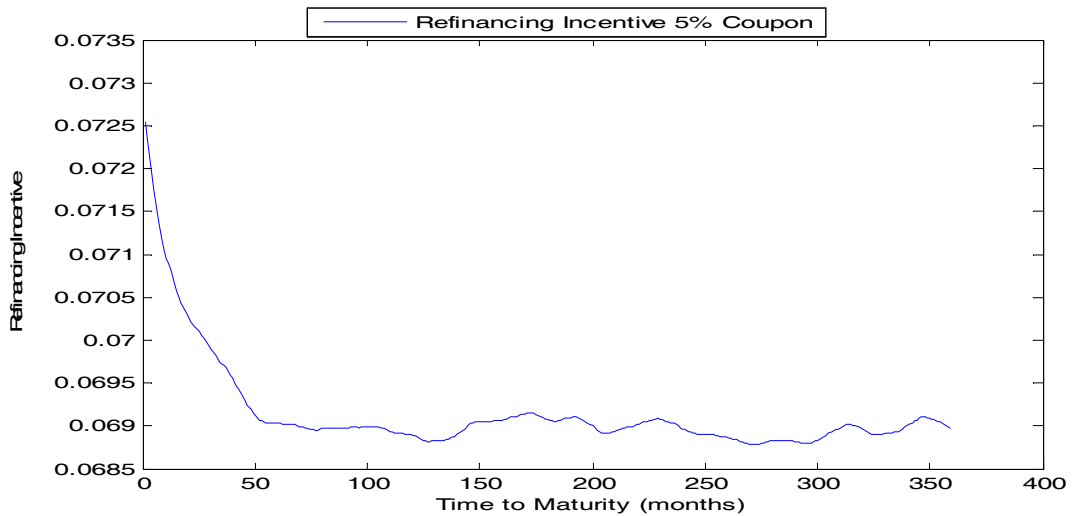
where:

$$RI_t = 0.28 + 0.14 \tan^{-1}[-8.571 + 430(WAC - r_{10}(t))]$$

$$AGE_t = \min\left(1, \frac{t}{30}\right)$$

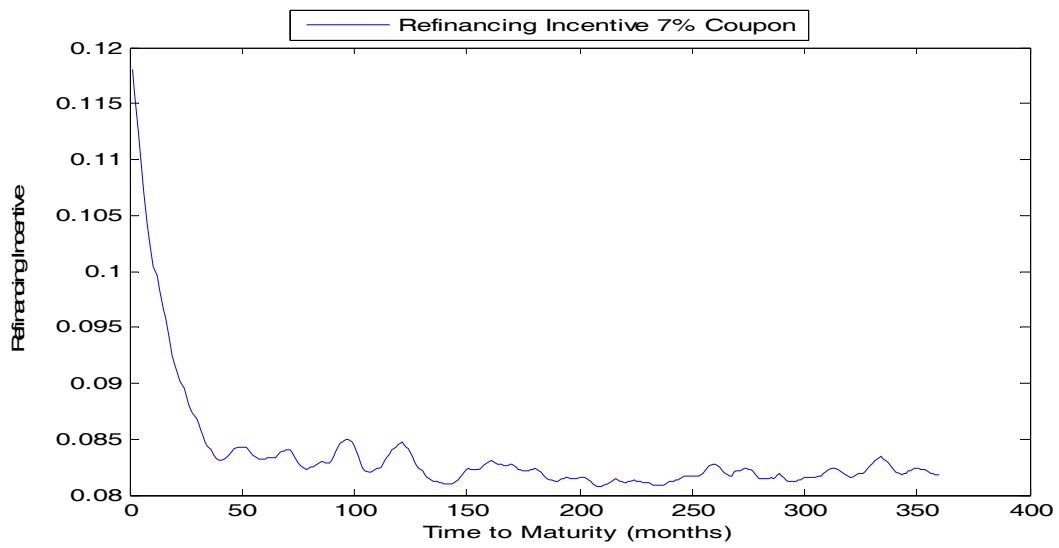
$$BM_t = 0.3 + 0.7\left(\frac{B_{t-1}}{B_0}\right)$$

$MM_t$  takes the following values, which start from January and end in December: (0.94, 0.76, 0.74, 0.95, 0.98, 0.92, 0.98, 1.1, 1.18, 1.22, 1.23, 0.98),  $r_{10}$  is 10-year Treasury rate, and  $WAC$  is the weighed average coupon rate.



**Figure 3.**





**Figure 4.**

Figure 3 and 4 above show the refinancing incentive function for 5% and 7% coupon rates. Borrowers have a higher incentive to exercise the prepayment option and refinance the mortgage when the coupon rate is higher than interest rates. This is shown in Figure 4.

