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U.S. Monetary and Fiscal Policies—Conflict or Cooperation?*

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Abstract

We estimate a model in which both fiscal and monetary policy behavior arise from the optimizing behavior of distinct monetary and fiscal authorities. Optimal time-consistent policy behavior fits U.S. time series at least as well as rules-based behavior. American policy makers have often been in conflict. After the Volcker disinflation, policies did not achieve the conventional mix of a conservative monetary policy paired with a debt-stabilizing fiscal policy. If credible, a conservative central bank that follows a time-consistent fiscal policy leader would come close to mimicking the cooperative Ramsey policy. Enhancing cooperation between policy makers without an ability to commit would be detrimental to welfare.

Keywords: Bayesian Estimation, Monetary and Fiscal Policy Interactions, Optimal Policy, Markov Switching
JEL Codes: C11, E31, E63

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1 Introduction

A large literature analyzes shifts in monetary policy regime. One important branch assesses how much of the "Great Moderation" in output and inflation volatility was simply "good luck"—a favorable shift in shock volatilities—or "good policy"—a desirable change in monetary policy rule parameters [Sims and Zha (2006)]. Many researchers attribute the improvement in policy making to the Volcker disinflation in 1979 or shortly after. Very little work examines the role fiscal policy played in altering inflation trends. This neglect is surprising in light of the co-movements in inflation, real interest rates, and fiscal variables including the debt-to-GDP ratio. The upward trend in inflation before the 1980s is associated with a downward trend in the debt-to-GDP ratio, while the moderation in inflation arose with a step increase in the real interest rate and a rising debt-to-GDP ratio, at least until 1995 [see figure 1].

Bianchi (2012) and Bianchi and Ilut (2017) are notable exceptions. They build on the policy interactions in Leeper (1991) to allow for switches in the combinations of monetary and fiscal policy rules over time.[1,2] Bianchi and Ilut find that a combination of passive monetary policy and active fiscal policy produced higher inflation and lower debt before the Volcker disinflation. A period of policy conflicts follows with both monetary and fiscal policy following active rules. Eventually, fiscal policy turns passive to stabilize debt in the face of the Fed's anti-inflationary actions. This benign policy mix—active money/passive fiscal—explains the steady decline in inflation and rise in debt in the 1980s.

This paper builds on that analysis in several ways. First, we consider other types of policy making in addition to simple policy rules. We allow monetary policy to be conducted optimally, but under time-consistent policy with fluctuations in the degree of inflation conservatism, as in Chen, Kirsanova, and Leith (2017). We permit fiscal policy to choose among active, passive, and optimal time-consistent fiscal rules, where the fiscal authority acts as a Stackelberg leader in a game with the optimizing monetary authority. Surprisingly, optimizing policies fit data well, a fit comparable to the usual rules-based menu. The paper develops a new algorithm to solve the strategic policy game between the monetary and fiscal policy makers in the face of regime switching.

Second, optimal policy's fit to data introduces a fresh narrative of how policies have evolved in the post-war period. Under time-consistent optimal policy the movement between regimes is more nuanced and it is rare that policy combinations conform to something akin to the usual active/passive pairings. We do not find that the Volcker disinflation was followed by a permanent shift to a debt-stabilizing fiscal policy, as conventional rules-based estimates do.

Third, we conduct a rich set of counterfactual exercises that combine optimal and rules-based policies. We ask what role the monetary-fiscal mix played in explaining the major

[1] Leeper (1991) characterizes monetary policy as being active (AM) or passive (PM) depending on whether or not it satisfies the Taylor principle. A fiscal policy that adjusts the deficit to ensure fiscal sustainability is labelled passive (PF), while failing to do is an active policy (AF). The consensus policy assignment is AM/PF, though the permutation of MP/AF still ensures a determinant equilibrium. AM/AF generates instability and PM/PF indeterminacy.

[2] Related papers include Davig (2004) and Davig and Leeper (2011), which allow for regime switching in estimated fiscal policy. Traum and Yang (2011) and Leeper, Traum, and Walker (2017) implicitly consider switches in monetary and fiscal policy by estimating a DSGE model with fixed policy rules over sub-samples.

trends in macroeconomic outcomes that figure 1 describes and how those trends would have differed if alternative policies had been followed. We emphasize the three major shifts in outcomes that appear in data: the reduction in the debt-GDP ratio and rise in inflation before the 1980s, the Volcker disinflation and rise in the debt-GDP ratio in the 1980s, and the stabilization of the debt-GDP ratio from the mid 1990s until the financial crisis.

Finally, we assess the welfare implications of alternative policy regimes. The mix of a conservative central bank that follows an optimizing fiscal authority who acts as Stackelberg leader will come close to mimicking cooperative Ramsey policies. But the Stackelberg leadership regime must be credible, and not expected to shift to another potential policy regime. Credibility is important because there can be substantial spillovers across regimes, with a fiscal authority behaving optimally, taking into account possible future switches to a passive fiscal rule. And the inflationary impacts of an active fiscal regime are affected by the possibility of switching to a passive fiscal policy that raises distorting tax rates to stabilize debt. This latter phenomenon arises from the inflationary impacts of alternative distorting tax policies, a fiscal consideration missing in Bianchi (2012) and Bianchi and Ilut (2017). It turns out that enhancing cooperation between policy makers can reduce welfare relative to the strategic interactions that our estimates deliver.

2 The Model

Households, a monopolistically competitive production sector, and the government populate the economy. A continuum of goods enter the households' consumption basket. Households form external consumption habits at the level of the consumption basket as a whole, what Ravn, Schmitt-Gróhe, and Uribe (2006) call "superficial" habits.[3] The economy is subject to both price and inflation inertia. Both effects help to capture the hump-shaped responses of output and inflation to shocks evident in VAR-based studies, and are often employed in empirical applications of the New Keynesian model [Smets and Wouters (2003) and Christiano, Eichenbaum, and Evans (2005)].

On the fiscal side, the government levies a tax on firms' sales revenue, which is equivalent to a tax on all labor and profit income in this model. These revenues finance government consumption, pay for transfers to households and service the outstanding stock of government debt. Government issues a portfolio of bonds of different maturities subject to a geometrically declining maturity structure.

2.1 Households

There is a continuum of households indexed by k and of measure one. Households derive utility from consumption of a composite good, $C_t^k = \left(\int_0^1 \left(C_{it}^k\right)^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}}$, where η is the elasticity of substitution between the goods in this basket, and suffer disutility from hours spent working, N_t^k. Habits are both superficial and external: they are formed at the level of the aggregate consumption good and households fail to take account of the impact of their consumption decisions on the utility of others. To facilitate data-consistent detrending around a balanced growth path without restricting preferences to be logarithmic, we

[3] For a comparison of the implications for optimal policy of alternative forms of habits see Amato and Laubach (2004) and Leith, Moldovan, and Rossi (2012).

assume that consumption enters the utility function scaled by the economy wide technology trend [Lubik and Schorfheide (2006) and An and Schorfheide (2007)]. This implies that the household's consumption norms rise with technology and are affected by habits externalities. Households derive utility from the habit-adjusted composite good

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(X_t^k\right)^{1-\sigma} (\xi_t)^{-\sigma}}{1-\sigma} - \frac{\left(N_t^k\right)^{1+\varphi} (\xi_t)^{-\sigma}}{1+\varphi} \right] \tag{1}$$

where $X_t^k \equiv \frac{C_t^k}{A_t} - \theta \frac{C_{t-1}}{A_{t-1}}$ is the habit-adjusted consumption aggregate, θ is the habit persistence parameter $(0 < \theta < 1)$, and $C_{t-1} \equiv \int_0^1 C_{t-1}^k dk$ is the cross-sectional average of consumption. Households gain utility from consuming more than other households, are disappointed if their consumption doesn't grow in line with technical progress, and are subject to a taste shock, $\ln \xi_t = \rho_\xi \ln \xi_{t-1} + \sigma_\xi \varepsilon_{\xi,t}$. β is the discount factor $(0 < \beta < 1)$, and σ and φ are the inverses of the intertemporal elasticities of habit-adjusted consumption and work $(\sigma, \varphi > 0;\ \sigma \neq 1)$.

The process for technology is non-stationary

$$\begin{aligned} \ln A_t &= \ln \gamma + \ln A_{t-1} + \ln q_t \\ \ln q_t &= \rho_q \ln q_{t-1} + \sigma_q \varepsilon_{q,t} \end{aligned}$$

Households choose the composition of the consumption basket to minimize expenditure, so demand for individual good i is

$$C_{it}^k = \left(\frac{P_{it}}{P_t}\right)^{-\eta} C_t^k$$

where P_{it} is the price of good i, and $P_t = \left(\int_0^1 (P_{it})^{1-\eta} di\right)^{1-\eta}$ is the CES aggregate price index associated with the composite good consumed by households. By aggregating across all households, we obtain the overall demand for good i as

$$C_{it} = \int_0^1 C_{it}^k dk = \left(\frac{P_{it}}{P_t}\right)^{-\eta} C_t \tag{2}$$

Households choose the habit-adjusted consumption aggregate, X_t^k, hours worked, N_t^k, and the portfolio allocation, $B_t^{S,k}$ and $B_t^{M,k}$, to maximize expected lifetime utility (1), subject to the budget constraint

$$\int_0^1 P_{it} C_{it}^k di + P_t^S B_t^{S,k} + P_t^M B_t^{M,k} = B_{t-1}^{S,k} + (1 + \rho P_t^M) B_{t-1}^{M,k} + W_t N_t^k + \Phi_t + Z_t \tag{3}$$

and a no-Ponzi scheme condition. Period t income includes: wage income from providing labor services to goods producing firms, $W_t N_t^k$, a lump-sum transfer from the government, Z_t, dividends from the monopolistically competitive firms, Φ_t, and payoffs from the portfolio of assets, $B_t^{S,k}$ and $B_t^{M,k}$. Households hold two forms of government bonds. The first is the familiar one-period debt, B_t^S, whose price equals the inverse of the gross nominal interest rate, $P_t^S = R_t^{-1}$. The second type of bond is actually a portfolio of many bonds, which pays a declining premium of ρ^j, j periods after being issued where $0 < \rho < \beta^{-1}$ [Woodford (2001)]. The duration of the bond is $\frac{1}{1-\beta\rho}$, which means that ρ can be varied to capture changes in the maturity structure of debt. By using this simple structure we need to price only a single bond, since any existing bond issued j periods ago is worth ρ^j new bonds. When $\rho = 1$ these bonds become infinitely lived consols.

Household optimization yields the optimal allocation of consumption across time, based on the pricing of one period bonds,

$$\begin{aligned} 1 &= \beta E_t \left[\left(\frac{X_{t+1}^k \xi_{t+1}}{X_t^k \xi_t} \right)^{-\sigma} \frac{A_t}{A_{t+1}} \frac{P_t}{P_{t+1}} \right] R_t \\ &= E_t Q_{t,t+1} R_t \end{aligned}$$

where we have defined the stochastic discount factor as

$$Q_{t,t+s} \equiv \beta \left(\frac{X_{t+s}^k \xi_{t+s}}{X_t^k \xi_t} \right)^{-\sigma} \frac{A_t}{A_{t+s}} \frac{P_t}{P_{t+s}}$$

and the geometrically declining payoff consols

$$\begin{aligned} P_t^M &= \beta E_t \left[\left(\frac{X_{t+1}^k \xi_{t+1}}{X_t^k \xi_t} \right)^{-\sigma} \frac{A_t}{A_{t+1}} \frac{P_t}{P_{t+1}} (1 + \rho P_{t+1}^M) \right] \\ &= E_t Q_{t,t+1} (1 + \rho P_{t+1}^M) \end{aligned}$$

When all bonds have one-period duration, $\rho = 0$, the price of these bonds is $P_t^M = R_t^{-1}$. Outside of this special case, the longer term bonds introduce the term structure of interest rates to the model. The first-order condition for labor is

$$\frac{W_t}{P_t A_t} = N_t^{k\varphi} X_t^{k\sigma}$$

There is an associated transversality condition derived as follows. Define household wealth in period t as

$$D_t^k \equiv (1 + \rho P_t^M) B_{t-1}^{M,k} + B_{t-1}^{S,k}$$

and imposing the no-arbitrage conditions allows us to rewrite the budget constraint as

$$\int_0^1 P_{it} C_{it}^k di + E_t Q_{t,t+1} D_{t+1}^k = D_t^k + W_t N_t^k + \Phi_t + Z_t$$

Household optimization implies a transversality condition that combined with the no-Ponzi condition yields

$$\lim_{T \to \infty} E_t Q_{t,T} D_T^k = 0$$

2.2 Firms

Individual goods producers are subject to the constraints of Calvo (1983) contracts. With probability $1 - \alpha$ in each period, a firm can reset its price and with probability α the firm retains the price of the previous period. That price is indexed to the steady-state rate of inflation, following Yun (1996). When a firm can choose a new price, it can do so either to maximize the present discounted value of after-tax profits, $E_t \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s} \Phi_{it+s}$, or to follow a simple rule of thumb as in Galí and Gertler (1999). Profits are discounted by the s-step ahead stochastic discount factor $Q_{t,t+s}$ and by the probability of not being able to set prices in future periods. Forward-looking profit maximizers are constrained by the demand for their good, condition (2), and the condition that all demand must be satisfied at the chosen price. An autocorrelated shock affects the desired markup $\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \sigma_\mu \varepsilon_{\mu,t}$. Firm i's optimization problem is

$$\max_{\{P_{it},\ Y_{it}\}} E_t \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s} \left[\left((1-\tau_{t+s}) P_{it} \pi^s - \mu_{t+s} MC_{t+s} \right) Y_{it+s} \right]$$

subject to the demand curve

$$Y_{it+s} = \left(\frac{P_{it} \pi^s}{P_{t+s}} \right)^{-\eta} Y_{t+s}$$

Optimizing firms that are able to reset price choose P_t^f, whose relative price satisfies

$$\frac{P_t^f}{P_t} = \left(\frac{\eta}{\eta - 1} \right) \frac{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \left(X_{t+s} \xi_{t+s} \right)^{-\sigma} \mu_{t+s} mc_{t+s} \left(\frac{P_{t+s}\pi^{-s}}{P_t} \right)^{\eta} \frac{Y_{t+s}}{A_{t+s}}}{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \left(X_{t+s} \xi_{t+s} \right)^{-\sigma} (1-\tau_{t+s}) \left(\frac{P_{t+s}\pi^{-s}}{P_t} \right)^{\eta-1} \frac{Y_{t+s}}{A_{t+s}}}$$

where $mc_t = \frac{MC_t}{P_t} = \frac{W_t}{P_t A_t}$ is the real marginal cost, given the linear production function, $Y_{it} = A_t N_{it}$. Under flexible prices, $mc_t = (1-\tau_t)\frac{\eta-1}{\eta}$.

Inflation is inertial. Some firms use rules of thumb. When those firms are permitted to post a new price, they choose P_t^b to obey

$$P_t^b = P_{t-1}^* \pi_{t-1}$$

so they update their price using last period's rate of inflation rather than steady-state inflation. P_{t-1}^* denotes an index of the reset prices, defined by

$$\ln P_{t-1}^* = (1-\zeta) \ln P_{t-1}^f + \zeta P_{t-1}^b$$

where ζ is the proportion of firms that adopt rule-of-thumb pricing. With α share of firms keeping last period's price (but indexed to steady-state inflation) and $1-\alpha$ share of firms setting a new price, the law of motion of the aggregate price index is

$$(P_t)^{1-\eta} = \alpha \left(P_{t-1}\pi \right)^{1-\eta} + (1-\alpha) \left(P_t^* \right)^{1-\eta}$$

We derive a hybrid New Keynesian Phillips curve, as Leith and Malley (2005) detail. Combine the rule-of-thumb pricing with the optimal price setting to produce the Phillips curve

$$\widehat{\pi}_t = \chi_f \beta E_t \widehat{\pi}_{t+1} + \chi_b \widehat{\pi}_{t-1} + \kappa_c \left(\widehat{mc}_t + \frac{\tau}{1-\tau} \widehat{\tau}_t + \widehat{\mu}_t \right)$$

$\widehat{\pi}_t = \ln(P_t) - \ln(P_{t-1}) - \ln(\pi)$ is the deviation of inflation from its steady-state value, $\widehat{mc}_t + \frac{\tau}{1-\tau}\widehat{\tau}_t = \ln(W_t/P_t) - \ln A_t + \frac{\tau}{1-\tau}\widehat{\tau}_t - \ln((\eta-1)/\eta) + \ln(1-\tau)$, are log-linearized real marginal costs adjusted for the impact of the sales revenue tax, and the reduced-form parameters are defined as $\chi_f \equiv \frac{\alpha}{\Phi}$, $\chi_b \equiv \frac{\zeta}{\Phi}$, $\kappa_c \equiv \frac{(1-\alpha)(1-\zeta)(1-\alpha\beta)}{\Phi}$, with $\Phi \equiv \alpha(1+\beta\zeta) + (1-\alpha)\zeta$.

2.3 The Government

The flow budget identity of the federal government is

$$P_t^M B_t^M = (1+\rho P_t^M)B_{t-1}^M - P_t Y_t \tau_t + P_t G_t + P_t Z_t + P_t Y_t \xi_{tp,t}$$

We assume short bonds are in zero net supply, so $B_t^S \equiv 0$. $P_t^M B_t^M$ is the market value of debt, $P_t G_t$ and $P_t Z_t$ are government spending and transfers and $P_t Y_t \xi_{tp,t}$ is an *i.i.d.* shock to the budget constraint that arises from random fluctuations in the debt maturity structure.[4] Government can use distorting taxes to service government debt and to stabilize the economy. Divide through by nominal GDP, $P_t Y_t$ to rewrite the budget identity in terms of the ratio $b_t^M = \frac{P_t^M B_t^M}{P_t Y_t}$

$$b_t^M = \frac{(1+\rho P_t^M)}{P_{t-1}^M}\frac{Y_{t-1}}{\pi_t Y_t} b_{t-1}^M - \tau_t + g_t + z_t + \xi_{tp,t}$$

where $\xi_{tp,t} = \sigma_{tp}\varepsilon_{tp,t}$ and we assume that the government spending-to-GDP ratio, g_t, evolves according to

$$\ln g_t = (1-\rho_g)\ln g + \rho_g \ln g_{t-1} + \sigma_g \varepsilon_{g,t}$$

and the transfers-to-GDP ratio, z_t, follows a similar process

$$\ln z_t = (1-\rho_z)\ln z + \rho_z \ln z_{t-1} + \sigma_z \varepsilon_{z,t}$$

The fiscal shocks, $\varepsilon_{tp,t}$, $\varepsilon_{g,t}$ and $\varepsilon_{z,t}$ are all standard normally distributed.

2.4 The Complete Model

The complete system of non-linear equations that describe the equilibrium appear in appendix A. After log-linearizing around the deterministic steady-state, the model is summarized by[5]

$$\text{Labor Supply: } \sigma \widehat{X}_t + \varphi \widehat{N}_t = \widehat{w}_t \tag{4}$$

$$\text{Euler equation: } \widehat{X}_t = E_t \widehat{X}_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t \widehat{\pi}_{t+1} - E_t \widehat{q}_{t+1}\right) - \widehat{\xi}_t + E_t \widehat{\xi}_{t+1} \tag{5}$$

$$\text{Bond Prices: } \widehat{P}_t^M = \frac{\rho\beta}{\gamma\pi} E_t \widehat{P}_{t+1}^M - \widehat{R}_t \tag{6}$$

$$\text{Resource Constraint: } \widehat{y}_t = \widehat{N}_t = \widehat{c}_t + \frac{1}{1-g}\widetilde{g}_t \tag{7}$$

$$\text{Consumption Habits: } \widehat{X}_t = (1-\theta)^{-1}(\widehat{c}_t - \theta \widehat{c}_{t-1}) \tag{8}$$

$$\text{Phillips curve: } \widehat{\pi}_t = \chi_f \beta E_t \widehat{\pi}_{t+1} + \chi_b \widehat{\pi}_{t-1} + \kappa_c(\widehat{w}_t + \frac{1}{1-\tau}\widetilde{\tau}_t + \widehat{\mu}_t) \tag{9}$$

[4]This shock breaks a singularity that arises when all the other elements of the budget constraint are observables in estimation.

[5]The fiscal variables are normalized with respect to GDP, so $\widetilde{b}_t^M, \widetilde{\tau}_t$, $\widetilde{g}_t$, and $\widetilde{z}_t$ are defined as linear deviations from their steady states. Other variables are expressed as percentage deviations from steady state. Before linearizing, output, consumption and real wages are rendered stationary by scaling by technology, A_t.

$$\text{Govt Budget: } \widetilde{b}_t^M = \frac{1}{\beta}\widetilde{b}_{t-1}^M + \frac{b^M}{\beta}\left(\frac{\rho\beta}{\gamma\pi}\widehat{P}_t^M - \widehat{P}_{t-1}^M + \widehat{y}_{t-1} - \widehat{y}_t - \widehat{\pi}_t - \widehat{q}_t\right) - \widetilde{\tau}_t + \widetilde{g}_t + \widetilde{z}_t + \sigma_{tp}\varepsilon_{tp,t} \tag{10}$$

$$\text{Govt Spending: } \widetilde{g}_t = \rho_g\widetilde{g}_{t-1} + \sigma_g\varepsilon_{g,t} \tag{11}$$

$$\text{Transfers: } \widetilde{z}_t = \rho_z\widetilde{z}_{t-1} + \sigma_z\varepsilon_{z,t} \tag{12}$$

$$\text{Technology: } \widehat{q}_t = \rho_q\widehat{q}_{t-1} + \sigma_q\varepsilon_{q,t} \tag{13}$$

$$\text{Cost-Push/Markup: } \widehat{\mu}_t = \rho_\mu\widehat{\mu}_{t-1} + \sigma_\mu\varepsilon_{\mu,t} \tag{14}$$

$$\text{Preference: } \widehat{\xi}_t = \rho_\xi\widehat{\xi}_{t-1} + \sigma_\xi\varepsilon_{\xi,t} \tag{15}$$

To close the model we specify policy behavior.

3 Policy Making

Policy makers behave both optimally and strategically. We contrast the fit to data of this description of policy to a version of the model in which policy obeys the kinds of simple rules in existing literature. That rules-based benchmark appears in appendix C.

3.1 Optimal Policy

Now we describe our optimal policy specifications. Chen, Kirsanova, and Leith (2017) estimate monetary policy models of the U.S. economy to find that monetary policy is best described as optimal but time-consistent. The fit of that description dominates both the rules-based and the time-inconsistent Ramsey monetary policy. Extending this analysis to fiscal policy raises several considerations. First, the monetary and fiscal authorities should be considered to be independent policy makers with potentially different policy objectives. This leads us to model strategic interactions between the two policy makers; they play a game where either authority may be the Stackelberg leader—making policy decisions anticipating the reaction of the other—or a Nash equilibrium where each policy maker takes the other's policies as given when formulating their own plans. It is generally thought that fiscal leadership is the best description of the interactions between the monetary and fiscal authorities because in practice the monetary authority's response to shocks is well articulated and can be anticipated by the fiscal authorities [Beetsma and Debrun (2004)].[6] We adopt this timing assumption in what follows.[7] Second, while Chen, Kirsanova, and Leith (2017) find strong evidence that monetary policy has been conducted optimally, albeit with

[6] Fiscal leadership is not the same as fiscal dominance and does not imply that the fiscal authority is forcing the central bank to accommodate its actions. Leadership means that the central bank takes fiscal policy as given and it has a well-known reaction to the state of the economy, which the fiscal authority takes into account when setting policy. For example, the fiscal authority might anticipate that the central bank will act to stabilize inflation in the face of a fiscal stimulus. The relative frequency and ease of monetary and fiscal policy changes also support this leadership assumption.

[7] We also estimated our model under the alternative assumptions of monetary leadership and the Nash solution. Changing the nature of the strategic interaction can have a material impact in simple models. This is not the case in our model, which features habits, inflation inertia and a desire to smooth instruments. Results are available upon request.

switches in the degree of conservatism over time, it is not obvious that fiscal policy can be considered to have been similarly optimal. For this reason, we posit that monetary policy behaves optimally—with changes in degree of conservatism—while fiscal policy switches between rules-based and optimal time-consistent policy, as fit to data dictates.

An obvious benchmark for policy objectives would be the micro-founded welfare function based on the utility of the households that populate the economy.[8] But estimation with micro-founded weights is problematic. Because the micro-founded weights are functions of structural parameters, they place very tight cross-equation restrictions on the model, which are likely to interfere with fit to data. With standard estimates of the degree of price stickiness, for example, the micro-founded weight attached to inflation can be over 100 times that attached to the output terms [see Woodford (2003, chapter 6)]. Optimal policy based on such a strong anti-inflation objective would be wildly inconsistent with observed inflation volatility. Instead, we adopt a form of the objective function for each policy maker which is consistent with the representative agents' utility, but freely estimate the weights within that objective function. The objective function for the monetary authority is

$$\Gamma_0^M = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \omega_1 \left(\widehat{X}_t + \widehat{\xi}_t\right)^2 + \omega_2 \left(\widehat{y}_t - \frac{\sigma}{\varphi}\widehat{\xi}_t\right)^2 \\ +\omega_3 \left(\widehat{\pi}_t - \widehat{\pi}_{t-1}\right)^2 + \omega_{\pi,S_t}^M \widehat{\pi}_t^2 + \omega_R(\Delta \widehat{R}_t)^2 \end{array} \right\} \tag{16}$$

Under the optimal monetary policy specification, we consider potential switches in the weight attached to inflation stabilization, ω_{π,S_t}^M. That normalized weight can switch between $\omega_{\pi,S_t=1}^M = 1$ in the More-Conservative (MC) regime and $0 < \omega_{\pi,S_t=2}^M < 1$ in the Less-Conservative (LC) regime. We also allow the monetary policy authority to value smooth interest rates.

When fiscal policy is conducted optimally, the objective function for the fiscal authority is

$$\Gamma_0^F = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \omega_1 \left(\widehat{X}_t + \widehat{\xi}_t\right)^2 + \omega_2 \left(\widehat{y}_t - \frac{\sigma}{\varphi}\widehat{\xi}_t\right)^2 \\ +\omega_3 \left(\widehat{\pi}_t - \widehat{\pi}_{t-1}\right)^2 + \omega_{\pi}^F \widehat{\pi}_t^2 + \omega_\tau(\Delta \widetilde{\tau}_t)^2 \end{array} \right\} \tag{17}$$

The objective of the fiscal authority can differ from that of the monetary authority only in the weight attached to inflation, ω_π^F, and the presence of a tax rate-smoothing term. In essence, the two policy makers share the same conception of social welfare, but the government may appoint a monetary authority with an aversion to inflation which differs from that of society, to reflect Rogoff's (1985) arguments.

Habits externalities introduce the preference shock, $\widehat{\xi}_t$, into the objective functions. Habits confront policy makers with a trade-off. When $\widehat{\xi}_t$ is high, utility of consumption and disutility of work are low. Policy makers will want to induce more labor, but any higher consumption from that labor produces a lower utility gain.

3.2 Policy Rules

We adopt an agnostic view of fiscal behavior by not forcing it to be optimal at all times. When fiscal policy is not optimal and time-consistent—maximizing (17)—it obeys the tax

[8] See appendix B for the micro-founded welfare function.

rule

$$\widetilde{\tau}_t = \rho_{\tau,s_t}\widetilde{\tau}_{t-1} + (1-\rho_{\tau,s_t})\left(\delta_{\tau,s_t}\widetilde{b}^M_{t-1} + \delta_y \widehat{y}_t\right) + \sigma_\tau \varepsilon_{\tau,t} \tag{18}$$

where we assume the coefficient on debt, δ_{τ,s_t}, and the persistence of the tax rate, ρ_{τ,s_t} are subject to regime switching with $s_t = 2$ the Passive Fiscal (PF) regime and $s_t = 3$ the Active Fiscal (AF) regime. The value of the coefficient on debt determines fiscal regime, with $\delta_{\tau,s_t=2} > \frac{1}{\beta} - 1$ in the PF regime and $\delta_{\tau,s_t=3} = 0$ in the AF regime.

We assume transition matrices for monetary and fiscal policy regimes as follows

$$\Phi = \begin{bmatrix} \phi_{11} & 1-\phi_{22} \\ 1-\phi_{11} & \phi_{22} \end{bmatrix}, \ \Psi = \begin{bmatrix} \psi_{11} & 1-\psi_{22}-\psi_{23} & \psi_{31} \\ \psi_{12} & \psi_{22} & 1-\psi_{13}-\psi_{33} \\ 1-\psi_{11}-\psi_{12} & \psi_{23} & \psi_{33} \end{bmatrix}$$

where $\phi_{ii} = \Pr[S_t = i | S_{t-1} = i]$ and $\psi_{ii} = \Pr[s_t = i | s_{t-1} = i]$. The Optimal Fiscal (OF) regime corresponds to $s_t = 1$,while the PF and AF regimes correspond to $s_t = 2$ and $s_t = 3$, respectively.

We also permit fundamental shock volatilities to change, a feature of existing explanations of the Great Moderation. Failure to do so can bias the identification of shifts in policy [see Sims and Zha (2006)]. Standard deviations of technology (σ_{q,k_t}), preference (σ_{ξ,k_t}) and cost-push (σ_{μ,k_t}) shocks may switch independently, with $k_t = 1$ the low volatility regime and $k_t = 2$ the high volatility regime. The transition matrix for the shock volatilities is

$$H = \begin{bmatrix} h_{11} & 1-h_{22} \\ 1-h_{11} & h_{22} \end{bmatrix}$$

where $h_{ii} = \Pr[k_t = i | k_{t-1} = i]$.[9]

To solve the optimal policy problem, we develop a new algorithm that appendices D and E describe, with two policy makers under different structures of strategic interaction: when one policy maker can act as a Stackelberg leader in the policy game and when they move simultaneously as part of a Nash equilibrium. Our algorithm incorporates potential changes in policy makers' preferences over time.

3.3 Discussion of Optimal Policy Behavior

To understand our results, it is helpful first to review the benchmark of a Ramsey policy in which the two policy makers share a common objective and are able to credibly commit to future policy actions. In a New Keynesian economy that policy setting implies a variant of tax smoothing: the policy maker smooths the distortions associated with satisfying its budget constraint, using government debt as a shock absorber to do so. This doesn't mean that taxes themselves are smoothed, since tax rates will adjust to offset cost-push shocks; rather, policy smooths the distortions that would arise from not moving tax rates perfectly in line with cost-push shocks. This policy generates a random walk in debt as the short-run costs of reducing debt, once a given shock has dissipated, are exactly balanced by the long-run benefits of lower debt. In our model, the desire to reduce variations in the tax

[9] The joint transition matrix governing the monetary-fiscal-shock regime is then $P = \Phi \otimes \Psi \otimes H$. In total, there are twelve regimes under the optimal policy model.

rate ensures that government debt is eventually retired back to its steady-state even under commitment, but this is extremely gradual.[10] Another notable feature of outcomes under commitment is that although policy makers do utilize inflation surprises to help stabilize debt, reliance on such measures is limited [Leeper and Leith (2017)].

When we relax the assumption that the policy maker can commit, outcomes change radically [Leeper, Leith, and Liu (2019)]. Our economy has an efficient steady-state in which monopolistic competition and tax distortions balance the impact of the habits externality. Any level of debt outside of this steady-state value creates an incentive for the policy maker to use inflation surprises. Those surprises bring the decentralized equilibrium closer to the efficient allocation, both by influencing output in the sticky-price economy and by reducing debt. The incentive to inflation generates an inflationary bias problem outside of the steady-state as economic agents understand the policy maker's incentives. The policy maker can eliminate this bias by returning debt to steady-state. But the rapid return of debt to steady-state produces as "debt stabilization bias," as Leeper, Leith, and Liu (2019) label it. Returning debt to steady state is efficient in the absence of shocks, but it is inconsistent with the policy of smoothing distortions associated with the budget constraint, as a policy maker acting under commitment would do. This explains why welfare outcomes under discretion are so much worse than commitment: the policy makers return debt to steady state far too rapidly, failing to use debt as a shock absorber.

Optimal policy in our model also deviates from the Ramsey benchmark by assuming that the policy makers do not cooperate. Our policy makers act strategically with the fiscal authority the Stackelberg leader and the monetary authority the follower.[11] The separation of policy makers is actually beneficial from a societal perspective. The fiscal authority knows that if they aggressively try to reduce debt through taxation, the inflation-averse monetary authority will tighten monetary policy to reduce inflation. This moderates the use of taxes to stabilize debt, reducing the inflationary consequences of such a policy. Lower inflation prompts the monetary authority to refrain from tightening monetary policy. Looser monetary policy feeds back to encourage the fiscal authority to further delay fiscal stabilization because debt service costs are not as high. The net outcome from a lack of coordination is that inflation is closer to target and debt gets stabilized more gradually.

The final complication in our description of optimal policy, relative to the benchmark is that economic agents in our model expect there to be switches in policy regimes. Those expectations produce destabilizing spillovers from the additional regimes. The impact of a potential switch to passive fiscal behavior is particularly important. In this regime, taxes adjust to return debt to steady state. The more debt deviates from steady state, the more taxes adjust. Suppose debt is above target and fiscal policy is being conducted by an optimizing Stackelberg leader. Since policy can switch potentially to a passive regime, debt growth creates the expectation that a future change to passive behavior will raise the tax rate substantially, which drives up inflation. The optimizing fiscal authority responds to higher expected and current inflation by cutting current taxes. This worsens debt dynamics,

[10] Counterfactual outcomes under commitment and other forms of benchmark optimal policy are presented in figure 7 and described in subsection 6.1.

[11] We considered alternative timing assumptions—simultaneous moves and the monetary authority acting as leader. This does not materially affect outcomes and there is no clear preference for one approach over another in terms of the marginal data density.

raising inflation expectations still more to encourage further tax cuts, and so on. If the economy stayed permanently in the non-cooperative optimal regime, while economic agents continue to expect a switch to one of passive fiscal policy regimes, this would ultimately be destabilizing. Nevertheless, this description of policy can describe the data during specific episodes.

4 Estimation

The empirical analysis uses seven U.S. time series on real output growth (ΔGDP_t), annualized domestic inflation (INF_t), the federal funds rate (FFR_t), the annualized debt-to-GDP ratio (B_t/GDP_t), government spending ratio (G_t/GDP_t), transfers ratio (Z_t/GDP_t) and federal tax revenue ratio (T_t/GDP_t) from 1955Q1 to 2008Q3. All data are seasonally adjusted and at quarterly frequencies. Output growth is the log difference of real GDP, multiplied by 100. Inflation is the log difference of the GDP deflator, scaled by 400. The four fiscal variables—debt, government spending, transfers and taxes—are normalized with respect to GDP and multiplied by 100. Appendix F describes the dataset in detail.

The data are linked to the law of motion of states through the measurement equation

$$\begin{bmatrix} \Delta GDP_t \\ INF_t \\ FFR_t \\ G_t/GDP_t \\ T_t/GDP_t \\ Z_t/GDP_t \\ B_t/GDP_t \end{bmatrix} = \begin{bmatrix} \gamma^Q + \Delta\widehat{y}_t + \widehat{q}_t \\ \pi^A + 4\widehat{\pi}_t \\ r^A + \pi^A + 4\gamma^Q + 4\widehat{R}_t \\ 100g + \widetilde{g}_t \\ 100\tau + \widetilde{\tau}_t \\ 100z + \widetilde{z}_t \\ \frac{100}{4}b^M + \frac{1}{4}\widetilde{b}_t^M \end{bmatrix}$$

where parameters, γ^Q, π^A, r^A, g, τ, z and b^M represent the steady-state values of output growth, inflation, real interest rates the government spending to GDP ratio, transfers to GDP ratio, the tax rate and debt-to-GDP on a quarterly basis.

Steady-state values of fiscal variables and output growth are fixed at their means over the sample period. The government spending-to-GDP ratio (g) is 8%, transfers (z) is 9.19%, the federal tax revenues to GDP ratio (τ) is 17.5%, the federal debt to annualized output ratio (b^M) is 31% and quarterly output growth (γ^Q) is 0.46%. The steady-state real interest rate (r^A) is 1.8% and the inflation target (π^A) is 2%. The average real interest rate, r^A, is linked to the discount factor, β, such that $\beta = \left(1 + r^A/400\right)^{-1}$. Average maturity of outstanding government debt is 5 years [see Leeper and Zhou (2013, table 1)]. The inverse of Frisch elasticity of labor supply, φ, is set to 2.[12]

We approximate the likelihood function using Kim's (1994) filter, and then combine it with the prior distribution to obtain the posterior distribution. A random walk Metropolis-Hastings algorithm generates four chains of 540,000 draws each, after discarding the first 240000 draws, and saving 1 in every 100 draws. Brooks-Gelman-Rubin potential reduction scale factors, reported in appendix G, are all below the 1.1 an upper bound for convergence.

[12]It can be difficult to estimate the inverse of Frisch elasticity without using labor market data. The value $\varphi = 2$ is consistent with the estimate of Smets and Wouters (2007). This value is in line with microeconomic estimates using household level data as in MaCurdy (1981).

4.1 Prior Distributions

Table 1 reports the priors of the optimal policy model, which consists of priors that are common to the rules-based estimation in appendix C, as well as those for parameters specific to the optimal policy estimation, such as the weights on the objective function. The priors for most of the parameters are relatively loose and broadly consistent with the literature that estimates New Keynesian models. We choose the normal distribution for the inverse of the intertemporal elasticity of substitution, σ, with a prior mean of 2.5. Habits formation, indexation and the AR(1) parameters of the technology, cost-push, taste and transfer shocks and government spending process are assumed to follow a beta distribution with a mean of 0.5 and a standard deviation of 0.15. The Calvo parameter for the probability of no price change, α, is set so that the average length of the contract is around one year with a fairly tight prior around that value. Allowing a looser prior on this parameter tends to result in implausibly high estimates of the degree of price stickiness.

The parameters specific to the optimal policy estimation include the relative weights ($\omega_1, \omega_2, \omega_3,$ and ω_R) attached to the output, changes in inflation and interest rate smoothing terms on the monetary policy objective function. Those follow beta distributions. The normalized weight on inflation, ω^M_{π,S_t}, is one in the MC regime and obeys a beta distribution in the LC regime. For the fiscal policy objective function, we restrict the relative weights attached to the output terms to be the same as those on the monetary policy objective function, while we estimate the weight on the inflation stabilization term, ω^F_π, placed by the fiscal authority. We assume that ω^F_π follows a Gamma distribution with prior mean of 1 and a standard deviation of 0.3, so we do not presume that the fiscal authority will be either more or less inflation conservative than the central bank. We assume that the fiscal authority wants to avoid large variations in tax rates and a beta distribution is used for ω_τ.

4.2 Posterior Estimates

Table 1 presents the posterior parameter estimates when the monetary policy authority conducts optimal policy taking the policies of the fiscal authority as given, and where we allow that monetary authority's objective function to switch in its degree of inflation conservatism over time—which we label More (MC) or Less Conservative (LC). At the same time the fiscal authority acts as a Stackelberg leader in the game with the monetary authority so that the fiscal authority conducts policy anticipating the response of the Fed. Fiscal policy may switch between this leadership role (OF) and conducting policy through simple passive or active rules, which we label PF and AF. Six alternative policy regimes may arise in the optimal policy model: MC/OF, MC/PF, MC/AF, LC/OF, LC/PF and LC/AF.

Monetary policy is always assumed to be optimal, but time-consistent with the normalized weight attached to inflation stabilization, ω^M_{π,S_t}, estimated to be 0.61 in the LC regime, relative to one in the MC regime. When the fiscal authority acts as a Stackelberg leader, although the prior mean of ω^F_π is set to 1, the posterior reduces to 0.32, implying that the fiscal policy maker has a lower degree of inflation conservatism than that of monetary policy, even in the LC regime. These estimates are consistent with Rogoff's (1985) idea that the government should appoint a conservative central banker. The optimized degree of inflation conservatism that would be chosen by the government is greater than the government's underlying preference for inflation stabilization as measured by the fiscal authority's estimated objective function. When we compute the optimal degree of inflation conservatism for a delegated central bank given the estimated parameters, we find that the optimized weight of 1.4 lies above the normalized weight of one under the MC regime. These additional gains from conservatism, however, come from reducing inflation volatility below levels observed in data.

The estimates of the deep model parameters remain similar to those found under rules-based policy—see appendix C—with a modest rise in the intertemporal elasticity of substitution, σ, to 3.2, indexation, ζ, to 0.37, and the degree of habits, θ, to 0.81. The other significant difference is that the estimated degree of persistence of the cost-push shock process, ρ_μ, rises from 0.21 to 0.93 as we move from the rules-based estimation to the optimal policy estimation, while the variance of $i.i.d.$ innovations to the cost-push shock fall dramatically. The combined effect of these differences is that the standard deviation of the cost-push shock process is actually lower under the optimal policy estimation.[13] Although cost-push shocks generate a meaningful trade-off for policy makers by raising inflation and reducing output, they do not rise to implausible levels in explaining the data when policy is described optimally. Appendix H reports results from the Komunjer and Ng (2011) identification test, along with plots of the prior and posterior densities.

4.3 Model Comparison

This paper moves beyond a simple rules-based description of macroeconomic policy to model strategic interactions between optimizing policy makers. Does this modeling effort deliver a reasonable statistical fit to data? Table 2 reports the log marginal likelihood values for models closed with the rules-based policy and optimal strategic policy to provide a framework to compare models. We compute Geweke's (1999) modified harmonic mean estimator and the statistic that Sims, Waggoner, and Zha (2008) propose to draw similar conclusions. The latter method is designed for models with time-varying parameters, where the posterior density may be non-Gaussian.

We also present the marginal likelihood associated with an intermediate case in which we allow monetary policy to be time-consistent with switches in the degree of conservatism, while fiscal policy switches between active and passive rules, without the possibility of the fiscal authority behaving optimally.[14] The optimal policy model's fit is also comparable to the intermediate model's: episodes of fiscal Stackelberg leadership can help explain the data, even when those episodes occur relatively infrequently. Fiscal leadership is consistent with specific policy episodes. Fiscal leadership also affects fit because of the impact it has on other policy regimes through expectations. We discuss this issue below.

Model comparisons lead to a key finding that speaks to the bulk of the literature that estimates policy rules. Optimal policy fits data at least as well as policy rules or a combination of optimal monetary policy and fiscal rules. This is a surprising outcome in light of

[13] The unconditional standard deviation of the cost-push shock process under the rules based estimation is 4.9% (13%) and the low (high) volatility regimes, but is only 1.5% (4.2%) under the optimal policy estimation. This compares to an unconditional standard deviation of the cost-push process in Smets and Wouters (2007) of 14.7%.

[14] Parameter estimates of this intermediate model are available upon request.

the additional restrictions that policy optimization imposes. Optimal policy in this model features an optimizing monetary authority, with fluctuations in the degree of conservatism, following a fiscal authority that switches between optimizing as a Stackelberg leader and implementing active or passive fiscal rules.

4.4 Regime Switching

We model monetary policy as optimal and fluctuating between the more (MC) and less (LC) conservative regimes. Fiscal policy can move among optimal policy, a passive rule, and an active rule [figure 2]. Looking at monetary policy alone, periods of the LC regime capture those identified as passive in the rules-based estimation [see appendix C]. There are other periods in which monetary policy is less conservative. The late 1950s gave way to fluctuations in conservatism throughout the first half of the 1960s, which then turned less conservative from 1967 until 1982. The Volcker disinflation didn't really take hold until 1982, as in Chen, Kirsanova, and Leith (2017). From 1982 onwards, the MC regime becomes the dominant regime, with monetary policy temporarily shifting back to the LC regime after the stock market crash of 1987.

Fiscal policy is predominantly passive until the late 1960s when it turns active. Instances of non-active fiscal policy in the 1970s are associated with specific policy events. The Nixon tax reforms of 1970 appear as an example of a passive policy, which then turned optimal as fiscal policy was loosened before the 1972 election. Here the policy was optimal in the sense that reducing tax revenues as a share of GDP reduced the inflationary impact of distortionary taxation at a time when inflation was rising sharply. Similarly, Ford's tax rebate in 1975 appears as a fleetingly passive fiscal policy as the debt-to-GDP ratio had fallen below its steady-state value. Fiscal policy becomes optimal for a sustained period only in 1995, but loses that status around 2000 for a couple of years as rising tax revenues amount to too aggressive a stabilization of debt to constitute an optimal policy. Following Clinton, the Bush tax cuts signal a return to an active fiscal stance which turns passive as the pre-2007 boom generates rising tax revenues despite the tax cuts.

Finding that fiscal policy was optimal the second half of the 1990s gives fiscal policy a prominent role in producing the stable inflation of the period. Rules-based studies credit monetary policy fully with delivering those favorable inflation outcomes. In those studies, fiscal policy is benign, passively adjusting taxes to stabilize debt.

4.4.1 Welfare Gaps We gain further insight into which features of the data drive the identification of the various policy regimes by examining the welfare-relevant "gaps" the policy maker aims to close. We consider four gaps: inflation, output, taxation and debt, where inflation and debt gaps measure the deviation of the variable from its steady-state or target value. The output gap, $\widehat{y}_t - \widehat{y}_t^*$, computes the deviation of output from the level of output that would be chosen by the social planner, $\widehat{y}_t^*$ [see appendix I]. This output gap captures the extent to which the policy maker is unable to achieve the desired level of output due to nominal inertia, the habits externality, fiscal constraints, and time-consistency problems. It reflects the welfare trade-offs between inflation and the real economy embedded in the estimated objective function, but reduces those to a single measure. The tax gap, $\widetilde{\tau}_t - \widetilde{\tau}_t^*$, is the difference between the actual tax rate, $\widetilde{\tau}_t$, and the rate that a policy maker could choose to eliminate cost-push shocks, $\widetilde{\tau}_t^* = -(1-\tau)\widehat{\mu}_t$.

The first two panels of figure 3 plot the inflation and output gap alongside the probability that monetary policy is in the LC regime. This shows that the LC regime arises from periods of higher inflation for a given output gap. Although there is a sizeable negative output gap in the early 1970s, this was not as large relative to the levels of excess inflation found during the Volcker disinflation. This is why the latter period shows up as a switch to more conservative monetary policy. Similarly, a more conservative policy maker would not have suffered the modest rise in inflation which was associated with the loosening of monetary policy after the stock market crash of 1987.

The bottom two panels of figure 3 plot the tax and debt gaps, alongside the probabilities of being in the OF and PF fiscal regimes. The relatively rare OF regime corresponds to periods when the tax, output, and inflation gaps are modest, with debt returning to steady-state. Passive fiscal policy is associated with debt-stabilizing movements in taxation. Exit from the passive fiscal regime in 1968, for example, corresponds to a period of rising taxation that was not consistent with debt stabilization because debt had fallen below its steady-state value by then. Seen in this way, the 1970s were not a decade when fiscal authorities failed to generate sufficient tax revenues to stabilize debt; that decade, instead, was a time when active policy failed to cut taxes despite debt falling below its implicit steady-state. For that period to have been identified as the OF regime, tax rates would have had to have been dramatically reduced to offset the inflationary consequences of the large cost-push shocks experienced at the time. The Nixon tax cuts before the 1972 election, which coincided with relatively low debt levels and rising inflation, are briefly identified as optimal fiscal behavior. To extend the OF regime through the 1970s, taxes would need to have to continue to be reduced as inflationary pressures rose.

5 Policy Episodes

Figure 1 depicts three distinct debt episodes over the sample:

$$\begin{aligned} \text{I: } & 1954Q3 - 1981Q3 \\ \text{II: } & 1981Q4 - 1993Q3 \\ \text{III: } & 1993Q4 - 2008Q2 \end{aligned}$$

Episode I captures the gradual decline in debt-GDP after World War II and the rising inflation rate of the 1970s. Ronald Reagan's tax cuts and defense spending increases launched episode II, in which debt rose back to the levels at the beginning of the first episode while inflation fell and stabilized. Episode III begins with the Clinton budget, which reduced and stabilized debt until the financial crisis in 2008. The last episode saw inflation remain stable at a low level.

How does the estimated evolution of monetary-fiscal regimes account for these debt and inflation episodes? How might these episodes have played out under counterfactual policy mixes?

We conduct two pieces of analysis to answer these questions. First, we assess the prevalence of different policy regimes within each of the episodes. This demonstrates that the estimated optimal policies deliver a rich description of the switches in policy, a description that differs in significant ways from the narrative presented in existing literature. Second, we consider a series of counterfactuals for each sub-period to reveal why the data point toward particular policy regimes.

5.1 Prevalence of Policy Regimes Within Policy Episodes

Table 3 reports which regimes are estimated to be prevalent during each of these three episodes under the rules-based policy versus the optimal policy estimates. The duration of each policy regime is calculated using the smoothed regime transition probabilities. During episode I, 1954Q4–1981Q3, which saw post-war debt decline, the rules-based estimates find that the majority of the period was spent in the conventional policy assignment of AM/PF; policies passed through the AM/AF regime at the start of the 1970s, to settle into PM/AF for the remainder of the 1970s (25% of the sub-period). Optimal policy estimates contrast sharply: monetary policy was less, not more, conservative for three quarters of the episode, effectively mirroring the split between active and passive monetary regimes found in the rules-based estimation. The time spent in the AF regime during the first episode was similar across both estimates.

Episode II, 1981Q3–1993Q2, includes the Volcker disinflation against the backdrop of the Reagan/Bush tax cuts. Rules-based estimates uncover that doubly active policies were dominant, accounting for 81% of the episode. The remaining 19% of the period was spent in the AM/PF regime, suggesting short-lived attempts to raise taxes to stabilize debt. Optimal policy estimates conclude that there was no attempt to adopt a passive or an optimal fiscal policy in this period, and the Federal Reserve lost its conservatism for 27% of the time after the 1987 stock market crash.

Episode III follows the Clinton budget of 1993Q4. Optimal policy estimates report that monetary policy regained conservatism, while the fiscal authority spent 36% of the period behaving optimally, 26% acting passively, and 37% of the time failing to adjust taxes to stabilize debt. Rules-based estimates offer a simpler description: fiscal policy was passive 93% of the time and monetary policy actively targeted inflation 92% of the time. Prevalence of the AM/PF regime conforms to conventional wisdom about the great moderation period.

Table 3's differences between rules-based and optimal policy models arise from subtle dynamic interactions between monetary and fiscal policies. The two policy environments impose very different cross-equation restrictions on the models. Although the models fit data equally well, they produce strikingly different narratives of how American macroeconomic policies evolved over more than a half century.

5.2 Counterfactuals

Rules-based descriptions of policy are often forced into identifying a starker combination of policy regimes than our preferred description of optimal policy [see Appendix C]. This section reports a counterfactual for each episode: the economy is hit by the same estimated shock realizations in each episode, but the policy regime in place is fixed at each possible permutation of monetary and fiscal policy regime.[15] Counterfactuals generate intuition for

[15]Counterfactual exercises condition on remaining in a particular policy regime, but solves for equilibrium using decision rules derived from the estimated model with recurring regime change. Counterfactuals feed

why particular combinations of policy regime are identified as accounting for the observed movements in debt, inflation and policy instruments during the three episodes. At the end of each episode all state variables are returned to their actual data values, and then are allowed to evolve as they would have if that particular policy regime been in place through the remainder of that sub-period.

Figures 4–6, consider counterfactuals under active, passive and optimal fiscal policy respectively. Left columns correspond to more-conservative monetary policy and right columns to less-conservative. Although the high and volatile inflation of the 1970s is clearly associated with the less-conservative monetary policy regime across all three fiscal policy variants, there are only small differences in inflation outcomes outside of the 1970s across the more- or less-conservative monetary policy regimes. Low inflation during the late 1950s and the 1960s, is not inconsistent with the less-conservative monetary policy regime—the shocks and debt policy of the time delivered relatively low and stable inflation even though it was not a policy priority. This interpretation contrasts to the rules-based estimates which can discriminate between the particularly high inflation of the 1970s and the rest of the sample period only by adopting a passive monetary rule in the 1970s alone.

There are sharp differences in the counterfactual outcomes for debt and taxes across the three fiscal regimes—active, passive and optimal. Tax rates under optimal policy are driven by the estimated cost-push shocks, which begin the sample being counter-inflationary, before turning strongly inflationary in the 1970s, and then moderating after the Volcker disinflation. Under the optimal policy counterfactual in Figure 4, this gives rise to a tax policy that reduces (increases) distortionary taxation when inflation is high (low).[16] With debt dynamics driven by significant optimal movements in tax rates, data reject prolonged periods of optimal fiscal behavior, except during the Clinton presidency in the 1990s.

Identification of active versus passive fiscal regimes largely reflects which rule best fits the evolution of debt and the ratio of tax revenues to GDP. The optimal policy estimates find that fiscal policy shifted from a passive stance in the early part of episode I to an active one during the 1970s, similar to estimates under rules. Had fiscal policy not switched in this way, debt would have risen during the 1970s, rather than stabilized at a relatively low level, as Figure 5 shows. Figure 6 illustrates that estimates of active fiscal behavior during the Republican presidencies in the 1980s permits the model to closely track both the run-up of debt and the decline in tax rates. Active fiscal policy also explains George W. Bush's policies beginning in 2000, which cut taxes and reversed the decline in debt.

in the estimated realizations of shocks and begin each of the three episodes at the value of state variables observed in actual data. Section 6.1 considers the implications of policy regime credibility.

[16]This pattern is reinforced by spillovers from the passive fiscal policy regimes which are discussed in Section 6.

5.3 Decomposing Fiscal Adjustment

In our final analysis of the three episodes, we trace the sources of the trends in the debt-to-GDP ratio by decomposing the government's budget constraint into its components

$$\begin{aligned}\Delta\widetilde{b}_t^M &= \overbrace{\frac{1-\beta}{\beta}\widetilde{b}_{t-1}^M + \frac{b^M}{\beta}\left(\frac{\rho\beta}{\gamma\pi}\widehat{P}_t^M - \widehat{P}_{t-1}^M - \widehat{\pi}_t\right)}^{\text{Debt Service}} \\ &\quad -\overbrace{\frac{b^M}{\beta}\left(\widehat{y}_t - \widehat{y}_{t-1} + \widehat{q}_t\right)}^{\text{GDP Growth}} \\ &\quad -\overbrace{(\widetilde{\tau}_t - \widetilde{g}_t - \widetilde{z}_t)}^{\text{Deficit}} + \overbrace{\sigma_{tp}\varepsilon_{tp,t}}^{\text{Term Premium}}\end{aligned} \tag{19}$$

where the first term reflects real debt service costs which are rising in both the level of debt and the ex-post real interest rate, the second term captures the erosion of the debt-to-GDP ratio through the growth in GDP and the third term measures the government primary deficit. The residual is the term-premium shock $\sigma_{tp}\varepsilon_{tp,t}$. We decompose the movements in the ex-post real return to government debt, $\widehat{r}_t \equiv \frac{\rho\beta}{\gamma\pi}\widehat{P}_t^M - \widehat{P}_{t-1}^M - \widehat{\pi}_t$ into changes in ex-ante real interest rates on debt and surprise components

$$\widehat{r}_t - E_{t-1}\widehat{r}_t = -(\widehat{\pi}_t - E_{t-1}\widehat{\pi}_t) + \frac{\rho\beta}{\gamma\pi}(\widehat{P}_t^M - E_{t-1}\widehat{P}_t^M) \tag{20}$$

where the wedge between ex-ante and ex-post real rates captures current inflation surprises, $\widehat{\pi}_t - E_{t-1}\widehat{\pi}_t$, and revaluation effects due to movements in bond prices, $\widehat{P}_t^M - E_{t-1}\widehat{P}_t^M$. These two effects feature prominently in the fiscal theory of the price level with longer-term debt [Sims (2013), Leeper and Leith (2017), and Cochrane (2019)].

Table 4 calculates the contribution of each of these terms to the changes in the debt-to-GDP ratio observed over the three episodes. The post-war decline in debt during episode I was almost entirely due to the reduced debt service costs observed over the period. Reduced costs include both a reduction in ex-ante real interest rates and the revaluation effects stressed by the fiscal theory of the price level, with revaluation smoothed over time thanks to the longer-term maturity structure. Lower service costs were partially offset by a modest deficit.

The declining debt trend was reversed in the 1980s. Reversal arose from two elements. First, Volcker's tighter monetary policy raised ex-ante real interest rates and created capital gains for bond holders at the government's expense. Second, higher interest payments on outstanding debt combined with Reagan's deficits to raise debt growth. The reduction of the debt-to-GDP ratio in episode III, 1993Q4-2008Q2, came from primary surpluses coupled with lower ex-ante real interest rates, but offset somewhat by capital gains for bond holders. To summarize, the reduction in debt in the 1960s was driven by reduced debt service costs, while the 1990s debt reduction was due to a reduced deficit. Across all three periods, the direct impact of GDP growth on the evolution of the debt-to-GDP ratio was negligible.

6 Welfare

Differences in outcomes across policy regimes can have significant welfare implications. Table 5 reports the unconditional variances of key variables as well as the implied welfare cost of shocks under various policy regimes. To measure welfare, we use the fiscal authority's objective function, excluding the tax-rate smoothing term. We believe this is a more natural measure of social welfare than is the monetary authority's objective function. By design, central bank objectives reflect Rogoff's (1985) suggestion to appoint monetary policy makers with stronger aversion to inflation than society at large. The fiscal authority's dislike of inflation, by contrast, reflects society's. From this social welfare measure, we report the "welfare cost" as how much steady-state inflation the policy maker would be willing to accept to achieve the Ramsey allocations.

The results are grouped according to the degree of credibility they assume and are welfare-ranked within each group, while the final column gives the overall ranking. A credible regime constitutes a once-for-all switch in policy, so economic agents do not anticipate any movement away from that regime. A non-credible regime is one where economic agents anticipate fluctuations in regime in line with the estimated transition probabilities.

We begin by examining Table 5's grouping of "No Credibility." The "Estimated" case ranks 8th overall, with an equivalent inflation cost of 1.17% relative to Ramsey. This case reflects an environment where policy regimes switch in line with the estimated transition probabilities. Therefore, any regime ranked higher than 8th overall amounts to an improvement relative to historical policies. The other two non-credible regimes combine a passive fiscal policy with either a more or less conservative monetary policy and are assumed to be in place indefinitely, even although economic agents anticipate switches to other policy regimes in line with the estimated transition probabilities. It can be seen that adopting a passive fiscal rule, even if it lacks credibility, would lead to a marginal improvement over the estimated mix of regimes. All other permutations of regime imply an unstable path for debt if followed indefinitely, given that economic agents expect to switch to the other regimes, and, therefore, a welfare ranking cannot be obtained for these regimes.

Results differ sharply when the policy regime is fully credible. A credible combination of a conservative central bank following a Stackelberg leading fiscal authority comes closest to achieving the Ramsey outcome. Its inflation-equivalent cost is only 0.6%. This is striking because without credibility the same regime would be unable to stabilize debt. There is a slight deterioration in welfare if the monetary authority is less conservative, but still combined with an optimal fiscal policy. Otherwise the credible regimes that improve upon historical policies require that the monetary authority be more conservative. Any other credible regime featuring a less conservative monetary policy and either a passive or active fiscal rule deteriorates in welfare relative to the estimated benchmark, and sizably so in the LC/AF regime. The credible and cooperative, but time-consistent, discretionary policy also performs poorly, worsening outcomes relative to the estimated benchmark. This is because the discretionary policy suffers from the "debt stabilization bias" that Leeper and Leith (2017) and Leeper, Leith, and Liu (2019) discuss.

6.1 The "Best" Regimes

Figure 7 reports the final set of counterfactuals. The first column contrasts what our welfare analysis suggested was the "best" regime—namely, optimal and fully credible fiscal policy—alongside the data and the same policy regime (MC/OF) without credibility. The second column plots outcomes under the cooperative policies (Commitment and Discretion) alongside the data. In order to see the importance of credibility for generating desirable policy outcomes, unlike the previous counterfactuals, we have not split the sample into sub-periods.

The optimal fiscal policy regime works only under full credibility. When a regime is not credible, agents expect it will change eventually. That expectation creates spillovers across regimes that show up in the first column of figure 7. When regime change is possible, optimal fiscal policy implies that the fiscal authority anticipates the rise in taxes and therefore inflation that would arise whenever the economy switches to a passive fiscal rule. Expecting that switch, the fiscal authority cuts taxes today to offset the inflationary effects of anticipated increases in taxation in the future. This reduces inflation today, but raises debt accumulation and inflation volatility. In the absence of credibility, this regime would ultimately be unstable as progressively higher debt levels fuel inflation. Credible optimal fiscal leadership produces large sustained movements in debt that are ultimately stabilized; inflation does not deviate significantly from target.

The second column of the figure plots outcomes under the cooperative policies with and without commitment. Under the Ramsey/Commitment policy we obtain a dramatic stabilization of inflation in combination with an effective tax smoothing policy and substantial movements in government debt. Deviations from pure tax smoothing reflect the desire to offset cost-push shocks through variations in distortionary taxation. Substantial tax cuts in the 1970s largely offset the big cost-push shocks estimated to have hit the economy during that period. The increase in the tax rate in the mid-1980s reflects the reversal in a persistent cost-push shock from positive to negative, implying a desirable rise in taxation to offset the cost-push shock.

Under time-consistent discretionary policy, in contrast, there is a temptation to reduce debt through inflation surprises, whether induced by monetary policy or distortionary tax rises. That temptation which gives rise to an state-dependent inflationary bias problem, which Leeper and Leith (2017) label a "debt stabilization bias." The stabilization bias implies that the policy maker wants to return debt to steady state by raising taxes above the tax smoothing level. Time-consistent policy more rapidly stabilizes debt. Contrasting the outcomes under cooperative policy with those in the first column we see that the credible regime of fiscal leadership combined with a conservative monetary follower closely mimic the outcomes under commitment. This regime implements the kinds of inflation reducing tax cuts that would have occurred in the 1970s under commitment, and supported the sustained increase in debt this would have implied.

In summary, the welfare ranking highlights the importance of credibility: a fiscal authority credibly acting as a Stackelberg leader in a game with the monetary authority results in outcomes closest to those achieved under a cooperative Ramsey policy. Without credibility, such a policy mix would lead to an unstable debt path if pursued indefinitely when economic agents expect the policy regime to switch. Finally, strategic interaction between the monetary and fiscal authorities is generally beneficial when the policy makers are unable to commit, as the cooperative time-consistent outcome suffers from a debilitating debt stabilization bias. Cooperation can be detrimental for welfare.

7 Conclusions

The evolution of inflation dynamics in the United States, as seen through the lens of a conventional new Keynesian model, cannot be understood without explicitly modeling the stance of fiscal policy. A model that allows monetary policy to be optimal, but with potential switches between more- or less-conservative inflation aversion, and fiscal policy to switch among a passive and an active fiscal rule and time-consistent Stackelberg leadership fits post-war American data at least as well purely rules-based policies.

This environment offers a more nuanced interpretation of monetary and fiscal policy interactions than the rules-based model. The narrative that the switch in monetary policy at the time of the Volcker disinflation was associated with a similar switch in fiscal policy making from a regime where the fiscal authorities did not act to stabilize debt to one where they did, does not fit with our estimates. Instead, we find that the Volcker disinflation occurred around 1982, but wasn't supported by a debt stabilizing fiscal policy until 1995 and even then this policy has been subject to further revisions. Moreover, there are numerous switches between the various permutations of policy regime, with a passive fiscal policy still not clearly supporting the post-Volcker monetary conservatism observed in the data. Therefore, the implicit assumption that allows fiscal policy to be safely ignored in monetary policy models does not appear to be consistent with, even, the latest data in our sample.

In a series of counterfactuals we explore the policy mixes that underpinned the major trends in post-war fiscal and monetary policy outcomes. We find that the fall in the debt-to-GDP ratio until the early 1980s, was largely due to a reduction in debt service costs, supported by a less conservative monetary policy and active fiscal policy in the 1970s. The tightening of monetary policy following the appointment of Paul Volcker raised debt service costs, while an active fiscal policy under the Republican presidents of the 1980s resulted in a rise in the debt-to-GDP ratio. The Clinton presidency resulted in lower debt largely through a reduction in the deficit, rather than reduced debt service costs as in the 1960s/70s.

Counterfactuals also suggest that adopting an optimal fiscal policy can be welfare improving, but only if it is credible. The ideal time-consistent policy regime would be where the fiscal authority acts as a Stackelberg leader and the monetary authority is a conservative follower. Such a regime can come close to mimicking the outcomes that would have been observed under a cooperative Ramsey policy. However, this is contingent on the policy being fully credible in the sense that there is no expectation that policy will switch to any alternative policy combination. In contrast, enhancing cooperation can actually reduce welfare relative to the case of strategic interactions between distinct monetary and fiscal authorities.

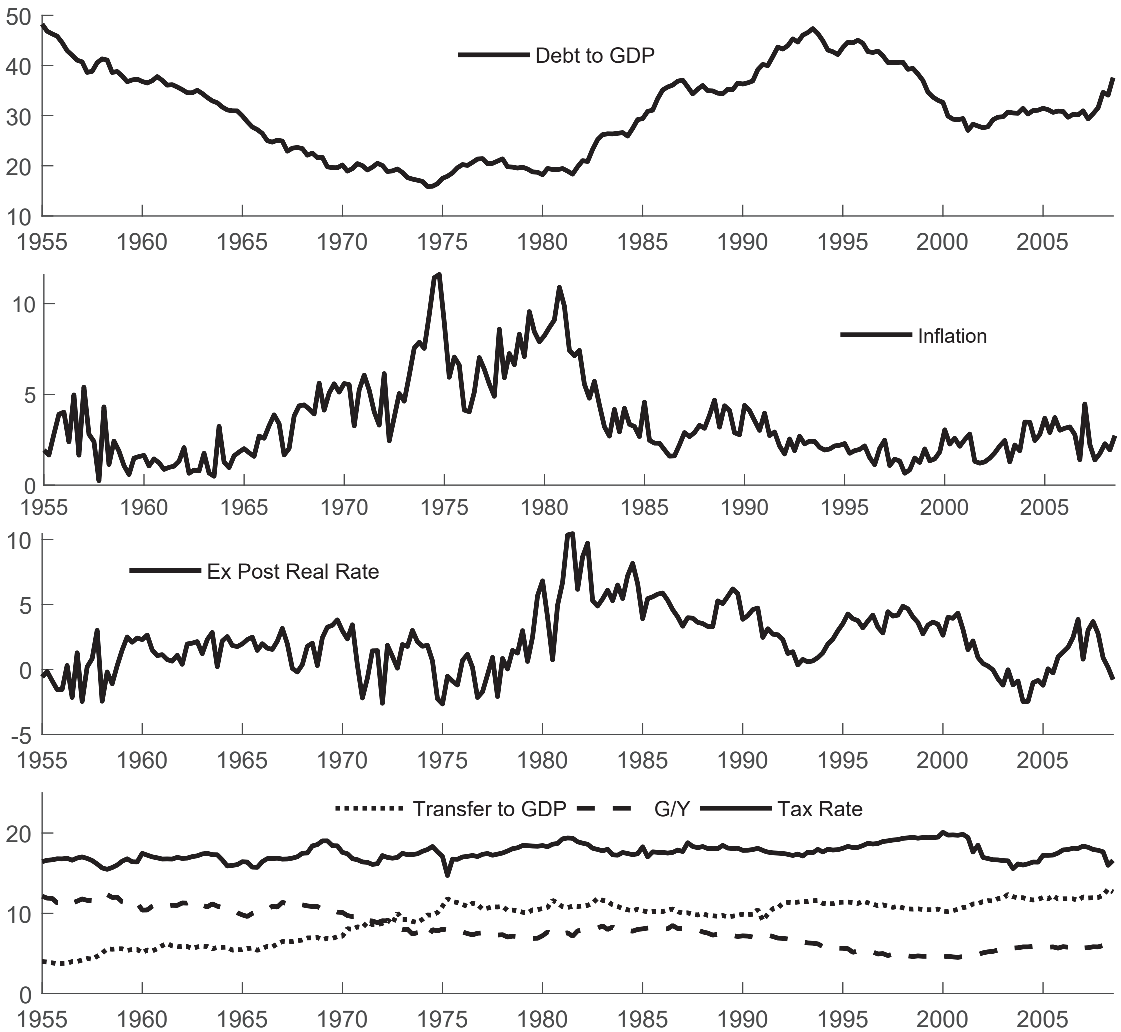

Figure 1: United States Data.

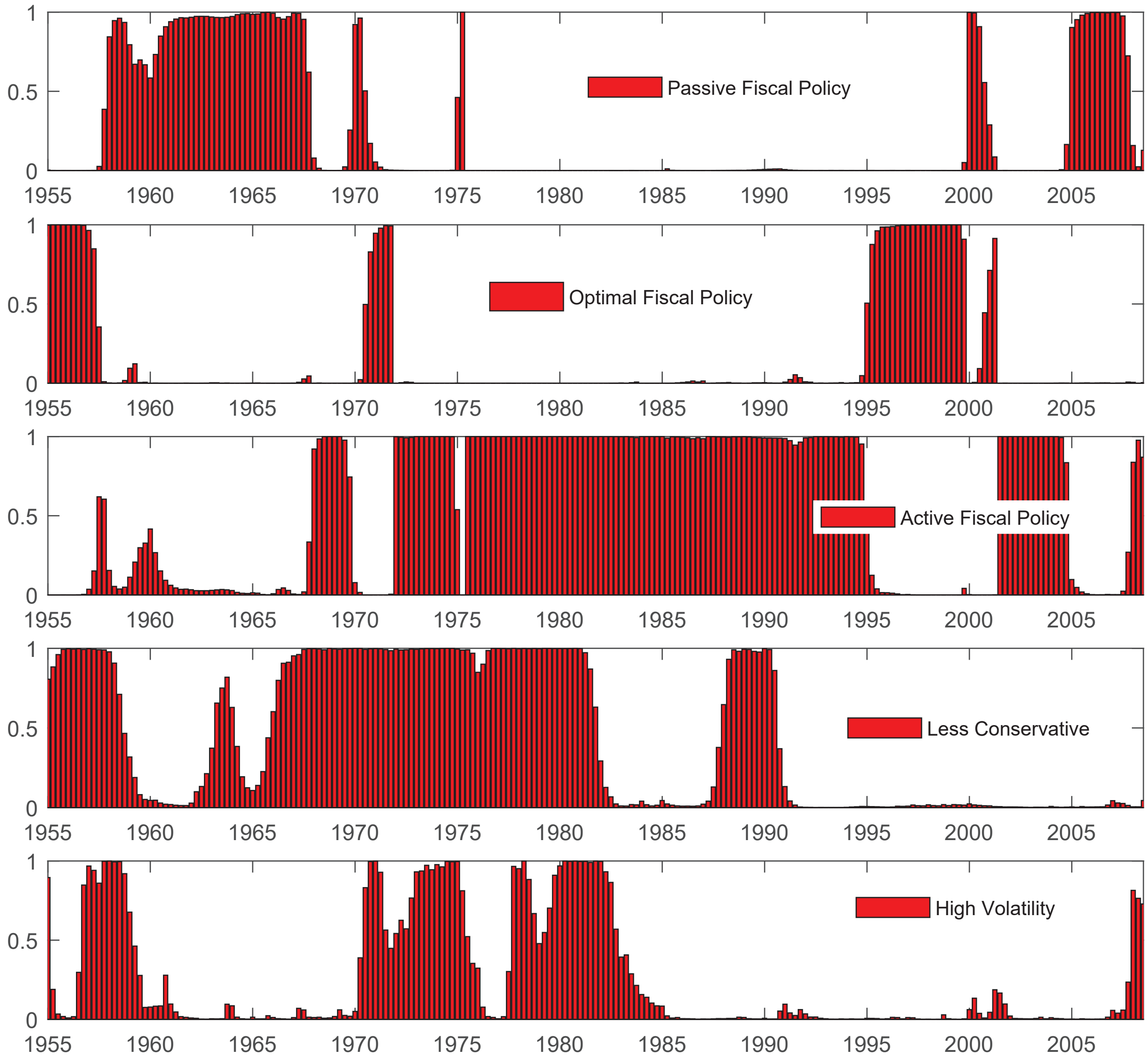

Figure 2: Markov Switching Probabilities: Policy and Volatility Switches under Optimal Strategic Policy

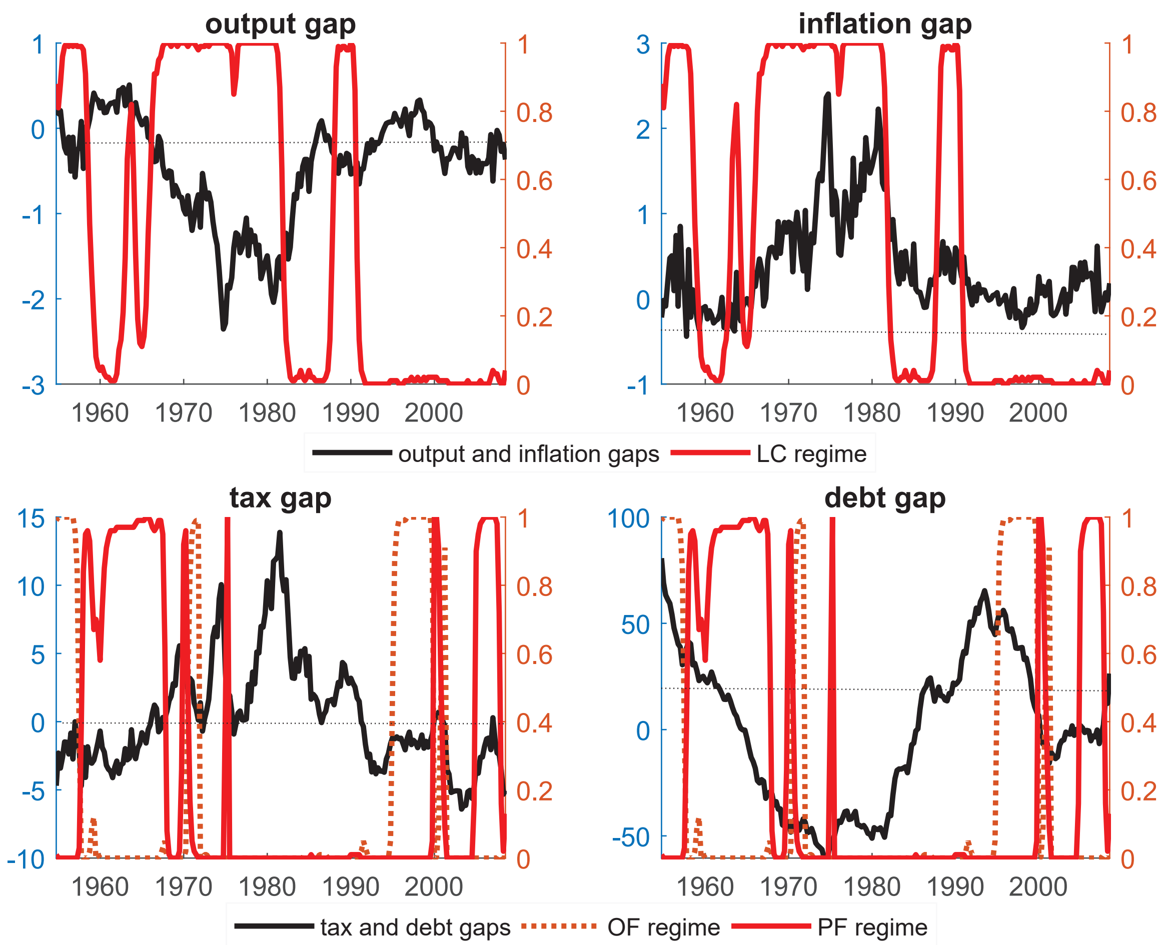

Figure 3: Output, Inflation, Tax, Debt and Policy Regimes. The output gap measures the difference between output and what would be chosen by a social planner given the estimated objective function as a percentage, as Appendix I describes. Inflation and debt gaps measure the deviation from steady-state and the tax gap is the difference between the percentage tax rate and the tax rate that would perfectly offset the inflationary impact of cost push shocks. All gaps are measured on the left scale and the probability of policy regimes on the right scale.

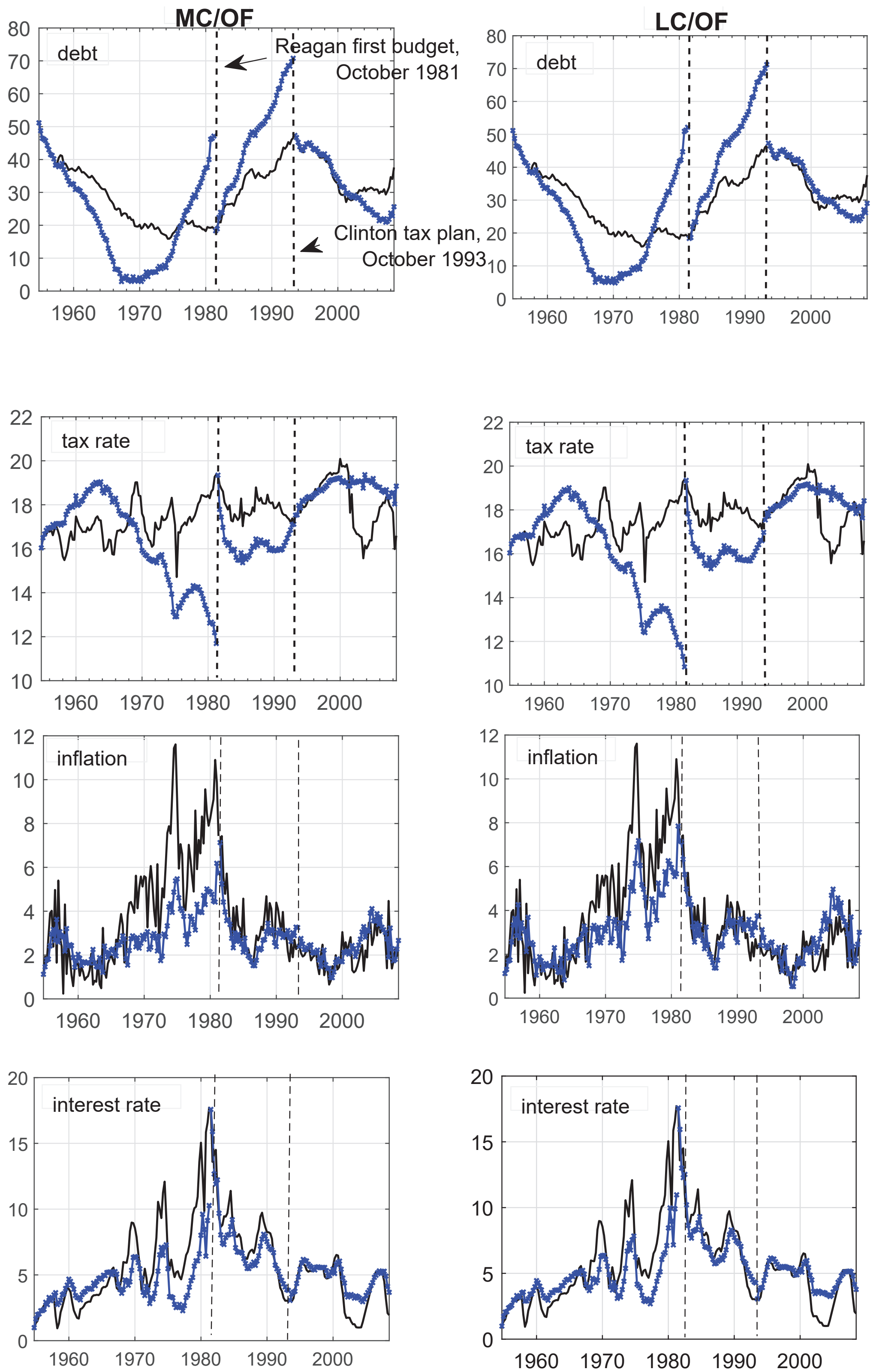

Figure 4: Optimal Fiscal Policy Counterfactual—Optimal Strategic Policy. Dashed vertical lines make episodes I–III. State variables are set equal to observed data values at the start of each episode. Data in solid line; counterfactual path in thick line.

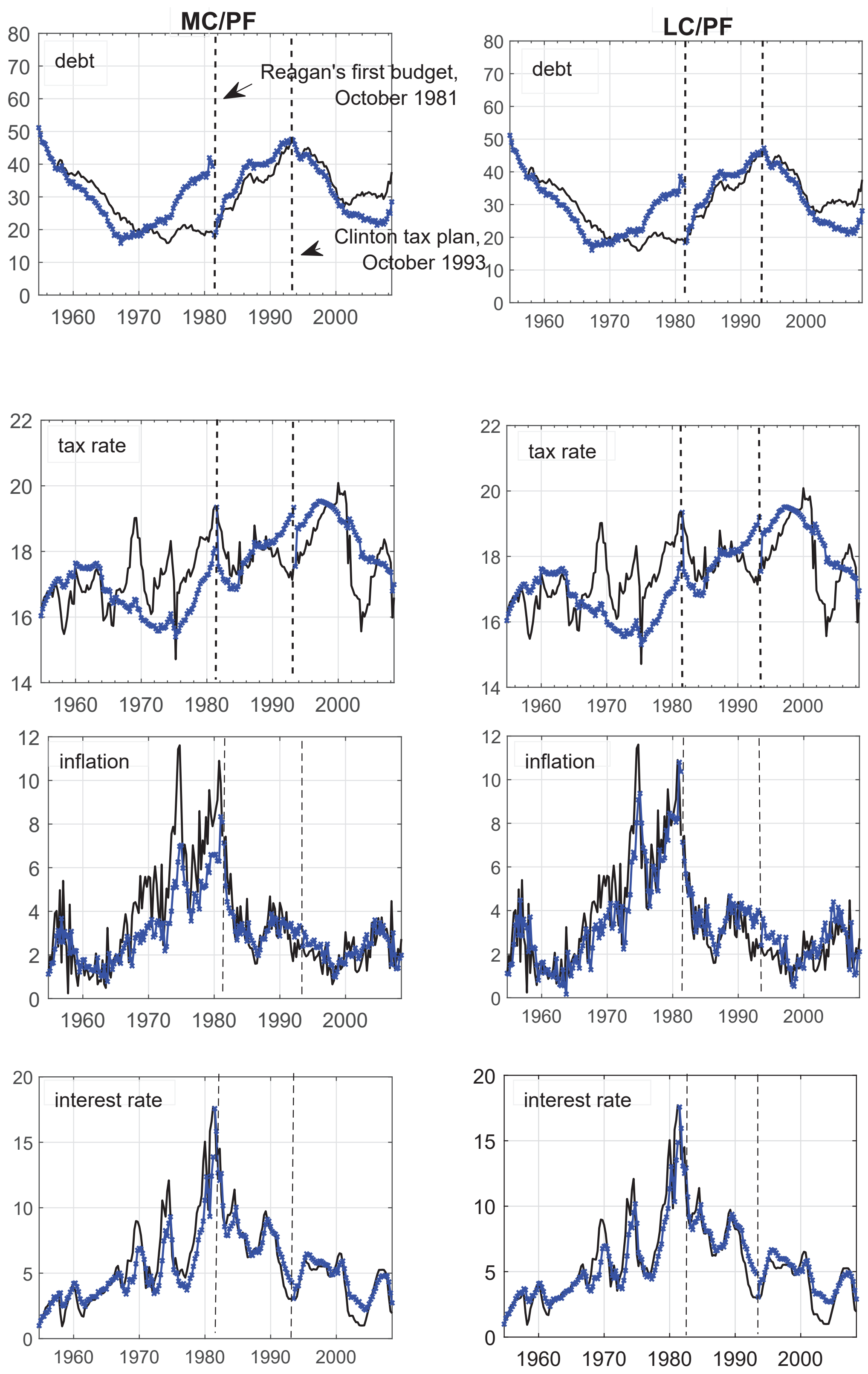

Figure 5: Passive Fiscal Policy Counterfactual—Optimal Strategic Policy. Dashed vertical lines make episodes I–III. State variables are set equal to observed data values at the start of each episode. Data in solid line; counterfactual path in thick line.

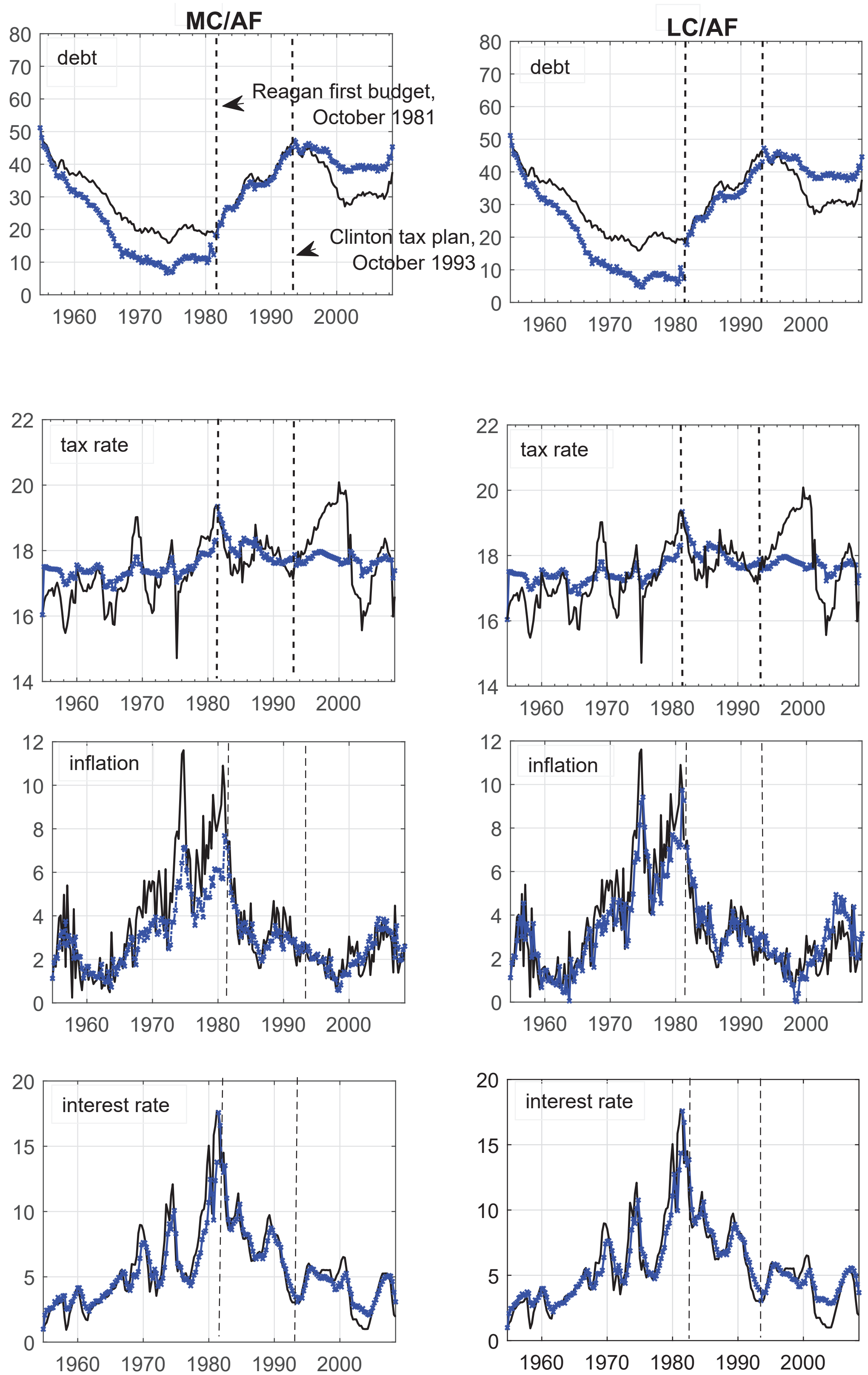

Figure 6: Active Fiscal Policy Counterfactual—Optimal Strategic Policy. Dashed vertical lines make episodes I–III. State variables are set equal to observed data values at the start of each episode. Data in solid line; counterfactual path in thick line.

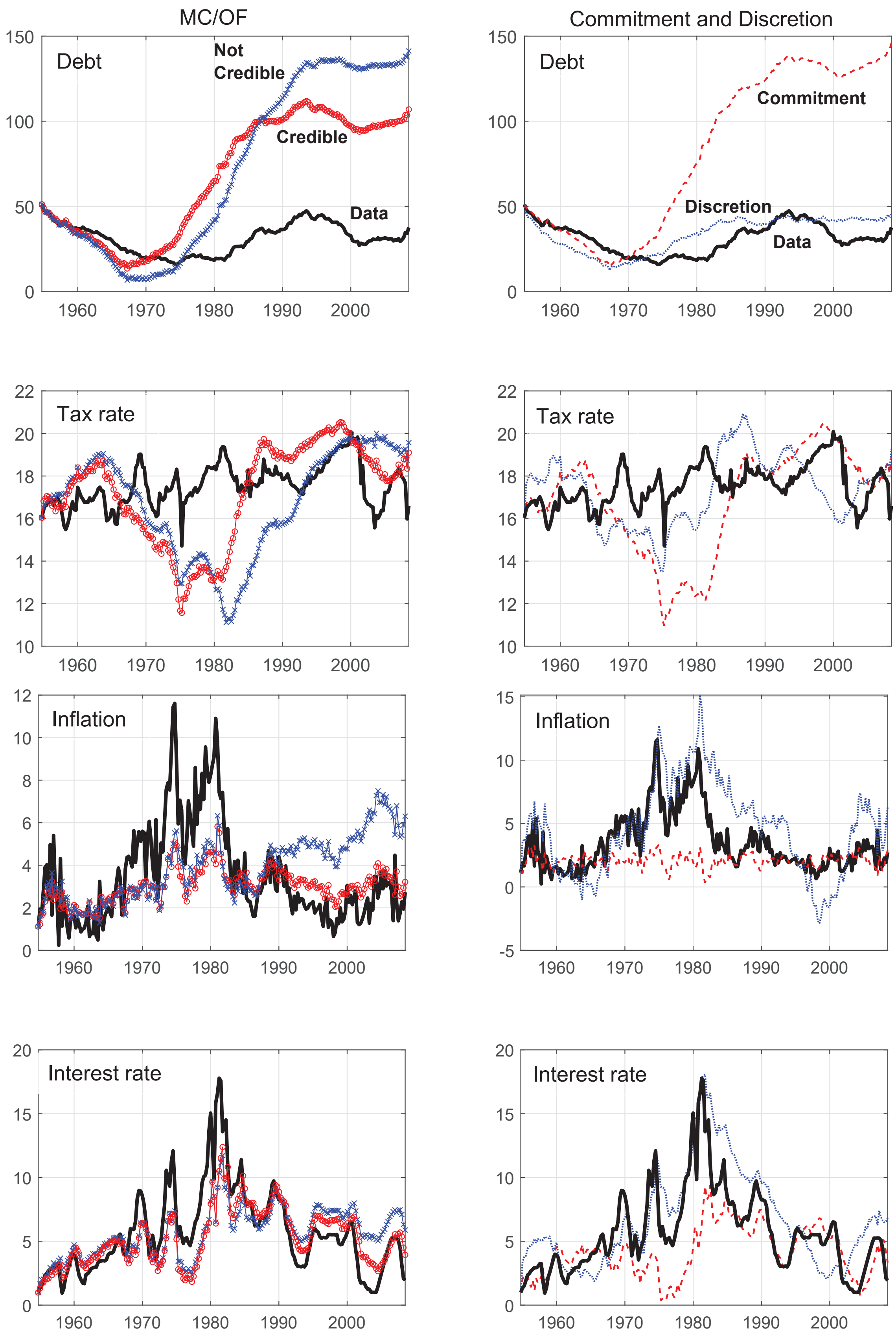

Figure 7: "Best" Policy Regimes Counterfactual—Optimal Strategic Policy.

	Posterior				Prior		
Parameters	Mode	Mean	5%	95%	Type	Mean	Std Dev
Optimal policy parameters							
ω_1, gap term, $\hat{X}_t - \hat{\xi}_t$,	0.221	0.208	0.135	0.280	B	0.50	0.10
ω_2, gap term, $\hat{y}_t - \frac{\sigma}{\varphi}\hat{\xi}_t$,	0.256	0.247	0.177	0.318	B	0.50	0.10
ω_3, change in inflation	0.422	0.420	0.271	0.588	B	0.50	0.10
$\omega^M_{\pi,S_t=1}$, inflation	1.00	1.00	-	-	Fixed	-	-
$\omega^M_{\pi,S_t=2}$, inflation	0.611	0.601	0.484	0.722	B	0.50	0.10
ω_R, change in interest	0.739	0.724	0.568	0.882	B	0.50	0.15
$\omega^F_{\pi,s_t=1}$, inflation	0.298	0.316	0.193	0.433	G	1.00	0.30
$\omega_{\tau,s_t=1}$, change in tax	0.699	0.659	0.491	0.812	B	0.50	0.15
$\rho_{\tau,s_t=2}$, lagged tax rate	0.964	0.950	0.924	0.971	B	0.70	0.15
$\rho_{\tau,s_t=3}$, lagged tax rate	0.932	0.935	0.914	0.960	B	0.50	0.15
$\delta_{\tau,s_t=2}$, tax rate resp. to debt	0.045	0.050	0.037	0.062	G	0.05	0.02
$\delta_{\tau,s_t=3}$, tax rate resp. to debt	0.00	0.00	-	-	Fixed	-	-
Deep parameters							
σ, Inverse of intertemp. elas. of subst.	3.102	3.208	2.759	3.631	N	2.50	0.25
α, Calvo parameter	0.780	0.774	0.751	0.797	B	0.75	0.02
ζ, inflation inertia	0.353	0.366	0.277	0.458	B	0.50	0.10
θ, habit persistence	0.802	0.810	0.736	0.885	B	0.50	0.10
φ, Inverse of Frisch elasticity	2.00	2.00	-	-	Fixed	-	-
Serial correlation of shocks							
ρ_ξ, AR coeff., taste shock	0.938	0.942	0.931	0.953	B	0.50	0.15
ρ_μ, AR coeff., cost-push shock	0.938	0.931	0.912	0.949	B	0.50	0.15
ρ_q, AR coeff., productivity shock	0.274	0.280	0.211	0.350	B	0.50	0.15
ρ_z, AR coeff., transfers	0.968	0.971	0.960	0.982	B	0.50	0.15
ρ_g, AR coeff., government spending	0.986	0.984	0.978	0.989	B	0.50	0.15

Table 1: Optimal Policy. Under optimal policy, we have six policy permutations: MC/OF, MC/PF, MC/AF, LC/OF, LC/PF, LC/AF. For monetary policy switches, $S_t = 1$ is the MC regime and $S_t = 2$ is the LC regime. For fiscal policy, the OF policy regime corresponds to $s_t = 1$, while the PF and AF regimes correspond to $s_t = 2$ and $s_t = 3$, respectively. Weights $\omega_1, \omega_2, \omega_3$ are constant across monetary and fiscal policy regimes.

	Posterior				Prior		
Parameters	Mode	Mean	5%	95%	Type	Mean	Std Dev
Standard deviation of shocks							
$\sigma_{\xi,k_t=1}$, taste shock	0.804	0.874	0.608	1.126	IG	0.50	2.00
$\sigma_{\xi,k_t=2,}$ taste shock	2.318	2.309	1.539	3.075	IG	0.50	2.00
$\sigma_{\mu,k_t=1,}$ cost-push shock	0.545	0.617	0.487	0.740	IG	0.50	2.00
$\sigma_{\mu,k_t=2,}$ cost-push shock	1.660	2.001	1.401	2.580	IG	0.50	2.00
$\sigma_{q,k_t=1,}$ productivity shock	0.684	0.680	0.605	0.759	IG	0.50	2.00
$\sigma_{q,k_t=2}$, productivity shock	1.218	1.286	1.055	1.507	IG	0.50	2.00
σ_{tp}, term premium shock	2.558	2.587	2.332	2.839	IG	2.00	2.00
σ_g, government shock	0.161	0.163	0.150	0.176	IG	0.50	2.00
σ_z, transfer shock	0.303	0.305	0.281	0.330	IG	0.50	2.00
σ_τ, tax rate shock	0.234	0.243	0.217	0.268	IG	0.50	2.00
Transition probabilities							
ϕ_{11}, monetary policy: remaining mc	0.962	0.962	0.942	0.983	B	0.95	0.05
ϕ_{22}, monetary policy: remaining lc	0.956	0.889	0.859	0.922	B	0.95	0.05
ψ_{11}, fiscal policy: remaining optimal	0.875	0.873	0.844	0.902	D	0.90	0.05
ψ_{12}, optimal to passive fiscal policy	0.004	0.008	0.000	0.016	D	0.05	0.05
ψ_{22}, fiscal policy: remaining passive	0.966	0.949	0.920	0.978	D	0.90	0.05
ψ_{23}, passive to active fiscal policy	0.007	0.013	0.000	0.025	D	0.05	0.05
ψ_{33}, fiscal policy: remaining active	0.916	0.912	0.889	0.936	D	0.90	0.05
ψ_{31}, active to optimal fiscal policy	0.001	0.005	0.000	0.010	D	0.05	0.05
h_{11}, volatility: remaining with lv	0.965	0.952	0.925	0.982	B	0.90	0.05
h_{22}, volatility: remaining with hv	0.894	0.943	0.906	0.979	B	0.90	0.05

Table 1: Optimal Policy (continued). For volatility, $k_t = 1$ is the low volatility regime and $k_t = 2$ is the high volatility regime.

	Log Marginal Data Density	
Model	Geweke (1999)	Sims, Waggoner, and Zha (2008)
Optimal Policy	−1410.627	−1410.502
Intermediate Model	−1416.304	−1416.392
Rules-Based Policy	−1418.116	−1418.541

Table 2: Model Comparison. The intermediate model treats monetary policy as time-consistent optimal policy with changes in the degree of inflation conservatism, while fiscal policy switches between the PF and AF regimes. The optimal policy model adds to the intermediate model the possibility that fiscal policy may switch to an additional OF regime.

	Rules-Based Policy								
	I:1954Q1-1981Q3			II:1981Q4-1993Q3			III:1993Q4-2008Q3		
	AM	PM	Duration Fiscal	AM	PM	Duration Fiscal	AM	PM	Duration Fiscal
PF	0.55	0.01	0.56	0.19	0.00	0.19	0.85	0.08	0.93
AF	0.19	0.25	0.44	0.81	0.00	0.81	0.07	0.00	0.07
Duration Monetary	0.74	0.26		1.00	0.00		0.92	0.08	
	Optimal Policy								
	MC	LC	Duration Fiscal	MC	LC	Duration Fiscal	MC	LC	Duration Fiscal
OF	0.00	0.15	0.15	0.00	0.00	0.00	0.36	0.00	0.36
PF	0.24	0.17	0.41	0.00	0.00	0.00	0.26	0.00	0.26
AF	0.00	0.44	0.44	0.73	0.27	1.00	0.37	0.00	0.37
Duration Monetary	0.24	0.76		0.73	0.27		1.00	0.00	

Table 3: Proportion of Time Spent in Different Regimes. For each episode I-III, the individual cells measure the proportion of time spent in each monetary-fiscal policy regime. "Duration" denotes total fraction of time fiscal (monetary) policy resides in specified regime, summing across monetary (fiscal) regimes.

	Percent of Change in Debt-GDP Due to					
Total Change	Real Debt Service Costs					
Debt-GDP (percentage points)	Initial Debt	Ex-Ante Real Return	Surprise Inflation	Surprise Bond Prices	GDP Growth	Primary Surplus
Episode I: 1954Q1-1981Q3						
-25.88	-1.77	-27.93	-0.68	-4.79	-0.15	9.22
Episode II: 1981Q4-1993Q3						
14.97	0.52	6.89	0.12	1.62	-0.23	6.03
Episode III: 1993Q41-2008Q3						
-20.15	1.29	-9.92	0.27	4.40	0.04	-16.24

Table 4: Fiscal Financing. The numbers are percentage changes in the debt-to-GDP ratio. Positive values imply an increase in debt, while negative values indicate a decline in debt. First column records the percentage point change in the debt-GDP ratio in each episode. Numbers exclude influence of term premium shocks on government budget identity, detailed in equations (19) and (20).

Regime	Output	Inflation	Interest	Tax	Welfare Cost	Ranking
No Credibility						
MC/PF	0.482	0.333	0.332	3.116	1.12	6
LC/PF	0.443	0.624	0.415	2.900	1.14	7
Estimated	0.458	0.468	0.367	2.546	1.17	8
Full Credibility						
Commitment/Ramsey	0.610	0.005	0.781	10.455	0.00	1
MC/OF	0.405	0.121	0.297	3.84	0.60	2
LC/OF	0.372	0.416	0.333	3.124	0.84	3
MC/AF	0.437	0.259	0.244	1.136	1.02	4
MC/PF	0.477	0.305	0.280	3.075	1.09	5
LC/PF	0.454	0.720	0.477	3.071	1.21	9
Discretion	0.596	0.910	2.426	192.075	1.41	10
LC/AF	1.522	11.076	8.295	0.403	3.80	11

Table 5: Unconditional Variances and Welfare With Regime Switching. Welfare cost is measured as the amount of steady-state inflation equivalent the policy maker would pay to move to the Ramsey outcome.

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Appendices for U.S. Monetary and Fiscal Policies—Conflict or Cooperation? For Online Publication*

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Appendices

A System of Non-Linear Equations

$$
\begin{aligned}
N_t^{k\varphi} X_t^{k\sigma} &= \frac{W_t}{A_t P_t} \equiv w_t \\
1 &= \beta E_t \left[\left(\frac{X_{t+1}^k \xi_{t+1}}{X_t^k \xi_t} \right)^{-\sigma} \frac{A_t}{A_{t+1}} \frac{P_t}{P_{t+1}} \right] R_t \\
P_t^M &= \beta E_t \left[\left(\frac{X_{t+1}^k \xi_{t+1}}{X_t^k \xi_t} \right)^{-\sigma} \frac{A_t}{A_{t+1}} \frac{P_t}{P_{t+1}} (1 + \rho P_{t+1}^M) \right] \\
N_t &= \left(\frac{Y_t}{A_t}\right) \int_0^1 \left(\frac{P(i)_t}{P_t}\right)^{-\eta} di \\
P_t Y_t &= P_t C_t + P_t G_t \\
\frac{P_t^f}{P_t} &= \left(\frac{\eta}{\eta - 1} \right) \frac{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s (X_{t+s}\xi_{t+s})^{-\sigma} \mu_{t+s} mc_{t+s} \left(\frac{P_{t+s}\pi^{-s}}{P_t} \right)^{\eta} \frac{Y_{t+s}}{A_{t+s}}}{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s (X_{t+s}\xi_{t+s})^{-\sigma} (1 - \tau_{t+s}) \left(\frac{P_{t+s}\pi^{-s}}{P_t} \right)^{\eta-1} \frac{Y_{t+s}}{A_{t+s}}} \\
mc_t &= \frac{W_t}{A_t P_t} \\
P_t^b &= P_{t-1}^* \pi_{t-1} \\
\ln P_{t-1}^* &= (1 - \zeta) \ln P_{t-1}^f + \zeta P_{t-1}^b \\
(P_t)^{1-\eta} &= \alpha (P_{t-1}\pi)^{1-\eta} + (1 - \alpha) (P_t^*)^{1-\eta} \\
b_t^M &= \frac{(1 + \rho P_t^M)}{P_{t-1}^M} \frac{Y_{t-1}}{\pi_t Y_t} b_{t-1}^M - \tau_t + g_t + z_t + \xi_{tp,t} \\
\ln g_t &= (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \sigma_g \varepsilon_{g,t} \\
\ln z_t &= (1 - \rho_z) \ln z + \rho_z \ln z_{t-1} + \sigma_z \varepsilon_{z,t} \\
\ln A_t &= \ln \gamma + \ln A_{t-1} + \ln q_t \\
\ln q_t &= \rho_q \ln q_{t-1} + \sigma_q \varepsilon_{q,t} \\
\ln \mu_t &= \rho_u \ln \mu_{t-1} + \sigma_\mu \varepsilon_t^\mu \\
\ln \xi_t &= \rho_\xi \ln \xi_{t-1} + \sigma_\xi \varepsilon_{\xi,t}
\end{aligned}
$$

The equation describing the evolution of price dispersion, $\int_0^1 (\frac{P(i)_t}{P_t})^{-\eta_t} di$ is not needed to tie down the equilibrium upon log-linearization.

In order to render this model stationary we need to scale certain variables by the non-stationary level of technology, A_t such that $k_t = K_t/A_t$ where $K_t = \{Y_t, C_t, W_t/P_t\}$. Fiscal variables (i.e. $P_t^M B_t^M / P_t$, G_t and Z_t) are normalized with respect to Y_t. All other real variables are naturally stationary. Applying this scaling, the steady-state equilibrium conditions reduce to:

$$
\begin{aligned}
N^{\varphi}X^{\sigma} &= w \\
1 &= \beta\left(R\pi^{-1}\right)/\gamma = \beta r/\gamma \\
P^{M} &= \frac{\beta}{\gamma\pi - \beta\rho} \\
y &= N \\
y &= \frac{c}{(1-g)} \\
X &= c(1-\theta) \\
mc &= w \\
\frac{\eta}{\eta-1} &= \frac{1-\tau}{mc} \\
b^{M} &= \left(\frac{\beta}{1-\beta}\right)s
\end{aligned}
$$

To determine the steady state value of labor, we substitute for X in terms of y and then, using the aggregate production function, we obtain the following expression,

$$
y^{\sigma+\varphi}\left[(1-g)(1-\theta)\right]^{\sigma} = \frac{\eta-1}{\eta}(1-\tau), \tag{A.1}
$$

where g is the steady state share of government spending in output. We shall contrast this with the labor allocation/output that would be chosen by a social planner to obtain a measure of the steady-state distortion inherent in this economy which features distortionary taxation, monopolistic competition and the habits externality.

B Derivation of Objective Functional Form

B.1 The Social Planner's Problem

In order to assess the scale of the steady-state inefficiencies caused by the monopolistic competition, tax and habits externalities it is helpful to contrast the decentralized equilibrium with that which would be attained under the social planner's allocation. The social planner ignores the nominal inertia and all other inefficiencies and chooses real allocations that maximize the representative consumer's utility subject to the aggregate resource constraint, the aggregate production function, and the law of motion for habits-adjusted consumption:

$$
\max_{\{X_t^*, C_t^*, N_t^*\}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{X_t^{*1-\sigma}\xi_t^{-\sigma}}{1-\sigma} + \chi\frac{(G_t^*/A_t)^{1-\sigma}(\xi_t)^{-\sigma}}{1-\sigma} - \frac{N_t^{*1+\varphi}\xi_t^{-\sigma}}{1+\varphi} \right)
$$

$$
\begin{aligned}
s.t.\ Y_t^* &= C_t^* + G_t^* \\
Y_t^* &= A_t N_t^* \\
X_t^* &= C_t^*/A_t - \theta C_{t-1}^*/A_{t-1}
\end{aligned}
$$

The optimal choice implies the following relationship between the marginal rate of substitution between labor and habit-adjusted consumption and the intertemporal marginal rate of substitution in habit-adjusted consumption

$$\frac{(N_t^*)^{\varphi}}{(X_t^*)^{-\sigma}} = \left[1 - \theta\beta E_t\left(\frac{X_{t+1}^*\xi_{t+1}}{X_t^*\xi_t}\right)^{-\sigma}\right].$$

The steady state equivalent of this expression can be written as,

$$(N^*)^{\varphi+\sigma}\left[(1 - \frac{G^*}{Y^*})(1-\theta)\right]^{\sigma} = (1-\theta\beta)\,. \tag{B.1}$$

where the optimal share of government consumption in output is given by,

$$\frac{G_t^*}{Y_t^*} = \chi^{\frac{1}{\sigma}}(\frac{Y_t^*}{A_t})^{-\frac{\sigma+\varphi}{\sigma}}$$

In steady state these can be combined to give the optimal share of government consumption in output,

$$\frac{G^*}{Y^*} = (1 + (1-\theta)^{-1}\chi^{-\frac{1}{\sigma}}(1-\theta\beta)^{\frac{1}{\sigma}})^{-1}$$

which can then used to get the steady state level of output under the social planner's allocation. We shall assume that the share of government spending in GDP in the data matches this, such that the data is calibrating the value of χ. Doing so facilitates the construction of a quadratic objective function.

If we contrast this with the allocation achieved in the steady-state of our decentralized equilibrium (A.1), assuming that the steady state share of government consumption to GDP is the same, we can see that the two will be identical whenever the following relationship between the markup, the tax rate and the degree of habits holds,

$$\frac{\eta}{\eta-1} = \frac{1-\tau}{1-\theta\beta}$$

Notice that in the absence of habits this condition could only be supported by a negative tax rate. However, for the data given level of taxation and the estimated degree of habits this condition will define our steady-state markup, enabling us to adopt an efficient steady-state and thereby avoiding a steady-state inflationary bias problem when describing optimal policy.

B.2 Quadratic Representation of Social Welfare

Individual utility in period t is

$$\frac{X_t^{1-\sigma}\xi_t^{-\sigma}}{1-\sigma} + \chi\frac{(G_t/A_t)^{1-\sigma}(\xi_t)^{-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}\xi_t^{-\sigma}}{1+\varphi}$$

where $X_t = C_t - \theta C_{t-1}$ is the habit-adjusted aggregate consumption. Before considering the elements of the utility function, we need to note the following general result relating to second order approximations

$$\frac{Y_t - Y}{Y_t} = \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2 + O[2]$$

where $\widehat{Y}_t = \ln\left(\frac{Y_t}{Y}\right)$ and $O[2]$ represents terms that are of order higher than 2 in the bound on the amplitude of the relevant shocks. This will be used in various places in the derivation of welfare. Now consider the second order approximation to the first term,

$$\frac{X_t^{1-\sigma}\xi_t^{-\sigma}}{1-\sigma} = \overline{X}^{1-\sigma}\left(\frac{X_t-\overline{X}}{\overline{X}}\right) - \frac{\sigma}{2}\overline{X}^{1-\sigma}\left(\frac{X_t-\overline{X}}{\overline{X}}\right)^2 - \sigma\overline{X}^{1-\sigma}\left(\frac{X_t-\overline{X}}{\overline{X}}\right)(\xi_t-1)+tip+O[2]$$

where *tip* represents 'terms independent of policy'. Using the results above this can be rewritten in terms of hatted variables

$$\frac{X_t^{1-\sigma}\xi_t^{-\sigma}}{1-\sigma} = \overline{X}^{1-\sigma}\left\{\widehat{X}_t + \frac{1}{2}(1-\sigma)\widehat{X}_t^2 - \sigma\widehat{X}_t\widehat{\xi}_t\right\} + tip + O[2].$$

In pure consumption terms, the value of X_t can be approximated to second order by:

$$\widehat{X}_t = \frac{1}{1-\theta}\left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right) - \frac{\theta}{1-\theta}\left(\widehat{c}_{t-1} + \frac{1}{2}\widehat{c}_{t-1}^2\right) - \frac{1}{2}\widehat{X}_t^2 + O[2]$$

and to a first order,

$$\widehat{X}_t = \frac{1}{1-\theta}\widehat{c}_t - \frac{\theta}{1-\theta}\widehat{c}_{t-1} + O[1]$$

which implies

$$\widehat{X}_t^2 = \frac{1}{(1-\theta)^2}\left(\widehat{c}_t - \theta\widehat{c}_{t-1}\right)^2 + O[2]$$

Therefore,

$$\frac{X_t^{1-\sigma}\xi_t^{-\sigma}}{1-\sigma} = \overline{X}^{1-\sigma}\left\{\frac{1}{1-\theta}\left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right) - \frac{\theta}{1-\theta}\left(\widehat{c}_{t-1} + \frac{1}{2}\widehat{c}_{t-1}^2\right) + \frac{1}{2}(-\sigma)\widehat{X}_t^2 - \sigma\widehat{X}_t\widehat{\xi}_t\right\}+tip+O[2]$$

Summing over the future,

$$\sum_{t=0}^{\infty}\beta^t\frac{X_t^{1-\sigma}\xi_t^{-\sigma}}{1-\sigma} = \overline{X}^{1-\sigma}\sum_{t=0}^{\infty}\beta^t\left\{\frac{1-\theta\beta}{1-\theta}\left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right) - \frac{1}{2}\sigma\widehat{X}_t^2 - \sigma\widehat{X}_t\widehat{\xi}_t\right\} + tip + O[2].$$

Similarly for the term in government spending,

$$\chi\frac{g_t^{1-\sigma}\xi_t^{-\sigma}}{1-\sigma} = \chi\overline{g}^{1-\sigma}\{\widehat{g}_t + \frac{1}{2}(1-\sigma)\widehat{g}_t^2 - \sigma\widehat{g}_t\widehat{\xi}_t\} + tip + O[2]$$

While the term in labour supply can be written as

$$\frac{N_t^{1+\varphi}\xi_t^{-\sigma}}{1+\varphi} = \overline{N}^{1+\varphi}\left\{\widehat{N}_t + \frac{1}{2}(1+\varphi)\widehat{N}_t^2 - \sigma\widehat{N}_t\widehat{\xi}_t\right\} + tip + O[2]$$

Now we need to relate the labour input to output and a measure of price dispersion. Aggregating the individual firms' demand for labour yields,

$$N_t = \left(\frac{Y_t}{A_t}\right)\int_0^1\left(\frac{P(i)_t}{P_t}\right)^{-\eta_t}di$$

It can be shown (see Woodford (2003, Chapter 6)) that

$$\begin{aligned}
\widehat{N}_t &= \widehat{y}_t + \ln[\int_0^1 (\frac{P(i)_t}{P_t})^{-\eta_t} di] \\
&= \widehat{y}_t + \frac{\eta}{2} var_i\{p(i)_t\} + O[2]
\end{aligned}$$

which implies

$$\widehat{N}_t^2 = \widehat{y}_t^2$$

so we can write

$$\frac{N_t^{1+\varphi}}{1+\varphi} = \overline{N}^{1+\varphi} \left\{ \widehat{y}_t + \frac{1}{2}(1+\varphi)\widehat{y}_t^2 - \sigma \widehat{y}_t \widehat{\xi}_t + \frac{\eta}{2} var_i\{p_t(i)\} \right\} + tip + O[2]$$

Welfare is then given by

$$\begin{aligned}
\Gamma_0 \;=\; & \overline{X}^{1-\sigma} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1-\theta\beta}{1-\theta} \left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2 \right) - \frac{1}{2}\sigma \hat{X}_t^2 - \sigma \widehat{X}_t \widehat{\xi}_t \right\} \\
& + \chi \overline{g}^{1-\sigma} E_0 \sum_{t=0}^{\infty} \beta^t \{ \widehat{g}_t + \frac{1}{2}(1-\sigma)\widehat{g}_t^2 - \sigma \widehat{g}_t \widehat{\xi}_t \} \\
& - \overline{N}^{1+\varphi} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \widehat{y}_t + \frac{1}{2}(1+\varphi)\widehat{y}_t^2 - \sigma \widehat{y}_t \widehat{\xi}_t + \frac{\eta}{2} var_i\{p_t(i)\} \right\} \\
& + tip + O[2]
\end{aligned}$$

From the steady-state of our model, and its comparison with the social planner's allocation we know that $\overline{X}^{1-\sigma}(1-\theta\beta) = (1-\theta)\frac{c}{y}\overline{N}^{1+\varphi}$. Similarly, assuming the same share of government spending in GDP across the social planner's and decentralized equilibrium, we also know that, $\chi \overline{g}^{1-\sigma} = \frac{g}{y}\overline{N}^{1+\varphi}$. Using the fact that,

$$\frac{c}{y}\widehat{c}_t = \widehat{y}_t - (1-\frac{c}{y})\widehat{g}_t - \frac{1}{2}\frac{c}{y}\widehat{c}_t^2 - \frac{1}{2}(1-\frac{c}{y})\widehat{g}_t^2 + \frac{1}{2}\widehat{y}_t^2 + O[2]$$

we can collect the levels terms and write the sum of discounted utilities as:

$$\Gamma_0 = -\frac{1}{2}\overline{N}^{1+\varphi} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{\sigma(1-\theta)}{1-\theta\beta}\frac{c}{y}\left(\widehat{X}_t + \widehat{\xi}_t\right)^2 + \sigma\frac{g}{y}\left(\widehat{g}_t + \widehat{\xi}_t\right)^2 \\ +\varphi\left(\widehat{y}_t - \frac{\sigma}{\varphi}\widehat{\xi}_t\right)^2 \\ +\eta var_i\{p_t(i)\} \end{array} \right\} + tip + O[2]$$

Using the result from Fabian Eser and Wren-Lewis (2009) that

$$\sum_{t=0}^{\infty} \beta^t var_i[p_t(i)] = \frac{\alpha}{(1-\beta\alpha)(1-\alpha)} \sum_{t=0}^{\infty} \beta^t \left[\widehat{\pi}_t^2 + \frac{\zeta\alpha^{-1}}{(1-\zeta)}(\widehat{\pi}_t - \widehat{\pi}_{t-1})^2 \right] + O[2].$$

we can write the discounted sum of utility as,

$$\Gamma_0 = -\frac{1}{2}\overline{N}^{1+\varphi} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{\sigma(1-\theta)}{1-\theta\beta}\frac{c}{y}\left(\widehat{X}_t + \widehat{\xi}_t\right)^2 + (\varphi)\left(\widehat{y}_t - \frac{\sigma}{\varphi}\widehat{\xi}_t\right)^2 \\ + \frac{\alpha\eta}{(1-\beta\alpha)(1-\alpha)}\left[\widehat{\pi}_t^2 + \frac{\zeta\alpha^{-1}}{(1-\zeta)}(\widehat{\pi}_t - \widehat{\pi}_{t-1})^2\right] \end{array} \right\} + tip + O[2]$$

where we have put the terms in public consumption into tip since they are treated as an exogenous process and therefore independent of policy.

After normalising the coefficient on inflation to one, we can write the microfounded objective function as,

$$\Gamma_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \Phi_1 \left(\widehat{X}_t + \widehat{\xi}_t \right)^2 + \Phi_2 \left(\widehat{y}_t - \frac{\sigma}{\varphi} \widehat{\xi}_t \right)^2 \\ + \frac{\zeta \alpha^{-1}}{(1-\zeta)} \left(\widehat{\pi}_t - \widehat{\pi}_{t-1} \right)^2 + \widehat{\pi}_t^2 \end{array} \right\} \tag{B.2}$$

where the weights on the two real terms are functions of model structural parameters, where $\Phi_1 = \frac{\sigma(1-\theta)}{1-\theta\beta} \frac{(1-\beta\alpha)(1-\alpha)}{\alpha\eta} \frac{c}{y}$ and $\Phi_2 = \frac{\varphi(1-\beta\alpha)(1-\alpha)}{\alpha\eta}$.

C Rules-Based Estimation

In this section we undertake an estimation of our model when describing policy using simple rules. This serves to create a set of benchmark results which we can contrast with our estimates which allow for strategic interactions between monetary and fiscal policy. In doing so it is important to note that while we extend the analysis of Bianchi (2012) and Bianchi and Ilut (2017) in some ways, this does not overturn their key results. Bianchi and Ilut argue that restricting the number and transition pattern of regimes is data-preferred largely as a result of the fact that the PM/AF and PM/PF regimes are very similar in terms of their dynamic responses to shocks. This is no longer the case when taxes are assumed to be distortionary where the inflationary impact of variation in taxes becomes a key ingredient in identifying policy regimes. Nevertheless, this results in a similar narrative in terms of the evolution of monetary and fiscal policy to the existing literature - fiscal policy turns active in the late 1960s and monetary policy turns active shortly afterwards, only regaining its activism following the Volcker disinflation in 1982. However, under our Rules-Based estimation the transition to a complementary passive fiscal regime was, unlike Bianchi and Ilut, not decisively achieved in 1982, and really only emerged a decade later in 1992.

When considering policy described by simple rules, we assume fiscal policy follows a simple tax rule,

$$\widetilde{\tau}_t = \rho_{\tau,s_t} \widetilde{\tau}_{t-1} + (1 - \rho_{\tau,s_t}) \left(\delta_{\tau,s_t} \widetilde{b}_{t-1}^M + \delta_y \widehat{y}_t \right) + \sigma_\tau \varepsilon_{\tau,t}$$

where we assume the coefficient on debt, δ_{τ,s_t}, and the persistence of the tax rate, ρ_{τ,s_t} are subject to regime switching with $s_t = 1$ indicating the Passive Fiscal (PF) regime and $s_t = 2$ being the Active Fiscal (AF) regime. The fiscal policy regimes are determined by the value of coefficient on debt with $\delta_{\tau,s_t=1} > \frac{1}{\beta} - 1$ in the PF regime and $\delta_{\tau,s_t=2} = 0$ in the AF regime.

When U.S. monetary policy is described as a generalized Taylor rule, we specify this rule following An and Schorfheide (2007),

$$\widehat{R}_t = \rho_{R,S_t} \widehat{R}_{t-1} + (1 - \rho_{R,S_t}) [\psi_{1,S_t} \widehat{\pi}_t + \psi_{2,S_t} (\Delta \widehat{y}_t + \widehat{q}_t)] + \sigma_R \varepsilon_{R,t}$$

where the Fed adjusts interest rates in response to movements in inflation and deviations of output growth from trend. We allow the rule parameters $(\rho_{R,S_t}, \psi_{1,S_t}, \psi_{2,S_t})$ to switch between active and passive policy regimes. The Active Monetary (AM) policy regime corresponds to

$S_t = 1$, while the Passive Monetary (PM) policy regime corresponds to $S_t = 2$. The labeling implies that $\psi_{1,S_t=1} > 1$ and $0 < \psi_{1,S_t=2} < 1$.

By considering both fiscal and monetary policy changes, we can distinguish four policy regimes under Rules-Based policy. They are AM/PF, AM/AF, PM/PF and PM/AF. Leeper (1991) shows that, in the absence of regime switching, the existence of a unique solution to the model depends on the nature of the assumed policy regime. A unique solution can be found under both the AM/PF and PM/AF regimes, what Leeper and Leith (2017) refer to as the M and F-regimes, respectively. In the former monetary policy actively targets inflation and fiscal policy adjusts taxes to stabilize debt, while under the latter combination the fiscal authority does not adjust taxes to stabilize debt and the monetary authority does not actively target inflation in order to facilitate the stabilization of debt. In contrast, no stationary solution and multiple equilibria are obtained under the AM/AF and PM/PF regimes, respectively. However, when regime switching is considered, the existence and uniqueness of a solution also depends on the transition probabilities of the potential regime changes as economic agents anticipate the transition to different policy regimes. Specifically, we allow monetary and fiscal policy rule parameters to switch independently of each other. The transition matrices for monetary policy and fiscal policy are as follows

$$P = \begin{bmatrix} p_{11} & 1-p_{22} \\ 1-p_{11} & p_{22} \end{bmatrix}, \ Q = \begin{bmatrix} q_{11} & 1-q_{22} \\ 1-q_{11} & q_{22} \end{bmatrix},$$

where $p_{ii} = \Pr\left[S_t = i | S_{t-1} = i\right]$ and $q_{ii} = \Pr\left[s_t = i | s_{t-1} = i\right]$. In addition, we also account for a possible shift in fundamental shock volatilities which has been used as a potential explanation of the Great Moderation. Failure to do so could potentially bias the identification of shifts in policy (see Sims and Zha (2006)). Therefore, we allow for independent regime switching in the standard deviations of technology (σ_{q,k_t}), preference (σ_{ξ,k_t}) and cost-push (σ_{μ,k_t}) shocks, with $k_t = 1$ being in the low volatility regime and $k_t = 2$ in the high volatility regime. The transition matrix for the shock volatilities is as follows

$$H = \begin{bmatrix} h_{11} & 1-h_{22} \\ 1-h_{11} & h_{22} \end{bmatrix},$$

where $h_{ii} = \Pr\left[k_t = i | k_{t-1} = i\right]$.[1]

We adopt the solution algorithm proposed by Farmer, Waggoner, and Zha (2011) to solve the model with Markov-switching in policy rule parameters. Since this algorithm implies that economic agents anticipate the Markov switching between different policy rules, there will be spillovers across policy regimes which will turn out to be crucial in determining the relative performance of alternative policies.

Table C.1 presents the priors and posterior estimates for the Rules-Based policy. For the interest rate rule parameters, we set symmetric priors for the parameter of the lagged interest rate and the parameter of output growth, whereas asymmetric and truncated priors are used for the parameter of inflation to ensure that $\psi_{1,S_t=1} > 1$ in the AM regime and $0 < \psi_{1,S_t=2} < 1$ in the PM regime. Similarly, for the tax rule, a symmetric prior is used for the parameter of lagged tax rate, while the parameter of debt is restricted to be zero in the AF

[1] The joint transition matrix governing the monetary-fiscal-shock regime is then $P = P \otimes Q \otimes H$. In total, there are eight regimes in the Rules-Based model.

regime and positive in the PF regime. Overall, the priors of the policy rule parameters imply four distinct fiscal and monetary policy regimes: AM/PF, AM/AF, PM/PF and PM/AF. In addition, variances of shocks are chosen to be highly dispersed inverted Gamma distributions to generate realistic volatilities for the endogenous variables.

	Posterior				Prior		
Parameters	Mode	Mean	5%	95%	Type	Mean	Std Dev
			AM/PF				
$\rho_{R,S_t=1}$, lagged interest rate	0.880	0.880	0.853	0.906	B	0.50	0.15
$\psi_{1,S_t=1}$, interest rate resp. to inflation	3.068	2.898	2.098	3.657	G	2.00	0.50
$\psi_{2,S_t=1}$, interest rate resp. to output	0.719	0.670	0.421	0.977	G	0.50	0.25
$\rho_{\tau,s_t=1}$, lagged tax rate	0.955	0.957	0.929	0.985	B	0.70	0.15
$\delta_{\tau,s_t=1}$, tax rate resp. to debt	0.032	0.036	0.014	0.057	G	0.05	0.02
δ_y, tax rate resp. to output	0.000	0.078	0.000	0.177	G	0.10	0.10
			AM/AF				
$\rho_{R,S_t=1}$, lagged interest rate	0.610	0.609	0.525	0.694	B	0.50	0.15
$\psi_{1,S_t=1}$, interest rate resp. to inflation	1.454	1.485	1.289	1.688	G	2.00	0.50
$\psi_{2,St=1}$, interest rate resp. to output	0.686	0.695	0.483	0.926	G	0.50	0.25
$\rho_{\tau,s_t=2}$, lagged tax rate	0.763	0.725	0.610	0.846	B	0.70	0.15
$\delta_{\tau,s_t=2}$, tax rate resp. to debt	0.00	0.00	-	-	Fixed	-	-
δ_y, tax rate resp. to output	0.000	0.078	0.000	0.177	G	0.10	0.10
			PM/PF				
$\rho_{R,S_t=2}$, lagged interest rate	0.869	0.856	0.819	0.896	B	0.50	0.15
$\psi_{1,S_t=2}$, interest rate resp. to inflation	0.982	0.904	0.810	0.990	G	0.80	0.15
$\psi_{2,St=2}$, interest rate resp. to output	0.581	0.583	0.288	0.938	G	0.50	0.25
$\rho_{\tau,s_t=1}$, lagged tax rate	0.437	0.466	0.308	0.623	B	0.70	0.15
$\delta_{\tau,s_t=1}$, tax rate resp. to debt	0.077	0.083	0.055	0.112	G	0.05	0.02
δ_y, tax rate resp. to output	0.000	0.078	0.000	0.177	G	0.10	0.10
			PM/AF				
$\rho_{R,S_t=2}$, lagged interest rate	0.869	0.856	0.819	0.896	B	0.50	0.15
$\psi_{1,S_t=2}$, interest rate resp. to inflation	0.982	0.904	0.810	0.990	G	0.80	0.15
$\psi_{2,St=2}$, interest rate resp. to output	0.581	0.583	0.288	0.938	G	0.50	0.25
$\rho_{\tau,s_t=2}$, lagged tax rate	0.763	0.725	0.610	0.846	B	0.70	0.15
$\delta_{\tau,s_t=2}$, tax rate resp. to debt	0.00	0.00	-	-	Fixed	-	-
δ_y, tax rate resp. to output	0.000	0.078	0.000	0.177	G	0.10	0.10

Table C.1: Rules-Based Policy. Under the Rules-Based policy, we have four alternative policy permutations: AM/PF, AM/AF, PM/PF and PM/AF. For monetary policy switches, $S_t = 1$ is the AM regime and $S_t = 2$ is the PM regime. For fiscal policy switches, $s_t = 1$ is the PF regime and $s_t = 2$ is the AF regime. δ_y is assumed to be time-invariant across regimes.

	Posterior				Prior		
Parameters	Mode	Mean	5%	95%	Type	Mean	Std Dev
Deep parameters							
σ, Inv. of intertemp. elas. of subst.	2.500	2.509	2.134	2.898	N	2.50	0.25
α, Calvo parameter	0.798	0.800	0.772	0.827	B	0.75	0.02
ζ, inflation inertia	0.387	0.339	0.206	0.458	B	0.50	0.10
θ, habit persistence	0.464	0.524	0.359	0.658	B	0.50	0.10
φ, Inverse of Frisch elasticity	2.00	2.00	-	-	Fixed	-	-
Serial correlation of exogenous processes							
ρ_ξ, AR coeff., taste shock	0.893	0.886	0.844	0.927	B	0.50	0.15
ρ_μ, AR coeff., cost-push shock	0.153	0.209	0.078	0.346	B	0.50	0.15
ρ_q, AR coeff., productivity shock	0.427	0.406	0.293	0.519	B	0.50	0.15
ρ_z, AR coeff., transfers	0.977	0.976	0.966	0.987	B	0.50	0.15
ρ_g, AR coeff., government spending	0.981	0.980	0.970	0.99	B	0.50	0.15
Standard deviations of exogenous processes							
$\sigma_{\xi,k_t=1}$, taste shock	0.555	0.532	0.375	0.684	IG	0.50	2.00
$\sigma_{\xi,k_t=2}$ taste shock	1.235	1.215	0.883	1.532	IG	2.00	2.00
$\sigma_{\mu,k_t=1,}$ cost-push shock	4.845	4.051	3.080	5.000	IG	0.50	2.00
$\sigma_{\mu,k_t=2,}$ cost-push shock	12.734	11.772	7.851	15.792	IG	2.00	2.00
$\sigma_{q,k_t=1,}$ productivity shock	0.510	0.572	0.479	0.660	IG	0.50	2.00
$\sigma_{q,k_t=2,}$ productivity shock	1.111	1.275	1.059	1.462	IG	2.00	2.00
σ_{tp}, term premium shock	3.258	3.293	2.996	3.570	IG	2.00	2.00
σ_g, government spending shock	0.246	0.249	0.229	0.269	IG	0.10	2.00
σ_z, transfers shock	0.300	0.303	0.279	0.327	IG	0.50	2.00
σ_τ, tax rate shock	0.359	0.361	0.330	0.393	IG	0.50	2.00
σ_R, interest rate shock	0.205	0.211	0.189	0.232	IG	0.50	2.00
Transition probabilities							
p_{11}, monetary policy: remaining active	0.972	0.971	0.955	0.989	B	0.95	0.02
p_{22}, monetary policy: remaining passive	0.933	0.915	0.877	0.956	B	0.95	0.02
q_{11}, fiscal policy: remaining passive	0.955	0.952	0.929	0.978	B	0.95	0.02
q_{22}, fiscal policy: remaining active	0.935	0.918	0.882	0.954	B	0.95	0.02
h_{11}, volatility: remaining with low volatility	0.958	0.951	0.926	0.977	B	0.95	0.02
h_{22}, volatility: remaining with high volatility	0.910	0.905	0.875	0.935	B	0.95	0.02

Table C.1: Rules-Based Policy (continued). For volatility, $k_t = 1$ is the low volatility regime and $k_t = 2$ is the high volatility regime.

C.1 Posterior Estimates: Rules-Based Policy

The posterior parameter estimates of the Rules-Based policy are reported in Table C.1. Our estimates of the structural parameters are broadly in line with other studies: an intertemporal elasticity of substitution, $\sigma = 2.5$; a measure of price stickiness, $\alpha = 0.8$, implying that price contracts typically last for just over one year; a degree of price indexation, $\zeta = 0.34$, and a significant estimate of the degree of habits, $\theta = 0.52$.

Under the Rules-Based policy, we have four alternative policy permutations: AM/PF, AM/AF, PM/PF and PM/AF. In order to allow for maximum flexibility in describing the policy regimes, we initially allowed for variations in rule parameters across the four policy regimes. Therefore, for example, the active monetary policy rule parameters in the AM/PF regime can differ from those in the AM/AF regime. Indeed, we find significant variations in the AM and PF regimes depending on which policy they are combined with. However, the PM and AF regimes appeared to be similar regardless of which policy they were paired with. Therefore, we restrict the PM and AF to be the same across their respective paired regimes. The resultant policy regimes imply that the passive monetary policy is inertial, $\rho_{R,S_t=2} = 0.86$, and only falling slightly short of the Taylor principle, $\psi_{1,S_t=2} = 0.9$, with a significant coefficient on output, $\psi_{2,S_t=2} = 0.58$. While an active monetary policy paired with a passive fiscal policy (AM/PF) is both inertial, $\rho_{R,S_t=1} = 0.88$, and very aggressive in targeting inflation, $\psi_{1,S_t=1} = 2.9$, with a relatively strong response to output, $\psi_{2,S_t=1} = 0.67$. When fiscal policy is active, then an associated active monetary policy is far less aggressive as interest rate inertia falls, $\rho_{R,S_t=1} = 0.61$, along with the response to inflation, $\psi_{1,S_t=1} = 1.48$, while the response to output increases, $\psi_{2,S_t=1} = 0.70$. Since the AM/AF regime is inherently unstable, it would appear that the conflict between the monetary and fiscal authority results in a moderation in the conservatism of monetary policy even while that policy remains active. Similarly, the passive fiscal policy is far more inertial, $\rho_{\tau,s_t=1} = 0.96$, and less responsive to debt, $\delta_{\tau,s_t=1} = 0.04$, when it is paired with an active monetary policy (AM/PF) than when the passive fiscal policy is paired with a passive monetary policy (PM/PF) where tax rate inertia falls, $\rho_{\tau,s_t=1} = 0.46$, and the response to debt rises, $\delta_{\tau,s_t=1} = 0.08$. These kinds of differences in estimation across regimes could reflect the nature of the interaction between monetary and fiscal policy. In the case of the AM/AF regime the policy is unstable and only rendered determinate because of spillovers from other policy permutations, so that the moderation in monetary policy would serve to mitigate the unstable debt dynamics caused by rising debt service costs under the active policy policy. Similarly, a passive fiscal policy which raises distortionary taxes to stabilize debt is likely to fuel inflation and lead to rising debt service costs when monetary policy is active. This is less of a danger when monetary policy is passive, so that fiscal policy can be relatively more aggressive in responding to debt in the latter case. These results suggest that the stance of one (or both) policy maker(s) is dependent on the policies of the other. This can be analyzed more formally by considering optimal policy where one policy maker takes into account the actions of the other.

C.2 Regime Switching Rules-Based Policy

Figure C.1 details the movements across fiscal and monetary policy regimes when the policy is described by Rules-Based policy. The first panel describes the probability of being in the passive fiscal policy regime, the second the active fiscal policy regime, and the third panel gives the probability of being in the passive monetary policy regime (with its complement being the active monetary regime). Taking these together, we observe that the conventional policy assignment (i.e. AM/PF) prevails right up until the late 1960s in contrast to the findings in Bianchi (2012) or Bianchi and Ilut (2017) who suggest that policy had already deviated from the textbook assignment by then. Fiscal policy then turns active in 1969, and monetary policy turns passive shortly afterwards. There is a brief attempt at disinflation in 1973, but we essentially stay in the PM regime until Volcker. Afterwards monetary policy stays active, and there are brief flirtations with passive fiscal policy around 1975 and 1981-1982, although none stick until 1992. Therefore, the AM/PF regime did not re-emerge until 1992. This result is consistent with Bianchi (2012) but different with Bianchi and Ilut (2017). Finally, we find a brief relaxation of monetary policy in the aftermath of the bursting of the dot com bubble around 2001, while fiscal policy remains passive.

Our estimates suggest that regimes that are determinate because of the expectations of returning to either the AM/PF or PM/AF regime actually describe observed policy configurations for much of our sample period. The AM/PF and PM/AF regimes are estimated to be in place for 60% and 12% of the sample period, respectively, while the PM/PF regime appears to be the least frequently observed regime which is only present for 2% of the time. This is consistent with Bianchi and Ilut (2017) in that the PM/PF regime does not appear to be a significant regime. The remaining 26% of the sample period is described by the AM/AF regime, which is inherently unstable in the absence of expectations that we would return to either the AM/PF or PM/AF regimes.

In short, the Rules-Based estimation is consistent with a narrative where fiscal policy ceases to act to stabilize debt in the 1970s, with monetary policy turning passive shortly afterwards. Monetary policy then actively targets inflation following the appointment of Paul Volcker, but fiscal policy does not decisively turn passive in support of that policy until the early 1990s. That the Rules-Based estimation would identify this pattern of regime change can easily be seen in the broad trends in inflation, interest rates and debt contained in Figure 1 in the paper. The PM/AF regime of the 1970s is associated with high inflation, the AM/AF regime of the 1980s with the tightening of monetary policy, falling inflation and rising debt, the AM/PF regime of the 1990s with the ongoing stabilization in inflation and the debt to GDP ratio. We shall see in the main text that the estimation based on optimal strategic policy allows for a more nuanced description of the evolution of policy regimes

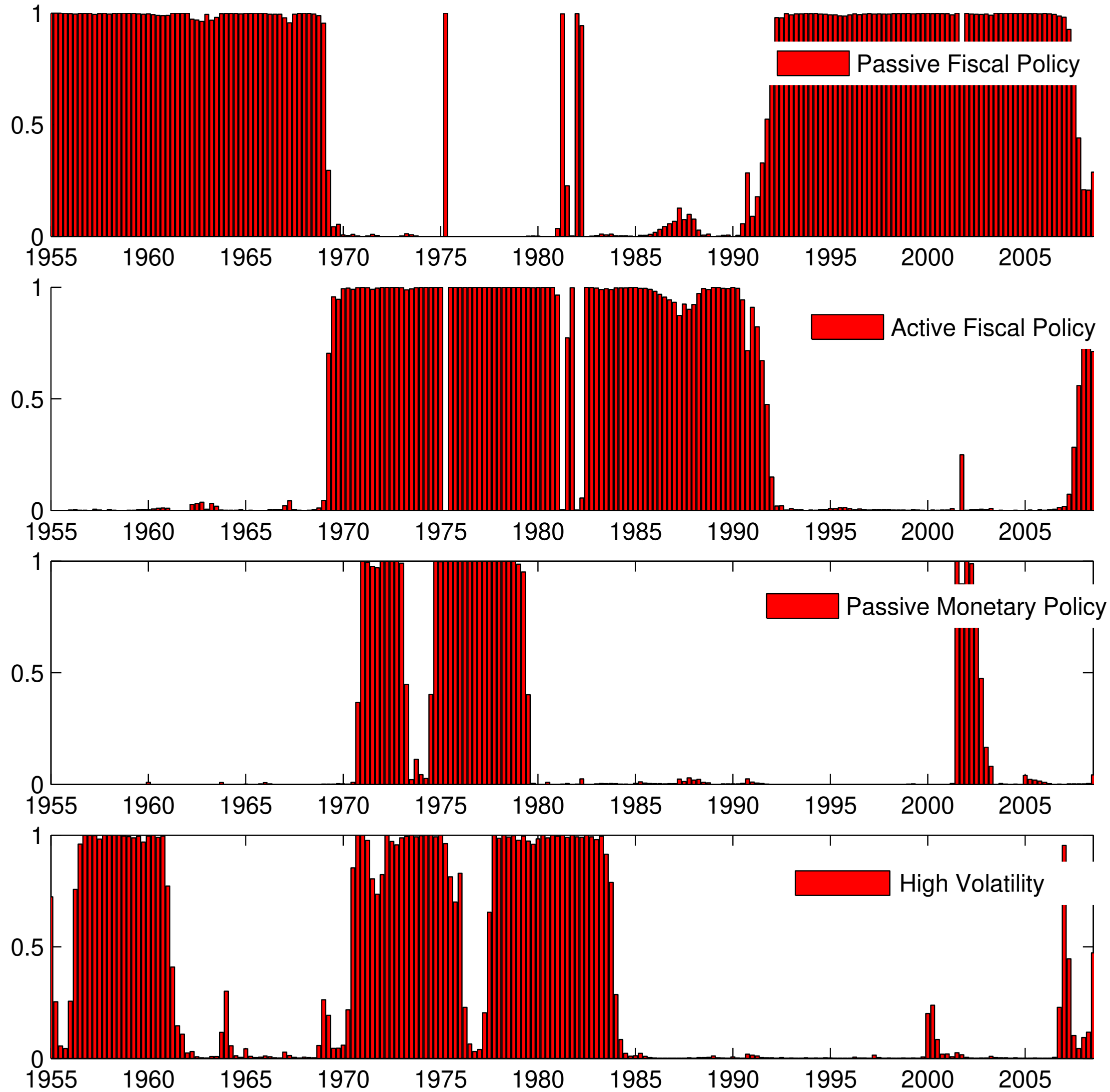

Figure C.1: Markov Switching Probabilities: Policy and Volatility Switches under Rules-Based Policy

C.3 Counterfactuals

We can see why the Rules-Based description of policy is often forced into identifying a starker combination of policy regimes than our preferred description of optimal policy by running a counterfactual for each sub-episode where the economy is hit by the same estimated shock processes in each sub-period, but the policy regime in place is fixed at each possible permutation of monetary and fiscal policy regime. In this way we can generate intuition as to why particular combinations of policy regime are identified as accounting for the observed movements in debt, inflation, output and policy instruments during these three episodes. Figures C.2 and C.3 plots the counterfactuals for the case of the Rules-Based description of policy, under passive and active fiscal rules, respectively. At the end of each episode all state variables a returned to their data values, and then are allowed to evolve as they would had that particular policy regime been in place through the remainder of that sub-period.

The first thing to note from Figures C.2 and C.3 are the outcomes for inflation across the different counterfactuals. When monetary policy is active inflation is typically closer to target and less volatile, while inflationary outcomes are less affected by the given fiscal regime. This means that the monetary policy regime is largely identified in line with inflationary outcomes - AM when inflation is low and relatively stable prior to the 1970s and following the Volcker disinflation, and is passive during the high inflation volatility of the 1970s. Differences in fiscal regime instead manifest themselves in marked differences in the trends in the debt to GDP ratio and movements in the tax rate, which in turn are affected by the monetary policy regime. Consider, for example, episode I in Figures C.2 and C.3, where the AM/PF regime implies a gradual reduction in debt without generating excessive inflation - this is therefore the data-preferred regime until the 1970s, where the higher inflation is explained by a switch to a passive monetary policy. As the estimated coefficients on the passive fiscal rule imply this rule is more aggressive when paired with a passive monetary policy, such a policy combination cannot explain the observed tax and debt data in the 1970s (see Figure C.3), such that the data prefers the combination of PM/AF.

The next episode, covering the Volcker disinflation and the Reagan and Bush budgets of the 1980s, is explained by the adoption of an active monetary policy which stabilizes inflation, while the failure to adopt a passive fiscal policy is consistent with the pace of the increase in government debt over this period. An earlier switch to a passive fiscal policy would have supported a more rapid disinflation under Volcker than was observed in the data, as distortionary taxes would have been cut to return the low levels of debt to their steady-state level. Finally, trends under the Clinton presidency are explained by the textbook combination of an active monetary policy and passive fiscal policy. This allows the gradual stabilization of debt without generating inflation, while the Presidency of George Bush II implied a switch to an active fiscal policy which ended the debt reduction observed under Clinton. During this sub-period, the stable inflation is consistent with the AM regime, and the movements in debt and taxation determine the choice of AF or PF regime in estimation.

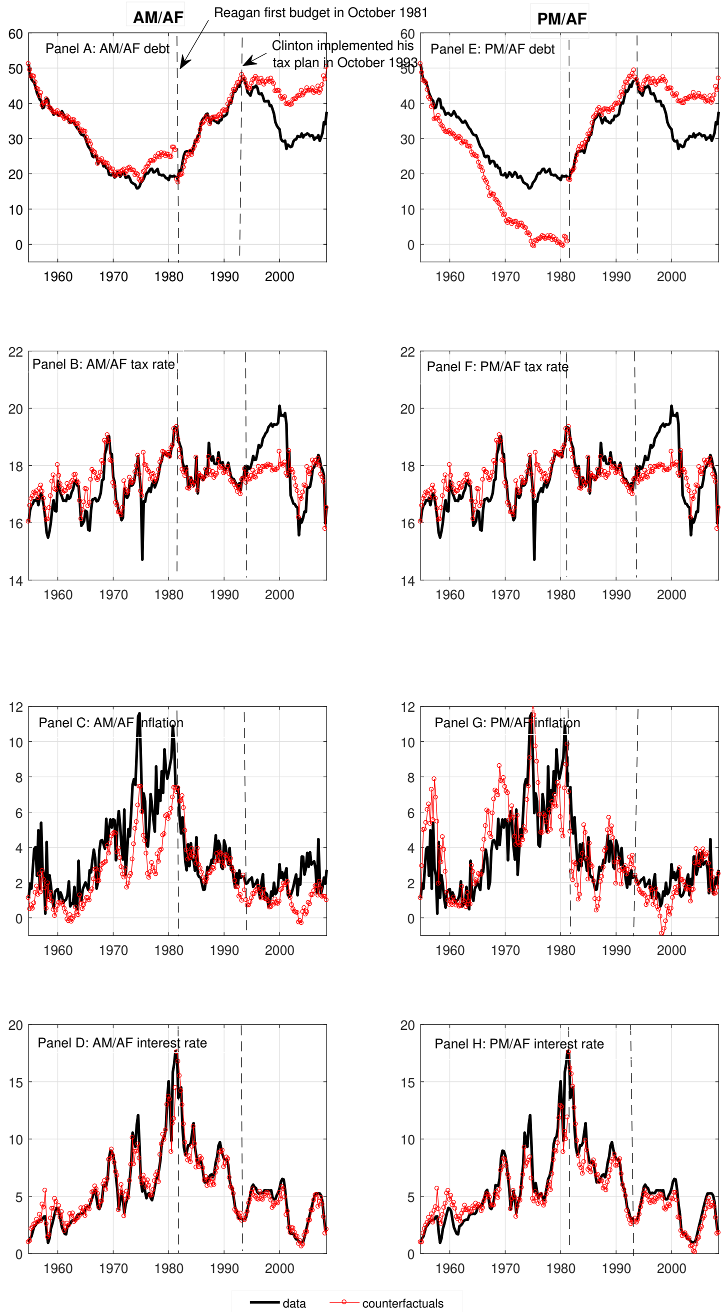

Figure C.2: Active Fiscal Policy Counterfactual—Rules-Based. All state variables are re-based to data values at the start of each episode I-III. The solid black line is the data.

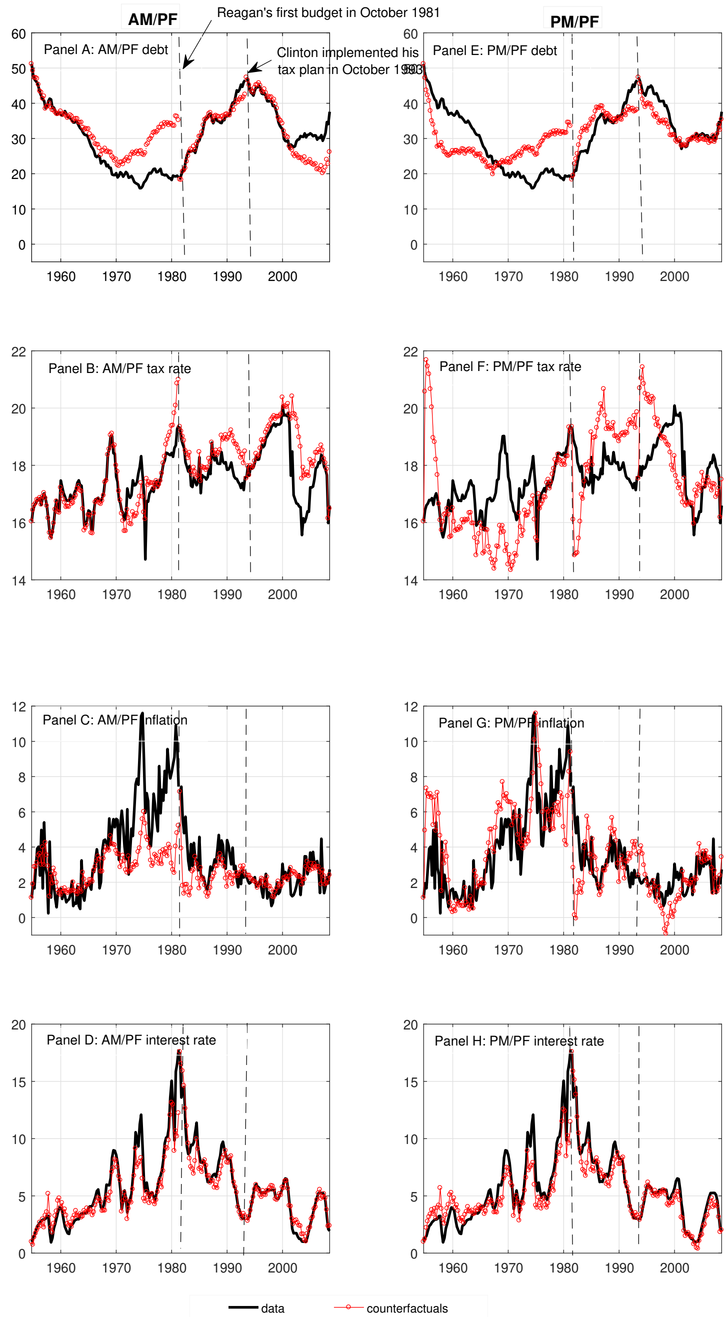

Figure C.3: Passive Fiscal Policy Counterfactual—Rules-Based. All state variables are re-based to data values at the start of each episode I-III. The solid black line is the data.

D Leadership Equilibria Under Discretion

This section demonstrates how to solve non-cooperative dynamic games in the Markov jump-linear quadratic systems. Consider an economy with two policy makers: a leader (L) and a follower (F).

$$X_{t+1} = A_{11k_{t+1}}X_t + A_{12k_{t+1}}x_t + B_{11k_{t+1}}u_t^L + B_{12k_{t+1}}u_t^F + C_{k_{t+1}}\varepsilon_{t+1}, \quad \text{(D.1)}$$
$$E_tH_{k_{t+1}}x_{t+1} = A_{21j_t}X_t + A_{22j_t}x_t + B_{21j_t}u_t^L + B_{22j_t}u_t^F. \quad \text{(D.2)}$$

where X_t is a n_1 vector of predetermined variables; x_t is a n_2 vector of forward-looking variables; u_t^L and u_t^F are the control variables, and ε_t contains a vector of zero mean $i.i.d.$ shocks. Without loss of generality, the shocks are normalized so that the covariance matrix of ε_t is an identity matrix, and the covariance matrix of the shocks to X_{t+1} is $C'_{k_{t+1}}C_{k_{t+1}}$.

The matrices $A_{11k_{t+1}}$, $A_{12k_{t+1}}$, $H_{k_{t+1}}$, $B_{11k_{t+1}}$, $B_{12k_{t+1}}$, A_{21j_t}, A_{22j_t}, B_{21j_t}, and B_{22j_t} can each take n different values, corresponding to the n modes $k_{t+1} = 1, 2, ..., n$ in period $t+1$, and $j_t = 1, 2, ..., n$ in period t. The modes follow a Markov process with constant transition probabilities:

$$P_{jk} = Pr\left\{k_{t+1} = k | j_t = j\right\}, \;\; j, k = 1, 2, ..., n$$

Let P denote the $n \times n$ transition matrix $[P_{jk}]$ and the $1 \times n$ vector $p \equiv (p_{1_t}, ..., p_{n_t})$ denote the probability distribution of the modes in period t,

$$p_{t+1} = p_tP.$$

Finally, the $1 \times n$ vector $\overline{p}$ denotes the unique stationary distribution of the modes,

$$\overline{p} = \overline{p}P.$$

We assume that the intertemporal loss functions of the two policy makers are defined by the quadratic loss function

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \frac{1}{2}\beta^{\tau} L^u_{j_{t+\tau}},$$

where $L^u_{j_t}$ is the period loss with $u = F$ for the follower and $u = L$ for the leader, respectively. The period loss, $L^u_{j_t}$, can take different value corresponding to the n modes in period t. The period loss satisfies

$$L^u_{j_t} = Y^{u\prime}_t \Lambda^u_{j_t} Y^u_t,$$

where $\Lambda^u_{j_t}$ is a symmetric and positive semi-definite weight matrix. Y^u_t are n_Y vectors of target variables for the follower and leader.

$$Y^u_t = D^u \begin{bmatrix} X_t \\ x_t \\ u^L_t \\ u^F_t \end{bmatrix}.$$

It follows that the period loss function can be rewritten as

$$L_{jt}^{u} = \begin{bmatrix} X_t \\ x_t \\ u_t^L \\ u_t^F \end{bmatrix}' W_{jt}^{u} \begin{bmatrix} X_t \\ x_t \\ u_t^L \\ u_t^F \end{bmatrix}, \tag{D.3}$$

where $W_{jt}^{u} = D^{u\prime}\Lambda_{jt}^{u} D^{u}$ is symmetric and positive semidefinite, and

$$W_{jt}^{u} = \begin{bmatrix} Q_{11j_t}^{u} & Q_{12j_t}^{u} & P_{11j_t}^{u} & P_{12j_t}^{u} \\ Q_{21j_t}^{u} & Q_{22j_t}^{u} & P_{21j_t}^{u} & P_{22j_t}^{u} \\ P_{11j_t}^{u\prime} & P_{21j_t}^{u\prime} & R_{11j_t}^{u} & R_{12j_t}^{u} \\ P_{12j_t}^{u\prime} & P_{22j_t}^{u\prime} & R_{12j_t}^{u\prime} & R_{22j_t}^{u} \end{bmatrix}$$

is partitioned with X_t, x_t, $u_t^L$and u_t^F.

The follower and leader decide their policy u_t^F and u_t^L in period t to minimize their intertemporal loss functions defined in (D.3) under discretion subject to (D.1), (D.2), X_t and j_t given. The follower also observes the current decision u_t^L of the leader. Furthermore, two policy makers anticipate that they will reoptimize in period $t+1$. Reoptimization will result in the two instruments and the forward-looking variables in period $t+1$ being functions of the predetermined variables and the mode in period $t+1$ according to

$$u_{t+1}^{L} = -F_{k_{t+1}}^{L} X_{t+1}, \tag{D.4}$$

$$u_{t+1}^{F} = -G_{k_{t+1}}^{F} X_{t+1} - D_{k_{t+1}}^{F} u_{t+1}^{L}, \tag{D.5}$$

$$x_{t+1} = -N_{k_{t+1}} X_{t+1}, \tag{D.6}$$

where $k_{t+1} = 1, ..., n$ are the n modes at period $t+1$. The dynamics of the predetermined variables will follow

$$X_{t+1} = M_{j_t k_{t+1}} X_t + C_{k_{t+1}} \varepsilon_{t+1},$$

where

$$M_{j_t k_{t+1}} = A_{11k_{t+1}} - A_{12k_{t+1}} N_{j_t} - B_{11k_{t+1}} F_{j_t}^{L} - B_{12k_{t+1}} G_{j_t}^{F} + B_{12k_{t+1}} D_{j_t}^{F} F_{j_t}^{L},$$

First, by (D.6) and (D.1) we have,

$$\begin{aligned} E_t H_{k_{t+1}} x_{t+1} &= -E_t H_{k_{t+1}} N_{k_{t+1}} X_{t+1} \\ &= -E_t H_{k_{t+1}} N_{k_{t+1}} \left(A_{11k_{t+1}} X_t + A_{12k_{t+1}} x_t + B_{11k_{t+1}} u_t^L + B_{12k_{t+1}} u_t^F \right) \end{aligned}$$

where $E_t H_{k_{t+1}} N_{k_{t+1}} = \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}}$, conditional on $j_t = 1, 2, ...n$ at the period. Combining this with (D.2) gives

$$\begin{aligned} & -E_t H_{k_{t+1}} N_{k_{t+1}} \left(A_{11k_{t+1}} X_t + A_{12k_{t+1}} x_t + B_{11k_{t+1}} u_t^L + B_{12k_{t+1}} u_t^F \right) \\ = \; & A_{21j_t} X_t + A_{22j_t} x_t + B_{21j_t} u_t^L + B_{22j_t} u_t^F. \end{aligned}$$

Solving for x_t we obtain

$$x_t = -J_{j_t} X_t - K^L_{j_t} u^L_t - K^F_{j_t} u^F_t, \tag{D.7}$$

where

$$\begin{aligned}
J_{j_t} &= \left(A_{22j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}} \right)^{-1} \left(A_{21j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{11k_{t+1}} \right), \\
K^L_{j_t} &= \left(A_{22j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}} \right)^{-1} \left(B_{21j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} B_{11k_{t+1}} \right), \\
K^F_{j_t} &= \left(A_{22j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}} \right)^{-1} \left(B_{22j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} B_{12k_{t+1}} \right).
\end{aligned}$$

We assume that $A_{22j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}}$ is invertible.

Second, substituting x_t from (D.1) using (D.7) gives

$$X_{t+1} = \widetilde{A}_{j_t k_{t+1}} X_t + \widetilde{B}^L_{j_t k_{t+1}} u^L_t + \widetilde{B}^F_{j_t k_{t+1}} u^F_t + C_{k_{t+1}} \varepsilon_{t+1}, \tag{D.8}$$

where

$$\begin{aligned}
\widetilde{A}_{j_t k_{t+1}} &= A_{11k_{t+1}} - A_{12k_{t+1}} J_{j_t}, \\
\widetilde{B}^L_{j_t k_{t+1}} &= B_{11k_{t+1}} - A_{12k_{t+1}} K^L_{j_t}, \\
\widetilde{B}^F_{j_t k_{t+1}} &= B_{12k_{t+1}} - A_{12k_{t+1}} K^F_{j_t}.
\end{aligned}$$

D.1 Policy of the Follower

Using (D.7) in the follower's loss function (D.3) gives

$$\begin{aligned}
L^F_{j_t} &= \begin{bmatrix} X_t \\ x_t \\ u^L_t \\ u^F_t \end{bmatrix}' \begin{bmatrix} Q^F_{11j_t} & Q^F_{12j_t} & P^F_{11j_t} & P^F_{12j_t} \\ Q^F_{21j_t} & Q^F_{22j_t} & P^F_{21j_t} & P^F_{22j_t} \\ P^{F\prime}_{11j_t} & P^{F\prime}_{21j_t} & R^F_{11j_t} & R^F_{12j_t} \\ P^{F\prime}_{12j_t} & P^{F\prime}_{22j_t} & R^{F\prime}_{12j_t} & R^F_{22j_t} \end{bmatrix} \begin{bmatrix} X_t \\ x_t \\ u^L_t \\ u^F_t \end{bmatrix} \\
&= \begin{bmatrix} X_t \\ u^L_t \\ u^F_t \end{bmatrix}' \begin{bmatrix} \widetilde{Q}^F_{j_t} & \widetilde{P}^F_{1j_t} & \widetilde{P}^F_{2j_t} \\ \widetilde{P}^{F\prime}_{1j_t} & \widetilde{R}^F_{11j_t} & \widetilde{R}^F_{12j_t} \\ \widetilde{P}^{F\prime}_{2j_t} & \widetilde{R}^{F\prime}_{12j_t} & \widetilde{R}^F_{22j_t} \end{bmatrix} \begin{bmatrix} X_t \\ u^L_t \\ u^F_t \end{bmatrix}
\end{aligned} \tag{D.9}$$

where

$$\begin{aligned}
\widetilde{Q}^F_{j_t} &= Q^F_{11j_t} - Q^F_{12j_t}J_{j_t} - J'_{j_t}Q^F_{21j_t} + J'_{j_t}Q^F_{22j_t}J_{j_t},\\
\widetilde{P}^F_{1j_t} &= P^F_{11j_t} - Q^F_{12j_t}K^L_{j_t} + J'_{j_t}Q^F_{22j_t}K^L_{j_t} - J'_{j_t}P^F_{21j_t},\\
\widetilde{P}^F_{2j_t} &= P^F_{12j_t} - Q^F_{12j_t}K^F_{j_t} + J'_{j_t}Q^F_{22j_t}K^F_{j_t} - J'_{j_t}P^F_{22j_t},\\
\widetilde{R}^F_{11j_t} &= K^{L\prime}_{j_t}Q^F_{22j_t}K^L_{j_t} - K^{L\prime}_{j_t}P^F_{21j_t} - P^{F\prime}_{21j_t}K^L_{j_t} + R^F_{11j_t},\\
\widetilde{R}^F_{12j_t} &= K^{L\prime}_{j_t}Q^F_{22j_t}K^F_{j_t} - K^{L\prime}_{j_t}P^F_{22j_t} - P^{F\prime}_{21j_t}K^F_{j_t} + R^F_{12j_t},\\
\widetilde{R}^F_{22j_t} &= K^{F\prime}_{j_t}Q^F_{22j_t}K^F_{j_t} - K^{F\prime}_{j_t}P^F_{22j_t} - P^{F\prime}_{22j_t}K^F_{j_t} + R^F_{22j_t}.
\end{aligned}$$

The optimal value of the problem in period t is associated with the symmetric positive semidefinite matrix $V^F_{k_{t+1}}$ and it satisfies the Bellman equation:

$$X_t V^F_{j_t} X_t = \min_{u^F_{j_t}} \left\{ L^F_{j_t} + \beta E_t \left[X'_{t+1} V^F_{k_{t+1}} X_{t+1} \right] \right\} \tag{D.10}$$

subject to (D.8) and (D.9). The first-order condition with respect to u^F_t is

$$\begin{aligned}
0 \;=\;& X'_t \widetilde{P}^F_{2j_t} + u^{L\prime}_t \widetilde{R}^F_{12j_t} + u^{F\prime}_t \widetilde{R}^F_{22j_t} + \beta E_t X'_t \widetilde{A}'_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}}\\
&+\beta E_t u^{L\prime}_t \widetilde{B}^{L\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}} + \beta E_t u^{F\prime}_t \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}}.
\end{aligned}$$

This leads to the optimal policy function u^F_t of the follower

$$u^F_t = -G^F_{k_{t+1}} X_{t+1} - D^F_{k_{t+1}} u^L_{t+1}, \tag{D.11}$$

where

$$\begin{aligned}
G^F_{k_{t+1}} &= \left(\widetilde{R}^F_{22j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}} \right)^{-1} \left(\widetilde{P}^{F\prime}_{2j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{A}_{j_t k_{t+1}} \right),\\
D^F_{k_{t+1}} &= \left(\widetilde{R}^F_{22j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}} \right)^{-1} \left(\widetilde{R}^{F\prime}_{12j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k+1} \widetilde{B}^L_{j_t k_{t+1}} \right).
\end{aligned}$$

Furthermore, using (D.4) and (D.11) in (D.7) gives

$$x_t = -N_{j_t} X_t, \tag{D.12}$$

where

$$N_{j_t} = J_{j_t} - K^L_{j_t} F^L_{j_t} - K^F_{j_t} G^F_{j_t} + K^F_{j_t} D^F_{j_t} F^L_{j_t},$$

and using (D.4) and (D.11) and (D.12) in (D.1) gives

$$X_{t+1} = M_{j_t k_{t+1}} X_t + C_{k_{t+1}} \varepsilon_{t+1},$$

where

$$M_{j_t k_{t+1}} = A_{11k_{t+1}} - A_{12k_{t+1}} N_{j_t} - B_{11k_{t+1}} F^L_{j_t} - B_{12k_{t+1}} G^F_{j_t} + B_{12k_{t+1}} D^F_{j_t} F^L_{j_t}.$$

Finally, using (D.4), (D.8), (D.9) and (D.11) in (D.10) results in

$$\begin{aligned}
V^F_{j_t} \equiv\ & \widetilde{Q}^F_{j_t} - \widetilde{P}^F_{1j_t} F^L_{j_t} - F^{L\prime}_{j_t} \widetilde{P}^{F\prime}_{1j_t} + F^{L\prime}_{j_t} \widetilde{R}^F_{11j_t} F^L_{j_t} \\
& + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right)' V^F_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right) \\
& - \left[\widetilde{P}^F_{2j_t} - \widetilde{R}^F_{12j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} + \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right)' V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}} \right] \\
& \left(\widetilde{R}^{F\prime}_{22j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}} \right)^{-1} \\
& \left[\widetilde{P}^{F\prime}_{2j_t} - \widetilde{R}^{F\prime}_{12j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} + \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right) \right],
\end{aligned}$$

D.2 Policy of the Leader

Using (D.7) and (D.11) in the leader's loss function (D.3) gives

$$\begin{aligned}
L^L_{j_t} &= \begin{bmatrix} X_t \\ x_t \\ u^L_t \\ u^F_t \end{bmatrix}' \begin{bmatrix} Q^L_{11j_t} & Q^L_{12j_t} & P^L_{11j_t} & P^L_{12j_t} \\ Q^L_{21j_t} & Q^L_{22j_t} & P^L_{21j_t} & P^L_{22j_t} \\ P^{L\prime}_{11j_t} & P^{L\prime}_{21j_t} & R^L_{11j_t} & R^L_{12j_t} \\ P^{L\prime}_{12j_t} & P^{L\prime}_{22j_t} & R^{L\prime}_{12j_t} & R^L_{22j_t} \end{bmatrix} \begin{bmatrix} X_t \\ x_t \\ u^L_t \\ u^F_t \end{bmatrix} \\
&= \begin{bmatrix} X_t \\ u^L_t \end{bmatrix}' \begin{bmatrix} \widetilde{Q}^L_{j_t} & \widetilde{P}^L_{j_t} \\ \widetilde{P}^{L\prime}_{j_t} & \widetilde{R}^L_{j_t} \end{bmatrix} \begin{bmatrix} X_t \\ u^L_t \end{bmatrix}, \qquad \text{(D.13)}
\end{aligned}$$

where

$$\begin{aligned}
\widetilde{Q}^L_{j_t} &= Q^L_{11j_t} - P^L_{12j_t} G^F_{j_t} - G^{F\prime}_{j_t} P^{L\prime}_{12j_t} + G^{F\prime}_{j_t} R^L_{22j_t} G^F_{j_t} - Q^L_{12j_t} \widetilde{J}_{j_t} \\
&\quad - \widetilde{J}'_{j_t} Q^L_{21j_t} + \widetilde{J}'_{j_t} Q^L_{22j_t} \widetilde{J}_{j_t} + \widetilde{J}'_{j_t} P^L_{22j_t} G^F_{j_t} + G^{F\prime}_{j_t} P^{L\prime}_{22j_t} \widetilde{J}_{j_t}, \\
\widetilde{P}^L_{j_t} &= P^L_{11j_t} - Q^L_{12j_t} \widetilde{K}_{j_t} - P^L_{12j_t} D^F_{j_t} + \widetilde{J}'_{j_t} Q^L_{22j_t} \widetilde{K}_{j_t} - \widetilde{J}'_{j_t} P^L_{21j_t} \\
&\quad + \widetilde{J}'_{j_t} P^L_{22j_t} D^F_{j_t} + G^{F\prime}_{j_t} P^{L\prime}_{22j_t} \widetilde{K}_{j_t} - G^{F\prime}_{j_t} R^{L\prime}_{12j_t} + G^{F\prime}_{j_t} R^L_{22j_t} D^F_{j_t}, \\
\widetilde{R}^L_{j_t} &= R^L_{11j_t} + \widetilde{K}'_{j_t} Q^L_{22j_t} \widetilde{K}_{j_t} - R^L_{12j_t} D^F_{j_t} - D^{F\prime}_{j_t} R^{L\prime}_{12j_t} + D^{F\prime}_{j_t} R^L_{22j_t} D^F_{j_t} \\
&\quad - \widetilde{K}'_{j_t} P^L_{21j_t} + \widetilde{K}'_{j_t} P^L_{22j_t} D^F_{j_t} - P^{L\prime}_{21j_t} \widetilde{K}_{j_t} + D^{F\prime}_{j_t} P^{L\prime}_{22j_t} \widetilde{K}_{j_t}.
\end{aligned}$$

and $\widetilde{J}_{j_t} = \left(J_{j_t} - K^F_{j_t} G^F_{j_t} \right)$ and $\widetilde{K}_{j_t} = \left(K^L_{j_t} - K^F_{j_t} D^F_{j_t} \right)$

The value of the problem in period t is associated with the symmetric positive semidefinite matrix $V^L_{k_{t+1}}$ and it satisfies the Bellman equation

$$X_t V^L_{j_t} X_t = \min_{u^L_{j_t}} \left\{ L^L_{j_t} + \beta E_t \left[X'_{t+1} V^L_{k_{t+1}} X_{t+1} \right] \right\}, \qquad \text{(D.14)}$$

subject to (D.8), (D.11) and (D.13) . The first-order condition with respect to u_t^L is

$$\begin{aligned} 0 &= X_t'\widetilde{P}_{j_t}^L + u_t^{L\prime}\widetilde{R}_{j_t}^L + \beta E_t X_t'\left(\widetilde{A}_{j_tk_{t+1}} - \widetilde{B}_{j_tk_{t+1}}^F G_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right) \\ &\quad + \beta E_t u_t^{L\prime}\left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right). \end{aligned}$$

This leads to the optimal policy function of the leader

$$u_t^L = -F_j^L X_t, \tag{D.15}$$

where

$$\begin{aligned} F_j^L &= \left[\widetilde{R}_{j_t}^L + \beta \sum_{k=1}^n P_{j_tk_{t+1}} \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)\right]^{-1} \\ &\quad \left[\widetilde{P}_{j_t}^{L\prime} + \beta \sum_{k=1}^n P_{j_tk_{t+1}} \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{A}_{j_tk_{t+1}} - \widetilde{B}_{j_tk_{t+1}}^F G_{j_t}^F\right)\right]. \end{aligned}$$

Furthermore, using (D.11) and (D.15) in (D.7) gives

$$x_t = -N_{j_t} X_t, \tag{D.16}$$

where

$$N_{j_t} = J_{j_t} - K_{j_t}^L F_{j_t}^L - K_{j_t}^F G_{j_t}^F + K_{j_t}^F D_{j_t}^F F_{j_t}^L,$$

and using (D.11), (D.15) and (D.16) in (D.1) gives

$$X_{t+1} = M_{j_tk_{t+1}} X_t + C_{k_{t+1}} \varepsilon_{t+1},$$

where

$$M_{j_tk_{t+1}} = A_{11k_{t+1}} - A_{12k_{t+1}} N_{j_t} - B_{11k_{t+1}} F_{j_t}^L - B_{12k_{t+1}} G_{j_t}^F + B_{12k_{t+1}} D_{j_t}^F F_{j_t}^L$$

Finally, using (D.8), (D.11), (D.13) and (D.15) in (D.14) results in

$$\begin{aligned} V_{j_t}^L &= \widetilde{Q}_{j_t}^L + \beta \sum_{k=1}^n P_{j_tk_{t+1}} \left(\widetilde{A}_{j_tk_{t+1}} - \widetilde{B}_{j_tk_{t+1}}^F G_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{A}_{j_tk_{t+1}} - \widetilde{B}_{j_tk_{t+1}}^F G_{j_t}^F\right) \\ &- \left[\widetilde{P}_{j_t}^L + \beta \sum_{k=1}^n P_{j_tk_{t+1}} \left(\widetilde{A}_{j_tk_{t+1}} - \widetilde{B}_{j_tk_{t+1}}^F G_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)\right] \\ &\left[\widetilde{R}_{j_t}^L + \beta \sum_{k=1}^n P_{j_tk_{t+1}} \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)\right]^{-1} \\ &\left[\widetilde{P}_{j_t}^{L\prime} + \beta \sum_{k=1}^n P_{j_tk_{t+1}} \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{A}_{j_tk_{t+1}} - \widetilde{B}_{j_tk_{t+1}}^F G_{j_t}^F\right)\right] \end{aligned}$$

To sum up, the first order conditions to the optimization problem (D.1), (D.2) and (D.3) can be written in the following form:

$$N_{j_t} = J_{j_t} - K^L_{j_t} F^L_{j_t} - K^F_{j_t} G^F_{j_t} + K^F_{j_t} D^F_{j_t} F^L_{j_t},$$

$$\begin{aligned}
V^F_{j_t} \equiv\ & \widetilde{Q}^F_{j_t} - \widetilde{P}^F_{1j_t} F^L_{j_t} - F^{L\prime}_{j_t} \widetilde{P}^{F\prime}_{1j_t} + F^{L\prime}_{j_t} \widetilde{R}^F_{11j_t} F^L_{j_t} \\
& + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right)' V^F_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right) \\
& - \left[\widetilde{P}^F_{2j_t} - \widetilde{R}^F_{12j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} + \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right)' V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}} \right] \\
& \left(\widetilde{R}^{F\prime}_{22j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}} \right)^{-1} \\
& \left[\widetilde{P}^{F\prime}_{2j_t} - \widetilde{R}^{F\prime}_{12j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} + \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right) \right],
\end{aligned}$$

$$\begin{aligned}
V^L_{j_t} = \widetilde{Q}^L_{j_t} & + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} G^F_{j_t} \right)' V^L_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} G^F_{j_t} \right) \\
& - \left[\widetilde{P}^L_{j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} G^F_{j_t} \right)' V^L_{k_{t+1}} \left(\widetilde{B}^L_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} D^F_{j_t} \right) \right] \\
& \left[\widetilde{R}^L_{j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{B}^L_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} D^F_{j_t} \right)' V^L_{k_{t+1}} \left(\widetilde{B}^L_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} D^F_{j_t} \right) \right]^{-1} \\
& \left[\widetilde{P}^{L\prime}_{j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{B}^L_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} D^F_{j_t} \right)' V^L_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} G^F_{j_t} \right) \right]
\end{aligned}$$

$$\begin{aligned}
F^L_j = & \left[\widetilde{R}^{L\prime}_{j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{B}^L_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} D^F_{j_t} \right)' V^L_{k_{t+1}} \left(\widetilde{B}^L_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} D^F_{j_t} \right) \right]^{-1} \\
& \left[\widetilde{P}^{L\prime}_{j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{B}^L_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} D^F_{j_t} \right)' V^L_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} G^F_{j_t} \right) \right],
\end{aligned}$$

$$\begin{aligned}
G^F_{k_{t+1}} &= \left(\widetilde{R}^{F\prime}_{22j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}} \right)^{-1} \left(\widetilde{P}^{F\prime}_{2j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{A}_{j_t k_{t+1}} \right), \\
D^F_{k_{t+1}} &= \left(\widetilde{R}^{F\prime}_{22j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}} \right)^{-1} \left(\widetilde{R}^{F\prime}_{12j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k+1} \widetilde{B}^L_{j_t k_{t+1}} \right).
\end{aligned}$$

The discretion equilibrium is a fixed point $\left(N, V^L, V^F\right) \equiv \left\{N_{j_t}, V^L_{j_t}, V^F_{j_t}\right\}^n_{j_t=1}$ of the mapping and a corresponding $\left(F^L, G^F, D^F\right) \equiv \left\{F^L_{j_t}, G^F_{j_t}, D^F_{j_t}\right\}^n_{j_t=1}$. The fixed point can be obtained as the limit of $\left(N_t, V^L_t, V^F_t\right)$ when $t \to -\infty$.

E Nash Equilibrium under Discretion

Consider an economy with two policy makers, A and B, who decide their policy simultaneously.

$$
\begin{aligned}
X_{t+1} &= A_{11k_{t+1}} X_t + A_{12k_{t+1}} x_t + B_{11k_{t+1}} u^A_t + B_{12k_{t+1}} u^B_t + C_{k_{t+1}} \varepsilon_{t+1}, &\text{(E.1)}\\
E_t H_{k_{t+1}} x_{t+1} &= A_{21j_t} X_t + A_{22j_t} x_t + B_{21j_t} u^A_t + B_{22j_t} u^B_t, &\text{(E.2)}
\end{aligned}
$$

where X_t is a n_1 vector of predetermined variables; x_t is a n_2 vector of forward-looking variables; u^A_t and u^B_t are the two policy makers' instruments, and ε_t contains a vector of zero mean $i.i.d.$ shocks. Without loss of generality, the shocks are normalized so that the covariance matrix of ε_t is an identity matrix, and the covariance matrix of the shocks to X_{t+1} is $C'_{j_t} C_{j_t}$.

The period loss function of policy makers, A and B, is defined as in equation (D.3) with $u = A$ and $u = B$, respectively. Policy makers A and B simultaneously decide their policy u^A_t and u^B_t in period t to minimize their intertemporal loss functions defined in (D.3) under discretion subject to (E.1), (D.2), X_t and j_t given. Reoptimization in period $t+1$ result in the two instruments and the forward-looking variables being functions of the predetermined variables and the mode as follows

$$
\begin{aligned}
u^A_{t+1} &= -F^A_{k_{t+1}} X_{t+1}, &\text{(E.3)}\\
u^B_{t+1} &= -F^B_{k_{t+1}} X_{t+1}, &\text{(E.4)}\\
x_{t+1} &= -N_{k_{t+1}} X_{t+1}. &\text{(E.5)}
\end{aligned}
$$

Combining equations (E.1), (E.2) and (E.5), we solve for x_t

$$
x_t = -J_t X_t - K^A_{j_t} u^A_t - K^B_{j_t} u^B_t, \qquad \text{(E.6)}
$$

where

$$
\begin{aligned}
J_{j_t} &= \left(A_{22j_t} + \sum_{k=1}^n P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}}\right)^{-1} \left(A_{21j} + \sum_{k=1}^n P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{11k_{t+1}}\right),\\
K^A_{j_t} &= \left(A_{22j_t} + \sum_{k=1}^n P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}}\right)^{-1} \left(B_{21j} + \sum_{k=1}^n P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} B_{11k_{t+1}}\right),\\
K^B_{j_t} &= \left(A_{22j_t} + \sum_{k=1}^n P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}}\right)^{-1} \left(B_{22j} + \sum_{k=1}^n P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} B_{12k_{t+1}}\right).
\end{aligned}
$$

By substituting x_t from (E.1) using (E.6) gives

$$X_{t+1} = \widetilde{A}_{j_t k_{t+1}} X_t + \widetilde{B}^A_{j_t k_{t+1}} u^A_t + \widetilde{B}^B_{j_t k_{t+1}} u^B_t + C_{k_{t+1}} \varepsilon_{t+1}, \tag{E.7}$$

where

$$\begin{aligned}
\widetilde{A}_{j_t k_{t+1}} &= A_{11k_{t+1}} - A_{12k_{t+1}} J_{j_t}, \\
\widetilde{B}^A_{j_t k_{t+1}} &= B_{11k_{t+1}} - A_{12k_{t+1}} K^A_{j_t}, \\
\widetilde{B}^B_{j_t k_{t+1}} &= B_{12k_{t+1}} - A_{12k_{t+1}} K^B_{j_t}.
\end{aligned}$$

E.1 Policy Maker A

Substitute (E.4) and (E.6) in the policy maker A's period loss function gives

$$\begin{aligned}
L^A_{j_t} &= \begin{bmatrix} X_t \\ x_t \\ u^A_t \\ u^B_t \end{bmatrix}' \begin{bmatrix} Q^A_{11j_t} & Q^A_{12j_t} & P^A_{11j_t} & P^A_{12j_t} \\ Q^A_{21j_t} & Q^A_{22j_t} & P^A_{21j_t} & P^A_{22j_t} \\ P^{A\prime}_{11j_t} & P^{A\prime}_{21j_t} & R^A_{11j_t} & R^A_{12j_t} \\ P^{A\prime}_{12j_t} & P^{A\prime}_{22j_t} & R^{A\prime}_{12j_t} & R^A_{22j_t} \end{bmatrix} \begin{bmatrix} X_t \\ x_t \\ u^A_t \\ u^B_t \end{bmatrix} \\
&= \begin{bmatrix} X_t \\ u^A_t \end{bmatrix}' \begin{bmatrix} \widetilde{Q}^A_{j_t} & \widetilde{P}^A_{j_t} \\ \widetilde{P}^{A\prime}_{j_t} & \widetilde{R}^A_{j_t} \end{bmatrix} \begin{bmatrix} X_t \\ u^A_t \end{bmatrix}
\end{aligned} \tag{E.8}$$

where

$$\begin{aligned}
\widetilde{Q}^A_{j_t} &= Q^A_{11j_t} - Q^A_{12j_t} \widetilde{J}^B_{j_t} - \widetilde{J}^{B\prime}_{j_t} Q^A_{21j_t} + \widetilde{J}^{B\prime}_{j_t} Q_{22} \widetilde{J}^B_{j_t} \\
&\quad + F^{B\prime}_{j_t} R^A_{22j_t} F^B_{j_t} + F^{B\prime}_{j_t} P^{A\prime}_{22j_t} \widetilde{J}^B_{j_t} + \widetilde{J}^{B\prime}_{j_t} P^A_{22j_t} F^B_{j_t} \\
&\quad - P^A_{12j_t} F^B_{j_t} - F^{B\prime}_{j_t} P^{A\prime}_{12j_t} \\
\widetilde{P}^A_{j_t} &= -Q^A_{12j_t} K^A_{j_t} + \widetilde{J}^{B\prime}_{j_t} Q^A_{22j_t} K^A_{j_t} + P^A_{11j_t} - \widetilde{J}^{B\prime}_{j_t} P^A_{21j_t} + F^{B\prime}_{j_t} P^{A\prime}_{22j_t} K^A_{j_t} - F^{B\prime}_{j_t} R^{A\prime}_{12j_t}, \\
\widetilde{R}^A_{j_t} &= K^{A\prime}_{j_t} Q^A_{22j_t} K^A_{j_t} - K^{A\prime}_{j_t} P^A_{21j_t} - P^{A\prime}_{21j_t} K^A_{j_t} + R^A_{11j_t}
\end{aligned}$$

and $\widetilde{J}^B_{j_t} = J_{j_t} - K^B_{j_t} F^B_{j_t}$.

The optimal value of the problem in period t is associated with the symmetric positive semidefinite matrix $V^A_{k_{t+1}}$ and it satisfies the Bellman equation:

$$X_t V^A_{j_t} X_t = \min_{u^A_{j_t}} \left\{ L^A_{j_t} + \beta E_t \left[X'_{t+1} V^A_{k_{t+1}} X_{t+1} \right] \right\} \tag{E.9}$$

subject to (E.4), (E.6) and (E.8). The first-order condition with respect to u^A_t is

$$0 = X'_t \widetilde{P}^A_{j_t} + u^{A\prime}_t \widetilde{R}^A_{j_t} + \beta E_t X'_t \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^B_{j_t k_{t+1}} F^B_{j_t} \right)' V^A_{k_{t+1}} \widetilde{B}^A_{j_t k_{t+1}} + \beta E_t u^{A\prime}_t \widetilde{B}^{A\prime}_{j_t k_{t+1}} V^A_{k_{t+1}} \widetilde{B}^A_{j_t k_{t+1}}.$$

The optimal policy function of the leader is given by

$$u_t^A = -F_{j_t}^A X_t, \tag{E.10}$$

where

$$\begin{aligned} F_{j_t}^A &= \left(\widetilde{R}_{j_t}^A + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{A\prime} V_{k_{t+1}}^A \widetilde{B}_{j_t k_{t+1}}^A \right)^{-1} \\ & \left[\widetilde{P}_{j_t}^{A\prime} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{A\prime} V_{k_{t+1}}^A \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^B F_{j_t}^B \right) \right] \end{aligned}$$

Furthermore, using (E.4) and (E.10) in (E.6) gives

$$x_t = -N_{j_t} X_t, \tag{E.11}$$

where

$$N_{j_t} = J_t - K_{j_t}^A F_{j_t}^A - K_{j_t}^B F_{j_t}^B$$

and using (E.4), and (E.10) and (E.11) in (23) gives

$$X_{t+1} = M_{j_t k_{t+1}} X_t + C_{k_{t+1}} \varepsilon_{t+1},$$

where

$$M_{j_t k_{t+1}} = A_{11 k_{t+1}} - A_{12 k_{t+1}} N_{j_t} - B_{11 k_{t+1}} F_{j_t}^A - B_{12 k_{t+1}} F_{j_t}^B$$

Finally, using (E.4), (E.7), (E.8) and (E.10) in (E.9) results in

$$\begin{aligned} V_{j_t}^A &= \widetilde{Q}_{j_t}^A + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^B F_{j_t}^B \right)' V_{k_{t+1}}^A \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^B F_{j_t}^B \right) \\ & - \left(\widetilde{P}_{j_t}^A + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^B F_{j_t}^B \right)' V_{k_{t+1}}^A \widetilde{B}_{j_t k_{t+1}}^A \right) \\ & \left(\widetilde{R}_{j_t}^A + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{A\prime} V_{k_{t+1}}^A \widetilde{B}_{j_t k_{t+1}}^A \right)^{-1} \\ & \left[\widetilde{P}_{j_t}^{A\prime} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{A\prime} V_{k_{t+1}}^A \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^B F_{j_t}^B \right) \right] \end{aligned}$$

E.2 Policy Maker B

Using (E.10) and (E.6) in policy maker B's period loss function gives

$$\begin{aligned} L_{j_t}^B &= \begin{bmatrix} X_t \\ x_t \\ u_t^A \\ u_t^B \end{bmatrix}' \begin{bmatrix} Q_{11j_t}^B & Q_{12j_t}^B & P_{11j_t}^B & P_{12j_t}^B \\ Q_{21j_t}^B & Q_{22j_t}^B & P_{21j_t}^B & P_{22j_t}^B \\ P_{11j_t}^{B\prime} & P_{21j_t}^{B\prime} & R_{11j_t}^B & R_{12j_t}^B \\ P_{12j_t}^{B\prime} & P_{22j_t}^{B\prime} & R_{12j_t}^{B\prime} & R_{22j_t}^B \end{bmatrix} \begin{bmatrix} X_t \\ x_t \\ u_t^A \\ u_t^B \end{bmatrix} \\ &= \begin{bmatrix} X_t \\ u_t^B \end{bmatrix}' \begin{bmatrix} \widetilde{Q}_{j_t}^B & \widetilde{P}_{j_t}^B \\ \widetilde{P}_{j_t}^{B\prime} & \widetilde{R}_{j_t}^B \end{bmatrix} \begin{bmatrix} X_t \\ u_t^B \end{bmatrix} \end{aligned} \tag{E.12}$$

where

$$\begin{aligned}
\widetilde{Q}^B_{j_t} &= Q^B_{11j_t} - Q^B_{12j_t}\widetilde{J}^A_{j_t} - \widetilde{J}^{A\prime}_{j_t}Q^B_{21j_t} + \widetilde{J}^{A\prime}_{j_t}Q^B_{22j_t}\widetilde{J}^A_{j_t} + F^{A\prime}_{j_t}R^B_{11j_t}F^A_{j_t} \\
&\quad + F^{A\prime}_{j_t}P^{B\prime}_{21j_t}\widetilde{J}^A_{j_t} + \widetilde{J}^{A\prime}_{j_t}P^B_{21j_t}F^A_{j_t} - P^B_{11j_t}F^A_{j_t} - F^{A\prime}_{j_t}P^{B\prime}_{11j_t}, \\
\widetilde{P}^B_{j_t} &= -Q^B_{12j_t}K^B_{j_t} + \widetilde{J}^{A\prime}_{j_t}Q^B_{22j_t}K^B_{j_t} - F^{A\prime}_{j_t}R^B_{12j_t} + P^B_{12j_t} - \widetilde{J}^{A\prime}_{j_t}P^B_{22j_t} + F^{A\prime}_{j_t}P^{B\prime}_{21j_t}K^B_{j_t}, \\
\widetilde{R}^B_{j_t} &= K^B_{j_t}Q^B_{22j_t}K^B_{j_t} - K^B_{j_t}P^B_{22j_t} - P^{B\prime}_{22j_t}K^B_{j_t} + R^B_{22j_t},
\end{aligned}$$

and $\widetilde{J}^A_{j_t} = \left(J_{j_t} - K^A_{j_t}F^A_{j_t}\right)$.

The optimal value of the problem in period t is associated with the symmetric positive semidefinite matrix $V^B_{k_{t+1}}$ and it satisfies the Bellman equation:

$$X_t V^B_{j_t} X_t = \min_{u^B_{j_t}} \left\{ L^B_{j_t} + \beta E_t \left[X'_{t+1} V^B_{k_{t+1}} X_{t+1} \right] \right\} \tag{E.13}$$

subject to (E.10), (E.6) and (E.12). The first-order condition with respect to u^B_t is

$$0 = X'_t\widetilde{P}^B_{j_t} + u^{B\prime}_t\widetilde{R}^B_{j_t} + \beta E_t X'_t\left(\widetilde{A}_{j_tk_{t+1}} - \widetilde{B}^A_{j_tk_{t+1}}F^A_{j_t}\right)' V^B_{k_{t+1}}\widetilde{B}^B_{j_tk_{t+1}} + \beta E_t u^{B\prime}_t\widetilde{B}^{B\prime}_{j_tk_{t+1}}V^B_{k_{t+1}}\widetilde{B}^B_{j_tk_{t+1}},$$

The optimal policy function of the follower is given by

$$u^B_t = -F^B_{j_t}X_t, \tag{E.14}$$

where

$$\begin{aligned}
F^B_{j_t} &= \left(\widetilde{R}^{B\prime}_{j_t} + \beta\sum_{k=1}^{n} P_{j_tk_{t+1}}\widetilde{B}^{B\prime}_{j_tk_{t+1}}V^B_{k_{t+1}}\widetilde{B}^B_{j_tk_{t+1}}\right)^{-1} \\
&\quad \left[\widetilde{P}^{B\prime}_{j_t} + \beta\sum_{k=1}^{n} P_{j_tk_{t+1}}\widetilde{B}^{B\prime}_{j_tk_{t+1}}V^B_{k_{t+1}}\left(\widetilde{A}_{j_tk_{t+1}} - \widetilde{B}^A_{j_tk_{t+1}}F^A_{j_t}\right)\right].
\end{aligned}$$

Furthermore, using (E.10) and (E.14) in (E.6) gives

$$x_t = -N_{j_t}X_t, \tag{E.15}$$

where

$$N_{j_t} = J_t - K^A_{j_t}F^A_{j_t} - K^B_{j_t}F^B_{j_t},$$

and using (E.10) and (E.14) and (E.15) in (23) gives

$$X_{t+1} = M_{j_tk_{t+1}}X_t + C_{k_{t+1}}\varepsilon_{t+1},$$

where

$$M_{j_tk_{t+1}} = A_{11k_{t+1}} - A_{12k_{t+1}}N_{j_t} - B_{11k_{t+1}}F^A_{k_{t+1}} - B_{12k_{t+1}}F^B_{j_t}$$

Finally, using (E.7), (E.10), (E.12) and (E.14) in (E.13) results in

$$
\begin{aligned}
V_{j_t}^{B} = {} & \widetilde{Q}_{j_t}^{B} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{k_{t+1}}^{A} \right)' V_{k_{t+1}}^{B} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{k_{t+1}}^{A} \right) \\
& - \left[\widetilde{P}_{j_t}^{B} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{j_t}^{A} \right)' V_{k_{t+1}}^{B} \widetilde{B}_{j_t k_{t+1}}^{B} \right] \\
& \left(\widetilde{R}_{j_t}^{B} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{B\prime} V_{k_{t+1}}^{B} \widetilde{B}_{j_t k_{t+1}}^{B} \right)^{-1} \\
& \left[\widetilde{P}_{j_t}^{B\prime} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{B\prime} V_{k_{t+1}}^{B} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{j_t}^{A} \right) \right]
\end{aligned}
$$

To sum up, the first order conditions to the optimization problem can be written in the following form:

$$
N_{j_t} = J_{j_t} - K_{j_t}^{L} F_{j_t}^{L} - K_{j_t}^{F} F_{j_t}^{F},
$$

$$
\begin{aligned}
V_{j_t}^{A} = {} & \widetilde{Q}_{j_t}^{A} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{B} F_{j_t}^{B} \right)' V_{k_{t+1}}^{A} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{B} F_{j_t}^{B} \right) \\
& - \left(\widetilde{P}_{j_t}^{A} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{B} F_{j_t}^{B} \right)' V_{k_{t+1}}^{A} \widetilde{B}_{j_t k_{t+1}}^{A} \right) \\
& \left(\widetilde{R}_{j_t}^{A} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{A\prime} V_{k_{t+1}}^{A} \widetilde{B}_{j_t k_{t+1}}^{A} \right)^{-1} \\
& \left[\widetilde{P}_{j_t}^{A\prime} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{A\prime} V_{k_{t+1}}^{A} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{B} F_{j_t}^{B} \right) \right]
\end{aligned}
$$

$$
\begin{aligned}
V_{j_t}^{B} = {} & \widetilde{Q}_{j_t}^{B} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{k_{t+1}}^{A} \right)' V_{k_{t+1}}^{B} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{k_{t+1}}^{A} \right) \\
& - \left[\widetilde{P}_{j_t}^{B} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{j_t}^{A} \right)' V_{k_{t+1}}^{B} \widetilde{B}_{j_t k_{t+1}}^{B} \right] \\
& \left(\widetilde{R}_{j_t}^{B} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{B\prime} V_{k_{t+1}}^{B} \widetilde{B}_{j_t k_{t+1}}^{B} \right)^{-1} \\
& \left[\widetilde{P}_{j_t}^{B\prime} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{B\prime} V_{k_{t+1}}^{B} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{j_t}^{A} \right) \right]
\end{aligned}
$$

$$F_{j_t}^A = \left(\widetilde{R}_{j_t}^{A\prime} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{A\prime} V_{k_{t+1}}^A \widetilde{B}_{j_t k_{t+1}}^A\right)^{-1} \left[\widetilde{P}_{j_t}^{A\prime} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{A\prime} V_{k_{t+1}}^A \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^B F_{j_t}^B\right)\right]$$

$$F_{j_t}^B = \left(\widetilde{R}_{j_t}^{B\prime} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{B\prime} V_{k_{t+1}}^B \widetilde{B}_{j_t k_{t+1}}^B\right)^{-1} \left[\widetilde{P}_{j_t}^{B\prime} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{B\prime} V_{k_{t+1}}^B \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^A F_{j_t}^A\right)\right]$$

The discretion equilibrium is a fixed point $\left(N, V^A, V^B\right) \equiv \left\{N_{j_t}, V_{j_t}^A, V_{j_t}^B\right\}_{j_t=1}^n$ of the mapping and a corresponding $\left(F^A, F^B\right) \equiv \left\{F_{j_t}^A, F_{j_t}^B\right\}_{j_t=1}^n$. The fixed point can be obtained as the limit of $\left(N_t, V_t^A, V_t^B\right)$ when $t \longrightarrow -\infty$.

F Data Appendix

We follow Bianchi and Ilut (2017) in constructing our fiscal variables. The data for government spending, tax revenues and transfers, are taken from National Income and Product Accounts (NIPA) Table 3.2 (Federal Government Current Receipts and Expenditures) released by the Bureau of Economics Analysis. These data series are nominal and in levels.

Government Spending. Government spending is defined as the sum of consumption expenditure (line 21), gross government investment (line 42), net purchases of nonproduced assets (line 44), minus consumption of fixed capital (line 45), minus wage accruals less disbursements (line 33).

Total tax revenues. Total tax revenues are constructed as the difference between current receipts (line 38) and current transfer receipts (line 16).

Transfers. Transfers is defined as current transfer payments (line 22) minus current transfer receipts (line 16) plus capital transfers payments (line 43) minus capital transfer receipts (line 39) plus subsidies (line 32).

Federal government debt. Federal government debt is the market value of privately held gross Federal debt, which is downloaded from Dallas Fed web-site

The above three fiscal variables are normalized with respect to Nominal GDP. **Nominal GDP** is taken from NIPA Table 1.1.5 (Gross Domestic Product).

Real GDP. Real GDP is take download from NIPA Table 1.1.6 (Real Gross Domestic Product, Chained Dollars)

The GDP deflator. The GDP deflator is obtained from NIPA Table 1.1.5 (Gross Domestic Product).

Effective Federal Funds Rate. Effective Federal Funds Rate is taken from the St. Louis Fed website.

G Convergence

A random walk Metropolis-Hastings algorithm is then used to generate four chains consisting of 540,000 draws each (with the first 240,000 draws being discarded and 1 in every 100 draws being saved). Brooks-Gelman-Rubin potential reduction scale factors (PRSF) are all below the 1.1 benchmark value used as an upper bound for convergence. FPSR values for Rules-Based Policy and Optimal Policy are presented in Table G.1.

Parameters	PSRF	Parameters	PSRF	Parameters	PSRF	Parameters	PSRF	Parameters	PSRF
				Rules-based policy					
AM/PF		PM/PF		σ	1.00	$\sigma_{\xi(k=1)}$	1.00	p_{11}	1.00
$\rho_{R,S_t=1}$	1.00	$\rho_{R,S_t=2}$	1.06	α	1.00	$\sigma_{\xi(k=2)}$	1.00	p_{22}	1.00
$\psi_{1,S_t=1}$	1.00	$\psi_{1,S_t=2}$	1.01	ζ	1.00	$\sigma_{\mu(k=1)}$	1.00	q_{11}	1.00
$\psi_{2,S_t=1}$	1.00	$\psi_{2,St=2}$	1.01	θ	1.00	$\sigma_{\mu(k=2)}$	1.00	q_{22}	1.01
$\rho_{\tau,s_t=1}$	1.00	$\rho_{\tau,s_t=1}$	1.00	φ	fixed	$\sigma_{q(k=1)}$	1.00	h_{11}	1.00
$\delta_{\tau,s_t=1}$	1.00	$\delta_{\tau,s_t=1}$	1.00	ρ_ξ	1.01	$\sigma_{q(k=2)}$	1.00	h_{22}	1.00
δ_y	1.02	δ_y	1.02	ρ_μ	1.00	σ_{tp}	1.00		
AM/AF		PM/AF		ρ_q	1.00	σ_g	1.00		
$\rho_{R,S_t=1}$	1.00	$\rho_{R,S_t=2}$	1.06	ρ_z	1.00	σ_z	1.00		
$\psi_{1,S_t=1}$	1.00	$\psi_{1,S_t=2}$	1.01	ρ_g	1.00	σ_τ	1.00		
$\psi_{2,St=1}$	1.00	$\psi_{2,St=2}$	1.01			σ_R	1.00		
$\rho_{\tau,s_t=2}$	1.02	$\rho_{\tau,s_t=2}$	1.02						
$\delta_{\tau,s_t=2}$	fixed	$\delta_{\tau,s_t=2}$	fixed						
δ_y	1.02	δ_y	1.02						
				Optimal policy					
ω_1	1.00	σ	1.00	$\sigma_{\mu(k_t=1)}$	1.01	ϕ_{11}	1.00		
ω_2	1.00	α	1.01	$\sigma_{\mu(k_t=2)}$	1.00	ϕ_{22}	1.01		
ω_3	1.00	ζ	1.00	$\sigma_{q(k_t=1)}$	1.00	ψ_{11}	1.01		
$\omega_{\pi,s_t=1}$	fixed	θ	1.02	$\sigma_{q(k_t=2)}$	1.01	ψ_{12}	1.01		
$\omega_{\pi,s_t=2}$	1.00	φ	fixed	$\sigma_{\xi(k_t=1)}$	1.01	ψ_{22}	1.00		
ω_R	1.00	ρ_ξ	1.02	$\sigma_{\xi(k_t=2)}$	1.00	ψ_{23}	1.02		
ω_π^f	1.00	ρ_μ	1.01	σ_{tp}	1.01	ψ_{33}	1.00		
ω_τ	1.01	ρ_q	1.00	σ_g	1.00	ψ_{31}	1.00		
$\rho_{\tau,S_t=2}$	1.03	ρ_z	1.02	σ_z	1.00	h_{11}	1.00		
$\rho_{\tau,S_t=3}$	1.01	ρ_g	1.01	σ_τ	1.01	h_{22}	1.00		
$\delta_{\tau,S_t=2}$	1.02								
$\delta_{\tau,S_t=3}$	fixed								
δ_y	1.00								

Table G.1: Brooks-Gelman-Rubin potential reduction scale factors.

H Model Identification

We apply the Komunjer and Ng (2011) identification test to analyze our optimal policy model. Komunjer and Ng (2011) study the local identification of a DSGE model from its linearized solution. Their test uses the restrictions implied by equivalent spectral densities to obtain rank and order conditions for identification. Minimality and left-invertibility are necessary and sufficient conditions for identification. It is important to note that Komunjer and Ng (2011) identification test only applies to covariance stationary processes. Therefore, the parameters associated with Markov-switching shock variances cannot be incorporated into the test.

Nevertheless we can test the identification of structural parameters and parameters associated with changes in monetary and fiscal policy. We can solve our model assuming that policy stays in one regime, while the private agents in the economy are aware that there are probabilities of policy switching to a different regime. For the optimal policy model, it has the parameter vector $\theta \equiv (\sigma,\, \alpha,\, \zeta,\, \theta,\, \rho_\xi,\, \rho_\mu,\, \rho_q,\, \rho_z,\, \rho_g,\, \sigma_{\xi(k_t=1)},\, \sigma_{\mu(k_t=1)},\, \sigma_{q(k_t=1)},\, \sigma_{tp},\, \sigma_g,\, \sigma_z,\, \sigma_\tau,\, \phi_{11},\, \phi_{22},\, \psi_{11},\, \psi_{12},\, \psi_{22},\, \psi_{23},\, \psi_{33},\, \psi_{31},\, \omega_1,\, \omega_2,\, \omega_3,\, \omega_{\pi,s_t=2},\, \omega_R,\, \omega_\pi^f,\, \omega_\tau,\, \rho_{\tau,S_t=2},\, \rho_{\tau,S_t=3},\, \delta_{\tau,S_t=2},\, \delta_y)$ of dimension $n_\theta = 35$. To apply the test, we solve for the model and find a minimal representation of the solution as follows

$$\begin{aligned} X_{1t+1} &= A(\theta)X_{1t} + B(\theta)\varepsilon_{t+1}, \\ Y_{t+1} &= C(\theta)X_{1t} + D(\theta)\varepsilon_{t+1}. \end{aligned}$$

Our model has 14 state variables, $\widetilde{z}_t$, $\widehat{\mu}_t$, $\widehat{\xi}_t$, $\widehat{q}_t$, $\widetilde{g}_t$, $\widehat{c}_t$, $\widehat{\pi}_t$, $\widehat{P}_t^M$, $\widehat{R}_t$, $\widetilde{\tau}_t$, $\widetilde{\tau}_{PF,t}$, $\widetilde{\tau}_{AF,t}$, $\widehat{y}_t$, and $\widetilde{b}_t^M$. However, $\widehat{y}_t$ and $\widetilde{b}_t^M$ are dependent on other state variables as follows

$$\widehat{y}_t = \widehat{c}_t + \frac{1}{1-g}\widetilde{g}_t,$$

$$\widetilde{b}_t^M = \frac{1}{\beta}\widetilde{b}_{t-1}^M + \frac{\rho b^M}{\gamma\pi}\widehat{P}_t^M - \frac{b^M}{\beta}\widehat{P}_{t-1}^M + \frac{b^M}{\beta}\widehat{y}_{t-1} - \frac{b^M}{\beta}\widehat{y}_t - \frac{b^M}{\beta}\widehat{\pi}_t - \frac{b^M}{\beta}\widehat{q}_t - \widetilde{\tau}_t + \widetilde{g}_t + \widetilde{z}_t + \sigma_{tp}\varepsilon_{tp,t}.$$

Therefore, the 14×14 matrix, $A(\theta)$, only has rank 12 (i.e. $n_X = 12$). $Y_{t+1} \equiv (\Delta GDP_t,\, INF_t,\, FFR_t,\, G_t/GDP_t,\, T_t/GDP_t,\, Z_t/GDP_t,\, B_t/GDP_t)$ and $\varepsilon_t \equiv (\varepsilon_{z,t},\, \varepsilon_{\mu,t},\, \varepsilon_{\xi,t},\, \varepsilon_{q,t},\, \varepsilon_{g,t},\, \varepsilon_t^\tau,\, \varepsilon_{tp,t})$ are the observables and shocks. As $n_Y = n_\varepsilon = 7$, the model is square. Proposition 2-S in Komunjer and Ng (2011) can be employed to assess identification.

Using the same notation as in Komunjer and Ng (2011), we check the rank of $\Delta^s(\theta_0)$ which is the matrix of partial derivatives of $\delta^s(\theta, T, U)$ evaluated at $(\theta_0, \mathbf{I}_{n_X}, \mathbf{I}_{n\varepsilon})$.

$$\begin{aligned} \Delta^s(\theta_0) &\equiv \left(\frac{\partial\delta^s(\theta, \mathbf{I}_{n_X}, \mathbf{I}_{n_\varepsilon})}{\partial\theta} \; \frac{\partial\delta^s(\theta, \mathbf{I}_{n_X}, \mathbf{I}_{n_\varepsilon})}{\partial vecT} \; \frac{\partial\delta^s(\theta, \mathbf{I}_{n_X}, \mathbf{I}_{n_\varepsilon})}{\partial vecU} \right)_{\theta=\theta_0} \\ &\equiv \left(\Delta_\Lambda^s(\theta_0)\; \Delta_T^s(\theta_0)\; \Delta_U^s(\theta_0)\right). \end{aligned}$$

The rank of $\Delta^s(\theta_0)$ required for identification is

$$\begin{aligned} rank(\Delta^s(\theta_0)) &= rank\left(\Delta_\Lambda^s(\theta_0)\; \Delta_T^s(\theta_0)\; \Delta_U^s(\theta_0)\right) \\ &= n_\theta + n_X^2 + n_\varepsilon^2. \\ &= 35 + 144 + 49 \end{aligned}$$

As in Komunjer and Ng (2011) we also consider 11 levels of tolerance along with the Matlab default to analyze the rank of $\Delta^s(\theta_0)$. We use the change in rank as tolerance tightens to identify problematic parameters. The Komunjer and Ng (2011) test does not indicate that any parameters are unidentified. In Table H.1 the required rank for identification of each model is presented, along with the Tol at which the model passes the rank requirement.

In addition, we plot draws from the the prior and posterior distributions of parameters for the optimal policy model in Figure H.1. Visual inspection reveals that the priors are widely dispersed around the respective means, whereas posteriors are more concentrated. In other words, the data are informative with respect to these parameters.

	Tolerance	Δ^s_Λ	Δ^s_T	Δ^s_U	Δ^s	Pass
	Required	35	144	49	228	
Regime 1	$e-3$	35	144	49	228	YES
Regime 2	$e-3$	35	144	49	228	YES
Regime 3	$e-4$	35	144	49	228	YES
Regime 4	$e-4$	35	144	49	228	YES
Regime 5	$e-4$	35	144	49	228	YES
Regime 6	$e-4$	35	144	49	228	YES

Table H.1: Komunjer and Ng (2011) Identification Test.

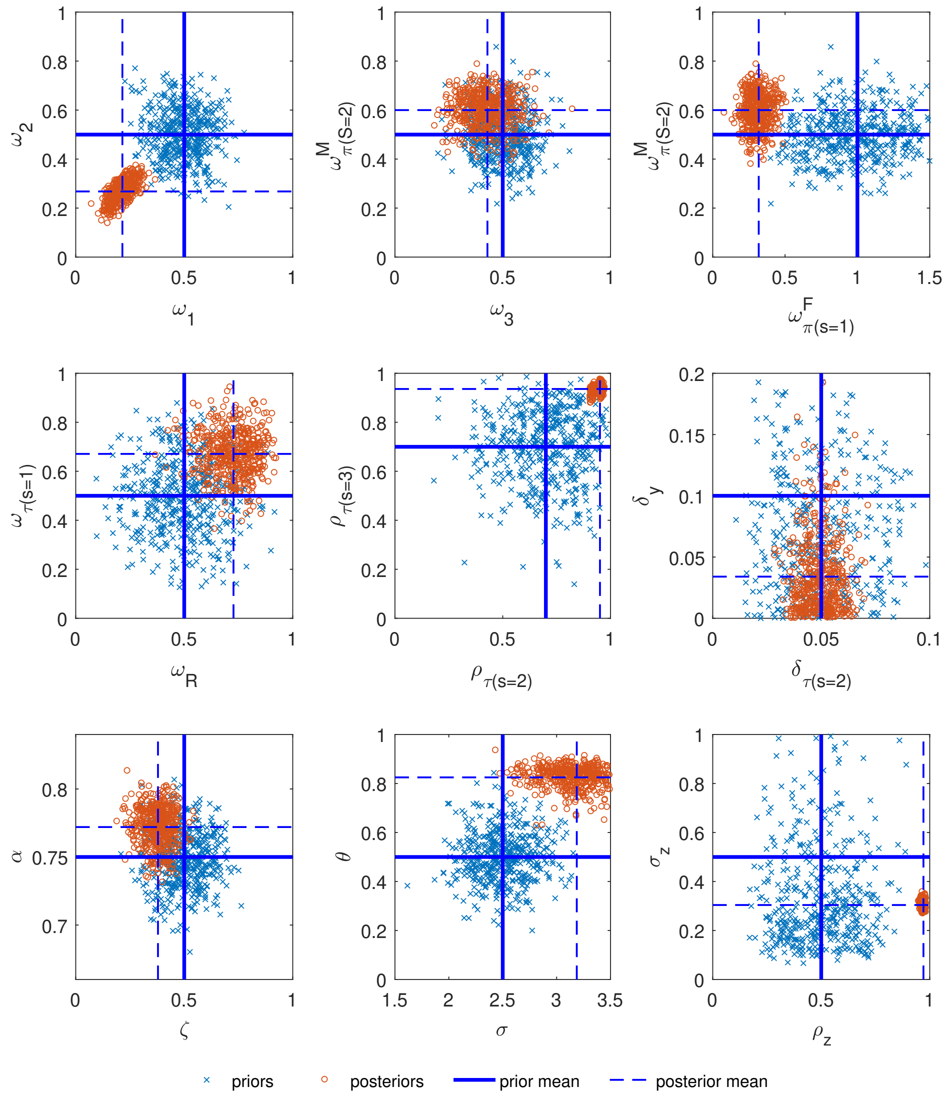

Figure H.1: Prior and Posterior Distributions of Parameters. The panels depict 500 draws from prior and posterior distributions from the estimates of our optimal policy model. The draws are plotted for pairs of estimated parameters and the intersections of lines signify prior (solid) and posterior (dashed) means, respectively.

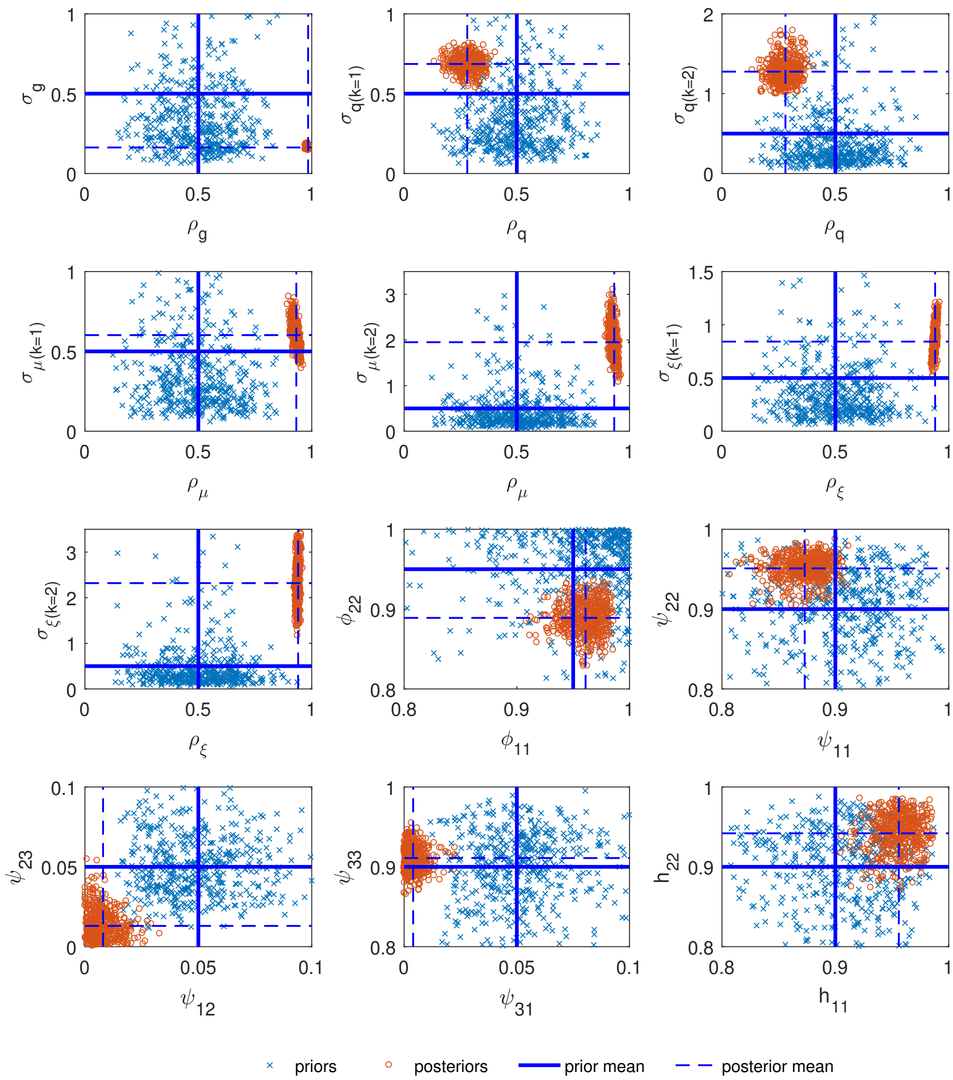

Figure H.1: Prior and Posterior Distributions of Parameters (continued). The panels depict 500 draws from prior and posterior distributions from the estimates of our optimal policy model. The draws are plotted for pairs of estimated parameters and the intersections of lines signify prior (solid) and posterior (dashed) means, respectively.

I Alternative Social Planner's Allocation

In this section we outline the social planner's allocation associated with our estimated model. Normally such an allocation would be obtained by maximising utility subject to resource and technology constraints as in Appendix B above. However, in order to generate insight into our policy maker's decisions we need to consider the estimated objective function. Therefore we maximise the following objective function,

$$L = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \omega_1 \left(\widehat{X}_t^* + \widehat{\xi}_t \right)^2 + \omega_2 \left(\widehat{y}_t^* - \frac{\sigma}{\varphi} \widehat{\xi}_t \right)^2 \right\},$$

subject to the definition of habits adjusted consumption,

$$\widehat{X}_t^* = (1-\theta)^{-1} (\widehat{y}_t^* - \frac{1}{1-g} \widetilde{g}_t - \theta \widehat{y}_{t-1}^* + \theta \frac{1}{1-g} \widetilde{g}_{t-1})$$

where the star superscripts denote the fact that we are considering the social planner's allocation. The first order condition this implies is given by,

$$\begin{aligned} & \omega_1 \left((1-\theta)^{-1} (\widehat{y}_t^* - \frac{1}{1-g} \widetilde{g}_t - \theta \widehat{y}_{t-1}^* + \theta \frac{1}{1-g} \widetilde{g}_{t-1}) + \widehat{\xi}_t \right) (1-\theta)^{-1} + \omega_2 \left(\widehat{y}_t^* - \frac{\sigma}{\varphi} \widehat{\xi}_t \right) \\ = \; & \theta \beta \omega_1 \left((1-\theta)^{-1} (E_t \widehat{y}_{t+1}^* - \frac{1}{1-g} \rho_g \widetilde{g}_t - \theta \widehat{y}_t^* + \theta \frac{1}{1-g} \widetilde{g}_t) + \rho^{\xi} \widehat{\xi}_t \right) (1-\theta)^{-1}. \end{aligned}$$

This describes the desired path for output that would be chosen by the social planner conditional on the exogenous path for government spending. This can be used to construct a welfare relevant output gap $\widehat{y}_t - \widehat{y}_t^*$ which captures the extent to which the policy maker is unable to achieve this desired level of output due to nominal inertia, the habits externality, fiscal constraints and time-consistency problems. Effectively, it reflects the welfare trade-offs between inflation and the real economy implied by the estimated objective function, but reduces those to a single measure.

In order to identify why the estimations adopts particular regimes at particular points of time it is also helpful to get a measure of various fiscal gaps, specifically the tax and debt gaps. The tax gap is the difference between $\widetilde{\tau}_t$ and the tax rate that the social planner would choose to eliminate cost-push shocks, $\widetilde{\tau}_t^* = -(1-\tau)\widehat{\mu}_t$, so that we have a tax gap, $\widetilde{\tau}_t - \widetilde{\tau}_t^*$.

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