# Monetary Policy Delegation and Equilibrium Coordination *

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**Abstract**

This paper revisits the argument that the stabilisation bias that arises under discretionary monetary policy can be reduced if policy is delegated to a policymaker with redesigned objectives. We study four delegation schemes: price level targeting, interest rate smoothing, speed limits and straight conservatism. These can all increase social welfare in models with a unique discretionary equilibrium. We investigate how these schemes perform in a model with capital accumulation where uniqueness does not necessarily apply. We discuss how multiplicity arises and demonstrate that no delegation scheme is able to eliminate all potential bad equilibria. Price level targeting has two interesting features. It can create a new equilibrium that is welfare dominated, but it can also alter equilibrium stability properties and make coordination on the best equilibrium more likely.

Key Words: Time Consistency, Discretion, Multiple Equilibria, Policy Delegation

JEL References: E31, E52, E58, E61, C61

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# 1 Introduction

This paper revisits the claim that the stabilization bias in monetary policy making – the welfare foregone by a central bank unable to commit to first-best monetary policy identified by Svensson (1997) – can be mitigated by delegating policy decisions to agencies with different priorities. Such delegation schemes effectively assign a modified objective to the monetary authority which is then pursued under discretion. Of the many proposals, leading candidates are to replace the inflation argument in the typical central bank mandate by a price level target (Svensson, 1997; Vestin, 2006); a speed-limit policy, in which the term in the level of the output gap is replaced by a term in its change (Walsh, 2003); an interest-rate smoothing policy (Woodford, 2003b) under which the normal mandate is augmented with an additional term in the change in the interest rate; and simply appointing a conservative central bank with a greater weight on inflation stabilization than is socially optimal (Rogoff, 1985) which was designed to mitigate static inflation bias but has power against its dynamic counterpart.

The intuition behind why any of these might work is easy to see once we characterize fully optimal but time inconsistent policy. In essence – and for simple enough models – it turns out to be optimal to establish a trade-off between non-zero *growth* in the output gap and inflation, despite preferences being specified over the *levels* of both. This policy is by definition at least as good as any discretionary equilibrium. Specifying policymaker preferences to generate such a trade off directly is a characteristic of several of the schemes outlined above. Indeed, both price level targeting and speed limit rules explicitly incorporate the trade off into the modified objective function, normalizing on output and inflation respectively. The explicitly inertial policy of interest rate smoothing can sometimes have a similarly beneficial impact if the lagged instrument proxies the implied trade off satisfactorily. All of these would seem better than a conservative central bank proposal, which simply mimics the discretionary equilibrium with different coefficients. A back-of-the-envelope analysis might rank the conservative central bank as least best alternative followed by interest rate smoothing and either price level targeting or speed limits best. Which proposal is better is simply a matter of degree.

However, this intuition is potentially flawed. The original analysis of the proposed bias-reduction schemes was largely conducted – for the sake of clarity and simplicity – in a simple New Keynesian model that abstracts from persistent endogenous state variables, like stocks of debt or capital. This abstraction was made with the assumption that nothing much by way of generality was lost by so doing. However, discretionary policy can result in expectations traps and multiple equilibria, see Albanesi et al. (2003), King and Wolman (2004) and Blake and Kirsanova (2012), and a simple New Keynesian model is not appropriate to investigate many consequences of policy delegation. As there is no particular reason to assume one equilibrium will prevail any more than another, even if the delegation schemes behave locally as intuition might suggest globally could be a different matter.

The central questions addressed in this paper follow on from this. How do delegation schemes affect the likelihood of obtaining multiple equilibria in more realistic models? If multiplicity survives, how should we quantify the welfare implications of choosing one of these delegation schemes? Can any of these schemes affect the likelihood that economic agents successfully coordinate on the best equilibrium?

To address these questions we study a version of the familiar sticky price model modified to incorporate capital accumulation (Sveen and Weinke, 2005; Woodford, 2005). This model contains all the features as at the heart of many DSGE models used in policy analysis which makes it a 'representative agent' for our policy analysis. Moreover, we choose a set up which encompasses the 'worst case' scenario: there are expectation traps and economic agents may be more likely to coordinate on the worst equilibrium (Dennis and Kirsanova, 2012).

We confirm the previous results that all these delegation schemes can improve social welfare, but we show that they only unequivocally improve welfare if the economy remains in the best equilibrium. At the same time, we demonstrate that none of the considered delegation schemes eliminates expectation traps, moreover, new equilibria arise under delegation. Whether there is a gain associated with a transition to a delegation scheme will depend on which equilibrium under the original unmodified objective one starts from and to which equilibrium one transitions.

More encouragingly, all delegation schemes reduce the range of parameters which ensure the unique worst equilibrium, thus increasing the likelihood of coordinating on the Pareto-preferred equilibrium. We also argue that a delegation scheme can change the way how agents coordinate on an equilibrium. In particular, we show that price level targeting increases the likelihood that the best equilibrium realizes.

The paper is organized as follows. In the next section we outline the model and discuss the calibration. Section 3 recapitulates the analysis in Blake and Kirsanova (2012) for our sticky-price model with capital, showing how multiplicity arises when monetary policy is conducted under discretion and with an unmodified monetary policy objective. Section 4 compares and contrasts four delegation schemes: conservative central bank, interest rate smoothing, speed-limit policy and price-level targeting. Section 5 concludes.

## 2 Model

We use a New Keynesian model with capital accumulation and complete markets, as in Sveen and Weinke (2005) and Woodford (2005). The model has monopolistic competition and sticky prices in goods markets. Capital accumulation is assumed to take place at the firm level and any additional capital resulting from an investment decision becomes productive with a one period delay. We assume a convex capital adjustment cost at the firm level. Since the details of the model are discussed in Sveen and Weinke (2005) we proceed directly to the equations that result from linearizing the equilibrium conditions around the steady state.

### 2.1 Linearized equilibrium conditions

From the standard household's optimization problem we obtain, respectively, an Euler equation and a labour supply equation[1]

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1} - \rho) \tag{1}$$

$$w_t = \phi n_t + \sigma c_t \tag{2}$$

where $\rho = -\log \beta$ is the time discount rate, $\sigma$ is the household's relative risk aversion or, equivalently, the inverse of the intertemporal elasticity of substitution and $\phi$ is the inverse of the Frisch labour supply elasticity. We denote the nominal interest rate at time $t$ as $i_t = \log R_t$, and $\pi_t = \log\left(\frac{P_t}{P_{t-1}}\right)$ is inflation. We also denote aggregate consumption as $c_t$, $n_t$ aggregate labour and $w_t$ average real wages. $\mathbb{E}_t$ is the expectations operator conditional on information available through time $t$.

The law of motion of capital is obtained from averaging and aggregating optimal investment decisions on the part of firms. This implies

$$\Delta k_{t+1} = \beta \mathbb{E}_t \Delta k_{t+2} + \frac{1}{\varepsilon_\psi}\left(\left(1 - \beta\left(1-\delta\right)\right)\mathbb{E}_t ms_{t+1} - \left(i_t - \mathbb{E}_t \pi_{t+1} - \rho\right)\right) \tag{3}$$

where aggregate capital is denoted by $k_t$ and $ms_t = w_t - k_t + n_t$ measures the average real marginal return to capital. $\beta$ is the subjective discount factor, $\delta$ the rate of depreciation and $\varepsilon_\psi$ measures capital adjustment costs at the firm level. The average real marginal return to capital is measured in terms of marginal savings in labour costs since firms are demand-constrained in this model.

[1] All variables expressed in terms of log deviations from their steady state values are lower case roman or greek script, with a zero target inflation rate.

The inflation equation takes the standard form

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa mc_t + v_t$$

where $mc_t = w_t - y_t + n_t$ denotes the average real marginal cost. If capital can be rented then $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta}$ where $\theta$ is the probability that a firm does not re-optimize its price in any given period. If there is no rental market then instead $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta} \frac{(1-\alpha)}{(1-\alpha+\varepsilon\alpha)} \frac{1}{\xi}$ where $\varepsilon$ is the elasticity of substitution between the differentiated goods, while $\xi$ is a function of the model's structural parameters, computed numerically using the method developed in Woodford (2005). Finally (and not in Sveen and Weinke (2005)) we introduce a cost-push shock $v_t$. It is common to interpret this shock as a temporary discretionary change in firms' desired margins.

The linearized aggregate production function is

$$y_t = \alpha k_t + (1-\alpha) n_t \tag{4}$$

where $\alpha$ denotes the capital share and $y_t$ aggregate output. The goods market clearing condition is

$$y_t = \zeta c_t + \frac{1-\zeta}{\delta} \left(k_{t+1} - (1-\delta) k_t\right) \tag{5}$$

where $\zeta = 1 - \frac{\delta\alpha(\varepsilon-1)}{\varepsilon(\rho+\delta)}$ is the steady state consumption to output ratio.

Finally, after eliminating all variables other than $\pi_t$, $c_t$, $k_t$, and $i_t$ the system can be expressed as

$$\begin{aligned}
\pi_t &= \beta \mathbb{E}_t \pi_{t+1} + \lambda_c c_t + \lambda_o k_{t+1} - \lambda_k k_t + v_t && (6)\\
k_{t+1} &= \nu_o \mathbb{E}_t k_{t+2} + \nu_k k_t + \nu_c c_t - \nu_r (i_t - \mathbb{E}_t \pi_{t+1}) && (7)\\
c_t &= \mathbb{E}_t c_{t+1} - \sigma (i_t - \mathbb{E}_t \pi_{t+1}) && (8)
\end{aligned}$$

with coefficients

$$\begin{aligned}
\bar{\nu} &= \left(\varepsilon_\psi (1+\beta) + \frac{(1-\beta(1-\delta))}{1-\alpha} \left(\frac{(\phi+1)(1-\zeta)(1-\delta)}{\delta} + \alpha\phi + 1\right)\right)^{-1}, \ \nu_k = \varepsilon_\psi \bar{\nu},\\
\nu_o &= \left(\varepsilon_\psi \beta + \frac{(1-\beta(1-\delta))(\phi+1)(1-\zeta)}{(1-\alpha)\delta}\right) \bar{\nu}, \ \nu_c = (1-\beta(1-\delta)) \left(\frac{(\phi+1)\zeta}{1-\alpha} + \frac{1}{\sigma}\right) \bar{\nu},\\
\nu_r &= \left(1 - (1-\beta(1-\delta)) \left(\frac{(\phi+1)\zeta\sigma}{(1-\alpha)} + 1\right)\right) \bar{\nu}, \ \lambda_c = \kappa \left(\frac{(\phi+\alpha)\zeta}{1-\alpha} + \frac{1}{\sigma}\right),\\
\lambda_o &= \kappa \frac{(\phi+\alpha)(1-\zeta)}{(1-\alpha)\delta}, \ \lambda_k = \kappa \left(\frac{(\phi+\alpha)(1-\zeta)(1-\delta)}{\delta(1-\alpha)} + \frac{\alpha(1+\phi)}{(1-\alpha)}\right)
\end{aligned}$$

## 2.2 Monetary policy

### 2.2.1 Policy objective

We assume that the central bank uses the nominal short-term interest rate $i_t$ as an instrument and acts under discretion.[2] We also assume that the social welfare function is well captured by the following discounted quadratic loss function

$$\frac{1}{2}\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\pi_s^2 + \omega y_s^2\right) \tag{9}$$

where $y_t$ is output gap (because we only consider cost push shocks). This welfare function has been shown by Woodford (2003a), Ch. 6, to approximate the aggregate of individual utility functions in a model without capital, but otherwise identical to the one we work with. In our model, this approximation will not hold up to the second order and so our policy objective function is to some degree *ad hoc*. However, as King and Wolman (2004) and Blake and Kirsanova (2012) argue, multiplicity under discretion is not a consequence of some 'unfortunate' form of social welfare, but rather a general property of discretionary policy which is brought about when the private sector and the policymaker make decisions based on forecasts of each other's actions. In what follows we simply refer to this objective as to the social objective. We also label the regime with social policy objective as 'inflation targeting'. Note that we do this for convenience and not to take a stand on the optimality or the precise nature of inflation targeting regimes as practiced in real life.

### 2.2.2 Discretionary policy

Our definition of discretionary policy is conventional and is widely used in the monetary policy literature, see e.g. Backus and Driffill (1986), Oudiz and Sachs (1985), Clarida et al. (1999), and Woodford (2003a). The optimal monetary policy and equilibrium reactions of the private sector can be written in the form of linear function of current states:

$$\begin{aligned} i_t &= \iota_v v_t + \iota_k k_t, & (10)\\ k_{t+1} &= k_v v_t + k_k k_t & (11)\\ c_t &= c_v v_t + c_k k_t & (12)\\ \pi_t &= \pi_v v_t + \pi_k k_t & (13) \end{aligned}$$

[2] We also compute the commitment soluition, but only as a reference solution.

and the policymaker's value function can be written as

$$V\left(v_t,k_t\right)=S_{vv}v_t^2+2S_{vk}v_tk_t+S_{kk}k_t^2. \tag{14}$$

At any time $t$, the policy maker reacts to the current state (10), knows that the private sector observes its action, and knows that the private sector expects all future policy makers will apply the same decision process and implement policy (10).

We lead (11)–(13) one period and use (1)–(5) to write private sector decisions as a response to the state and to policy

$$k_{t+1} = \hat{k}_v v_t+\hat{k}_k k_t+\hat{k}_\iota i_t, \tag{15}$$

$$\pi_t = \hat{\pi}_v v_t+\hat{\pi}_k k_t+\hat{\pi}_\iota i_t, \tag{16}$$

$$c_t = \hat{c}_v v_t+\hat{c}_k k_t+\hat{c}_\iota i_t, \tag{17}$$

where

$$\hat{k}_v = -\rho_v\left(\nu_r\pi_v+\nu_c c_v+\nu_o k_v+\sigma\nu_c\pi_v\right)/d \tag{18}$$

$$\begin{aligned}\hat{\pi}_v = {} & 1+\rho_v\left(\left(\nu_c\pi_v c_k-\nu_c\pi_k c_v-\pi_v-\nu_o\pi_k k_v+\nu_o\pi_v k_k\right)\beta\right.\\ & +\left(\nu_r\left(\pi_k c_v-\pi_v c_k\right)+\left(\nu_o k_k-1\right)\left(c_v+\sigma\pi_v\right)-\nu_o k_v\left(c_k+\sigma\pi_k\right)\right)\lambda_c\\ & \left.-\lambda_o\left(\nu_r\pi_v+\nu_o k_v+\nu_c\left(c_v+\sigma\pi_v\right)\right)\right)/d\end{aligned} \tag{19}$$

$$\hat{c}_v = \rho_v\left(\nu_r\left(\pi_k c_v-\pi_v c_k\right)+\left(\nu_o k_k-1\right)\left(c_v+\sigma\pi_v\right)-\left(c_k+\sigma\pi_k\right)\nu_o k_v\right)/d \tag{20}$$

$$\hat{k}_k = -\nu_k/d \tag{21}$$

$$\begin{aligned}\hat{\pi}_k = {} & \left(\lambda_k-\nu_k\lambda_o-\beta\nu_k\pi_k-\lambda_k\nu_r\pi_k-\lambda_c\nu_k c_k-\nu_c\lambda_k c_k\right.\\ & \left.-\lambda_k\nu_o k_k-\sigma\lambda_c\nu_k\pi_k-\sigma\nu_c\lambda_k\pi_k\right)/d\end{aligned} \tag{22}$$

$$\hat{c}_k = -\nu_k\left(c_k+\sigma\pi_k\right)/d \tag{23}$$

$$\hat{k}_\iota = \left(\nu_r+\sigma\nu_c\right)/d \tag{24}$$

$$\hat{\pi}_\iota = \left(\left(\sigma\left(1-\nu_o k_k\right)+\nu_r c_k\right)\lambda_c+\left(\lambda_o+\beta\pi_k\right)\left(\nu_r+\sigma\nu_c\right)\right)/d \tag{25}$$

$$\hat{c}_\iota = \left(\sigma\left(1-\nu_o k_k\right)+\nu_r c_k\right)/d \tag{26}$$

and $d=\nu_r\pi_k+\nu_c c_k+\nu_o k_k+\sigma\nu_c\pi_k-1=d\left(k_k,c_k,\pi_k\right)$. Written this way, equations (15)–(17) isolate the 'instantaneous' influence of policy on private sector decisions.

Optimal discretionary policy satisfies the following Bellman equation

$$\begin{aligned}V\left(v_t,k_t\right) = {} & \min_{i_t}\left(\left(\hat{\pi}_v v_t+\hat{\pi}_k k_t+\hat{\pi}_\iota i_t\right)^2+\omega\left(\left(\zeta\hat{c}_k+\frac{1-\zeta}{\delta}\left(\hat{k}_k-(1-\delta)\right)\right)k_t\right.\right.\\ & \left.\left.+\left(\zeta\hat{c}_v+\frac{1-\zeta}{\delta}\hat{k}_v\right)v_t+\left(\zeta\hat{c}_\iota+\frac{1-\zeta}{\delta}\hat{k}_\iota\right)i_t\right)^2+\beta\mathbb{E}_tV\left(v_{t+1},k_{t+1}\right)\right)\end{aligned} \tag{27}$$

where we take the intra-period leadership of the policy maker into account by substituting in constraints (15)-(17). Optimization yields the following feedback coefficients in (10)

$$\iota_v = -\frac{\hat{\pi}_\iota\hat{\pi}_v + \omega\left(\zeta\hat{c}_\iota + \frac{1-\zeta}{\delta}\hat{k}_\iota\right)\left(\zeta\hat{c}_v + \frac{1-\zeta}{\delta}\hat{k}_v\right) + \beta\rho_v S_{\nu k}\hat{k}_\iota + \beta S_{kk}\hat{k}_\iota\hat{k}_v}{\hat{\pi}_\iota^2 + \omega\left(\zeta\hat{c}_\iota + \frac{1-\zeta}{\delta}\hat{k}_\iota\right)^2 + \beta S_{kk}\hat{k}_\iota^2} \tag{28}$$

$$\iota_k = -\frac{\hat{\pi}_\iota\hat{\pi}_k + \omega\left(\zeta\hat{c}_\iota + \frac{1-\zeta}{\delta}\hat{k}_\iota\right)\left(\zeta\hat{c}_k + \frac{1-\zeta}{\delta}\left(\hat{k}_k - (1-\delta)\right)\right) + \beta S_{kk}\hat{k}_\iota\hat{k}_k}{\hat{\pi}_\iota^2 + \omega\left(\zeta\hat{c}_\iota + \frac{1-\zeta}{\delta}\hat{k}_\iota\right)^2 + \beta S_{kk}\hat{k}_\iota^2} \tag{29}$$

and the value-function coefficients satisfy

$$\begin{aligned} S_{kk} &= \omega\left(\zeta\hat{c}_k + \frac{1-\zeta}{\delta}\left(\hat{k}_k - (1-\delta)\right) + \left(\zeta\hat{c}_\iota + \frac{1-\zeta}{\delta}\hat{k}_\iota\right)\iota_k\right)^2 + \left(\hat{\pi}_k + \hat{\pi}_\iota\iota_k\right)^2 \\ &\quad +\beta S_{kk}\left(\hat{k}_k + \hat{k}_\iota\iota_k\right)^2 \end{aligned} \tag{30}$$

$$\begin{aligned} S_{\nu k} &= \omega\left(\zeta\hat{c}_k + \frac{1-\zeta}{\delta}\left(\hat{k}_k - (1-\delta)\right) + \left(\zeta\hat{c}_\iota + \frac{1-\zeta}{\delta}\hat{k}_\iota\right)\iota_k\right)\left(\zeta\hat{c}_v + \frac{1-\zeta}{\delta}\hat{k}_v\right. \\ &\quad \left. + \left(\zeta\hat{c}_\iota + \frac{1-\zeta}{\delta}\hat{k}_\iota\right)\iota_v\right) + \left(\hat{\pi}_v + \hat{\pi}_\iota\iota_v\right)\left(\hat{\pi}_k + \hat{\pi}_\iota\iota_k\right) \\ &\quad +\beta\left(S_{\nu k}\rho_v\left(\hat{k}_k + \hat{k}_\iota\iota_k\right) + S_{kk}\left(\hat{k}_v + \hat{k}_\iota\iota_v\right)\left(\hat{k}_k + \hat{k}_\iota\iota_k\right)\right) \end{aligned} \tag{31}$$

$$\begin{aligned} S_{\nu\nu} &= \left(\hat{\pi}_v + \hat{\pi}_\iota\iota_v\right)^2 + \omega\left(\zeta\hat{c}_v + \frac{1-\zeta}{\delta}\hat{k}_v + \left(\zeta\hat{c}_\iota + \frac{1-\zeta}{\delta}\hat{k}_\iota\right)\iota_v\right)^2 \\ &\quad +\beta\left(S_{\nu\nu}\rho_v^2 + 2S_{\nu k}\rho_v\left(\hat{k}_v + \hat{k}_\iota\iota_v\right) + S_{kk}\left(\hat{k}_v + \hat{k}_\iota\iota_v\right)^2\right). \end{aligned} \tag{32}$$

We substitute equation (10) into (15)–(17) and obtain coefficients in (11)–(13)

$$k_v = \hat{k}_v + \hat{k}_\iota\iota_v, \quad \pi_v = \hat{\pi}_v + \hat{\pi}_\iota\iota_v, \quad c_v = \hat{c}_v + \hat{c}_\iota\iota_v \tag{33}$$

$$k_k = \hat{k}_k + \hat{k}_\iota\iota_k, \quad \pi_k = \hat{\pi}_k + \hat{\pi}_\iota\iota_k, \quad c_k = \hat{c}_k + \hat{c}_\iota\iota_k \tag{34}$$

Any set of coefficients $\mathcal{D} = \{\iota_v, \iota_k, k_v, k_k, c_v, c_k, \pi_v, \pi_k, S_{\nu\nu}, S_{\nu k}, S_{kk}\}$ which satisfies (18)–(26), (28)–(34) describes the solution to the discretionary optimization problem outlined above.

# 3 Multiple discretionary equilibria with a benevolent policymaker

## 3.1 Existence of multiple equilibria

This model has three discretionary equilibria for the benchmark calibration when we assume a benevolent policymaker who does not delegate.[3] We can show this using the following steps. First,

[3] We set the capital share $\alpha = 0.36$. The risk aversion parameter $\sigma = 1$, and a unit elasticity of labour supply is assumed ($\phi = 1$). The elasticity of substitution between goods, $\varepsilon$, is set to 11. The rate of capital depreciation, $\delta$,

we dispense with the stochastic parts of the model. Note that equations (18)–(26), (28)–(34) are recursive. We can solve equations (21)–(26), (29), (30) and (34) for $\{\iota_k, k_k, c_k, \pi_k, S_{kk}\}$ first, only then solving the remainder of the system (18)–(26), (28)–(34) for the stochastic component of the solution. As Blake and Kirsanova (2012) show, because the system for any stochastic components is linear it has a unique solution (unless it is degenerate) and it is enough to demonstrate the multiplicity of solutions for the deterministic part alone.

The deterministic system (21)–(26), (29), (30) and (34) has only one predetermined endogenous state variable, capital $k_t$. Suppose the initial level of capital $k_0 > 0$. Suppose the policymaker's response to higher level of capital is $\iota_k$, which is not necessarily optimal. In response to this policy the private sector's response is given by system (21)–(26) and (34). It is straightforward to demonstrate[4] that this system has at most four solutions, i.e. sets $\{k_k, c_k, \pi_k\}$ each of which describes a rational expectations response of the private sector. For our calibration only two solutions are real for the range of realistic values of $\iota_k$. These two solutions are labelled $A$ and $B$ and are plotted in the left hand side charts in Panel I in Figure 1.

In order to understand this multiplicity of private sector responses better, recall that firms choose current-period prices based on marginal cost. Marginal cost can be written as

$$mc_t = \left(\zeta\frac{\phi+\alpha}{1-\alpha} + \sigma\right) c_t + \frac{(1-\zeta)(\phi+\alpha)}{\delta(1-\alpha)}\left(k_{t+1} - (1-\delta)k_t\right) - \alpha\frac{\phi+1}{1-\alpha}k_t.$$

It is apparent that for a given interest rate policy higher consumption raises inflation but it also makes profit maximizing firms increase their next-period capital stock in order to meet anticipated increased demand. Higher next-period capital then raises current-period inflation. The decisions to both increase consumption and the next-period capital stock are dynamic complements as defined, for example, in Cooper and John (1988). The existence of multiple *policy-induced private sector equilibria* becomes a likely outcome: the private sector may choose to react in several possible ways each of which is consistent with a corresponding policy forecast.[5]

Of course, the policymaker will react differently in response to different private sector actions. For each response of the private sector $\{k_k, c_k, \pi_k\}_j$, $j = 1, 2$ we can find the unique welfare-maximizing policy of the policymaker from (29)–(30), which we denote by $\left\{S^*_{kkj}, \iota^*_{kj}\right\}$. Of course, $\iota^*_k = \iota^*_{kj}(\iota_k)$, where $j = 1, 2$. By construction, all discretionary equilibria are among the points of

is assumed to be 0.025 and we set $\varepsilon_\psi = 30$. Finally, our value for the Calvo price stickiness parameter, $\theta$, is 0.75. We assume that the weight in the objective function is $\omega = \kappa/\varepsilon$. This relative weight is given in Woodford (2003a), Ch. 6 as a microfounded weight for a simple model without capital.

[4] Details are available from the authors.

[5] See King and Wolman (2004), Blake and Kirsanova (2012) for a discussion of policy induced private sector equilibria.

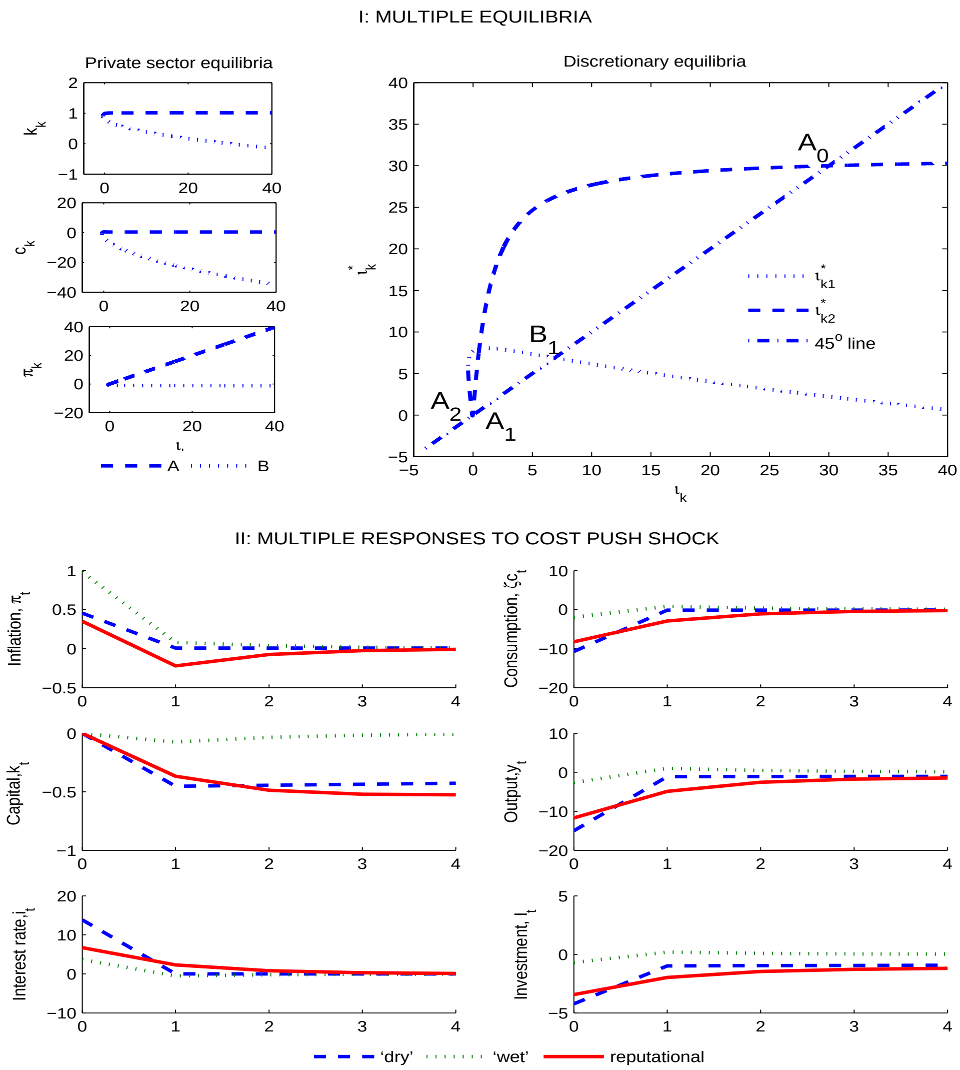


Figure 1: Multiple Discretionary Equilibria under Inflation Targeting

Table 1: Three discretionary equilibria

| Eqm. | Policy Reaction | Private sector Reaction | Loss matrix | Speed of adjustment | Loss |
|---|---|---|---|---|---|
| | $\begin{bmatrix} \iota_v & \iota_k \end{bmatrix}$ | $\begin{bmatrix} k_v & k_k \\ \pi_v & \pi_k \\ c_v & c_k \end{bmatrix}$ | $\begin{bmatrix} S_{vv} & S_{vk} \\ S_{vk} & S_{kk} \end{bmatrix}$ | $k_k$ | $L$ |
| $A_1$ | $\begin{bmatrix} 13.85 & -0.024 \end{bmatrix}$ | $\begin{bmatrix} -0.453 & 0.981 \\ 0.454 & -0.017 \\ -14.01 & 0.369 \end{bmatrix}$ | $\begin{bmatrix} 0.454 & -0.017 \\ -0.017 & 0.009 \end{bmatrix}$ | 0.981 | 1.297 |
| $A_2$ | $\begin{bmatrix} 13.70 & -0.056 \end{bmatrix}$ | $\begin{bmatrix} -0.446 & 0.978 \\ 0.475 & -0.050 \\ -13.84 & 0.354 \end{bmatrix}$ | $\begin{bmatrix} 0.475 & -0.050 \\ -0.050 & 0.049 \end{bmatrix}$ | 0.978 | 1.358 |
| $B_1$ | $\begin{bmatrix} 3.645 & 6.886 \end{bmatrix}$ | $\begin{bmatrix} -0.074 & 0.472 \\ 0.982 & -1.086 \\ -2.524 & -14.00 \end{bmatrix}$ | $\begin{bmatrix} 0.982 & -1.086 \\ -1.086 & 1.848 \end{bmatrix}$ | 0.472 | 2.805 |

Note: The figures express social losses (9) as a ratio to the social loss obtained under the commitment solution of a benevolent policymaker.

intersection of $\iota_k^* = \iota_{kj}^*(\iota_k)$ with $45^o$-degree line. The right chart in Panel I in Figure 1 illustrates this for the base-line calibration: we have two branches $\iota_k^* = \iota_{kj}^*(\iota_k)$, plotted with solid and dotted lines, which intersect the $45^o$-degree line in points labelled by $A$ and $B$ depending to which branch they belong.

Point $A_0$ is not a discretionary equilibrium, because parameter $d$ (see equations (18)–(26)) is equal to zero at this point, but all other points of intersection are discretionary equilibria. This can be verified by direct substitution of the corresponding set of coefficients into system (21)–(26), (29), (30) and (34).[6] We end up with three discretionary equilibria where the policymaker validates the private sector's beliefs in each case. In response to the same shock several adjustment paths are possible and coordination failure happens: the agents can coordinate on any of several possible paths. Some random exogenous event may then determine which equilibrium prevails.

[6] The error term is of order 10e-13.

Table 1 reports numerical characteristics, including the loss, of the three equilibria for the baseline calibration. Equilibria $A_1$ and $A_2$ have similar characteristics, which are very different from the characteristics of Equilibrium $B_1$.

Different characteristics of equilibria $A$ and $B$ imply different dynamics of the economy under discretionary policy. The second Panel in Figure 1 plots impulse responses of the economy to a unit cost-push shock in equilibria $A_1$ and $B_1$. The interest rate rises in response to a positive cost-push shock in both equilibria, but the amount by which interest rate rises is substantially greater in equilibrium $A_1$ than in $B_1$. As a consequence, in one equilibrium we see a larger fall in the output gap and a smaller rise in inflation than in the other equilibrium. We call equilibrium $A_1$ 'seemingly dry' and equilibrium $B_1$ 'seemingly wet', as it looks like the central bank has greater determination to combat the cost-push shock in the first equilibrium. Figure 1 also suggests that the 'seemingly dry' policymaker implements 'nearly optimal' solution as its actions are more similar to the ones under commitment than the actions of the 'seemingly wet' policymaker. For brevity we shall refer to these equilibria as to just 'dry' and 'wet' correspondingly, but we bear in mind that these two equilibria are generated by the same policy objective. Note that the 'dry' equilibrium is 'slow' and the 'wet' equilibrium is 'fast'. The slow adjustment results in higher social welfare and the 'dry' equilibrium $A_1$ is Pareto-preferred.

### 3.2 Coordination on the best equilibrium

The existence of multiple equilibria immediately raises two questions. Are all discretionary equilibria equally plausible? Can the agents coordinate on the best equilibrium? Dennis and Kirsanova (2012) introduce two coordination mechanisms which can help to understand which equilibria are more realistic and whether the policymaker can choose the Pareto-preferred equilibrium, building on work by Evans (1986); Bernheim et al. (1987); Bernheim and Whinston (1987).

The first coordination mechanism is iterative expectations (IE) stability under joint learning. Suppose the policymaker and private agents understand each other's optimization problem, and start learning about each other's actions with some joint guesses about equilibrium reactions, $x = \{\iota_k, k_k, c_k, \pi_k\}$. Their guess has to be consistent both with the optimization problem and with the evolution of the economy. Any discrepancy leads to an update of the initial guess, $\bar{x} = T(x)$. A discretionary equilibrium is IE-stable under joint learning if the revision process converges to this equilibrium, implying the ability of economic agents to learn and rationalize the RE equilibrium.

The second coordination mechanism is IE-stability under private sector learning. In response to the *given* equilibrium policy rule $\iota_k$ the private sector forms a guess about its own equilibrium reactions $x = \{k_k, c_k, \pi_k\}$. The guess has to be consistent with the evolution of the economy alone as the optimization problem is encapsulated in the rule. Again, a consistency check is used to update the initial guess, $\bar{x} = T(x)$. A discretionary equilibrium is IE-stable under the private sector learning if this revision process converges to this equilibrium, implying the ability of the private sector to learn and rationalize the RE equilibrium. Although this coordination mechanism only concerns the ability of *the private sector* to coordinate, this ability is crucial for the ability of several consequent policymakers to form a coalition and coordinate on the Pareto-preferred equilibrium in case there are multiple equilibria. If the Pareto-preferred equilibrium is IE-stable under the private sector learning, and the discretionary policymaker has a sufficiently long term in office, the policymaker can form a coalition with future selves and *guarantee* that such equilibrium will prevail.

For the baseline calibration equilibria $A_1$ and $B_1$ are IE-stable under joint learning, but only equilibrium $B_1$ is IE-stable under private sector learning. As a result, the Pareto-preferred 'dry' equilibrium $A_1$ is not self-enforceable.

### 3.3 Robustness of the results

It is the presence of capital in our model that drives the multiplicity of discretionary equilibria so we investigate the properties these equilibria and IE-stability for very wide ranges of the parameters which determine capital accumulation. These intervals are: for depreciation $\delta \in [0, 0.1]$, for the capital-labour ratio $\alpha \in [0, 1]$ and for capital adjustment costs $\varepsilon_\psi \in [0, 350]$.

For all combinations of the parameters considered (and for a wide range of $\iota_k$) there are always two real solutions to the system (21)–(26) and (34). This implies that there are always two lines, plotted as the dotted and dashed lines in Panel I in Figure 1. Either one or both of them intersect the $45^o$-degree line at points different from $A_0$. We can show numerically that only points of intersection of the dotted line $\iota_k^* = \iota_{k2}^*(\iota_k)$ – points labelled $B$ – are IE-stable under private sector learning. Welfare-preferred equilibria of 'type' $A$ are not IE-stable under private sector learning and, therefore, are not self-enforceable. Although economic agents can coordinate on equilibrium $A_1$ if they learn jointly, this does not ensure that this equilibrium will prevail unless equilibrium $A_1$ is unique.

Panel I in Figure 2 demonstrates the areas of multiplicity (and, by implication, the areas of IE-stability) under benevolent discretion. The 'x' in each chart marks the spot of our baseline calibration. The top left chart fixes the adjustment costs parameter at its baseline value and

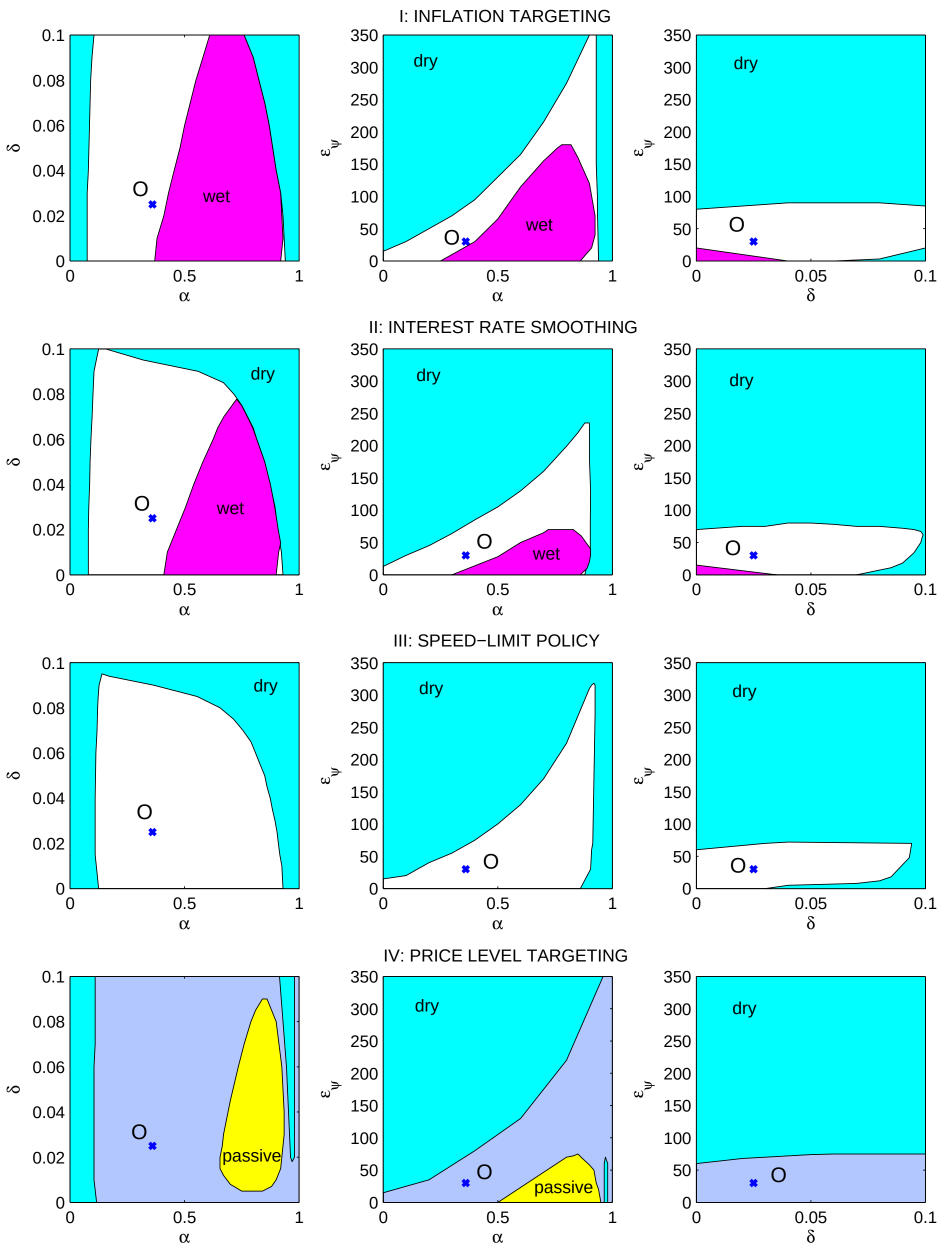


Figure 2: Areas of multiplicity for different delegation schemes.

varies the depreciation rate and the capital share. The other two charts in this row fix first the depreciation rate, then the capital share, allowing the other two to vary. The blue shaded regions in the charts are the regions of unique 'dry' equilibrium. It is apparent that multiplicity is widespread and we are likely to have a unique 'dry' equilibrium only if parameters of interest are pushed to their extreme to take on unrealistic values.

With higher adjustment cost of capital ($\varepsilon_\psi$) the evolution of capital is less affected by the behavior of economic agents and capital becomes more like an exogenous predetermined state. Then the rate of convergence of the economy back to the steady state is almost completely determined by the speed of adjustment of capital. No coordination failure happens and the 'dry' equilibrium is unique.

The region with a unique 'wet' equilibrium is shaded magenta. If $\alpha$ is not close to one but relatively large then efficient control of inflation in the 'dry' equilibrium requires more aggressive monetary policy. But a higher capital share requires less aggressive interest rate movements as they cause bigger investment and consumption movements and so the stabilization of the capital stock may become problematic. These two requirements conflict with each other. At some large value of $\alpha$ the 'seemingly dry' equilibrium does not exist. The region of the unique 'wet' equilibrium is very close to the spot of the baseline calibration, and the spot can be in the magenta area if we change the other parameters of the model – for example elasticities $\sigma$ and $\phi$ – only slightly.

As explained above, only the 'wet' equilibrium $B_1$ is IE-stable under the private sector learning for the baseline calibration. The Pareto-preferred 'dry' equilibrium $A_1$ is not IE-stable, whether or not it is unique. We argue that the 'dry' equilibrium is unlikely to realize for two reasons. First, it is unique only for unrealistic values of parameters and does not exist for the wide range of realistic parameters. Second, out of the two equilibria which are IE-stable under joint learning, the 'dry' equilibrium is not IE-stable under private sector learning, so no coalition of policymakers can 'select' it and economic agents are less likely to coordinate on it than on the 'wet' equilibrium.

# 4 Policy delegation and coordination on the best equilibrium

From these results the central questions of this paper naturally arise. First, how do delegation schemes – like speed limits, interest rate smoothing, or price level targeting – affect the likelihood of obtaining multiple equilibria? Second, if multiplicity survives – and, as we have already stated, it does – how can we quantify the welfare implications of choosing one of these delegation schemes? Finally, can any of these schemes affect the likelihood of successful coordination of economic agents

Table 2: Coefficient definitions for the delegation regimes

| Delegation Scheme | | Constraints | | | |
|---|---|---|---|---|---|
| | | $\omega_\pi$ | $\omega_y$ | $\omega_{\Delta y}$ | $\omega_i$ |
| Benevolent Discretion | | 1 | 1 | 0 | 0 |
| Conservative central bank | | 1 | $0 < \omega_y < 1$ | 0 | 0 |
| Interest rate smoothing | | 1 | 1 | 0 | $\omega_i > 0$ |
| Speed limit policy | (i) strict | 1 | 1 | $\omega_{\Delta y} = 1$ | 0 |
| | (ii) hybrid | | | $0 < \omega_{\Delta y} < 1$ | |
| Price level targeting | (i) strict | $\omega_\pi = 0$ | 1 | 0 | 0 |
| | (ii) hybrid | $0 < \omega_\pi < 1$ | | | |

Note: The coefficients $\omega_j$, $j \in \{\pi, y, \Delta y, i\}$ correspond to equation (35).

on the best equilibrium?

### 4.1 Overview of delegation regimes

As our paper is a reassessment of the gains associated with various delegation schemes for monetary policy we need to make these regimes concrete. They are associated with particular penalty functions assigned to the central bank that differ from the benchmark welfare function. We can encompassed all of them using the objective function

$$\frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}\left(\omega_\pi \pi_s^2 + (1-\omega_\pi)\,p_s^2 + \omega\,(1-\omega_{\Delta y})\,\omega_y y_s^2 + \omega\omega_{\Delta y}\Delta y_s^2 + \omega_i(\Delta i_s)^2\right). \tag{35}$$

With this we can implement four modifications to benevolent discretion by the following choices of the weights $\omega_j$, $j \in \{\pi, y, \Delta y, i\}$ in Table 2.

The various proposals are nested in the following way. Price level targeting (the focus of Svensson, 1999; Vestin, 2006) is implemented by *replacing* the term in the inflation rate in the social welfare function by a term in the price level; we call $\omega_\pi$ the regime-defining parameter. The conservative central bank proposal (Rogoff, 1985) is implemented by increasing the *relative* weight on inflation with the regime-defining parameter now $\omega_y$. Interest rate smoothing (Woodford, 2003b) is implemented by *adding* to the social welfare function a term in the change in the interest rate, with $\omega_i$ the regime-defining parameter. Finally, the speed limit policy (Walsh, 2003) is achieved by *replacing* the term in the level of the output gap in the social welfare function with a term in the change in the output gap; $\omega_{\Delta y}$ is the regime-defining parameter. For price level targeting and speed limit policies we also consider their hybrid forms, using the full range of regime-defining parameters, as this allows a clearer comparison with benevolent discretion/commitment.

In the next two subsections we present our findings about which equilibria arise under these delegation schemes. We use the benevolent commitment regime as a benchmark. As the behavior of the economy is qualitatively similar under the conservative central bank, interest rate smoothing and speed limit regimes, we discuss them together. However, the behavior of the economy is different in an important way if price level targeting is implemented so we treat that separately.

We classify the equilibria that then exist as follows. Denote the array of parameters that characterize a delegation scheme as $\boldsymbol{\omega} = (\omega_\pi, \omega_y, \omega_{\Delta y}, \omega_i)$. Benevolent discretion we denote as $\boldsymbol{\omega}_0 = (1, 1, 0, 0)$. Any discretionary equilibrium can be characterized by the set of deterministic components of policy functions and responses of the private sector $\mathcal{D} = \{\iota_k, k_k, c_v, c_k, \pi_k, S_{kk}\}$. Of course, $\mathcal{D}$ is a function of parameters of the system that include policy parameters $\boldsymbol{\omega}$. Suppose we discover an equilibrium $\mathcal{D}(\boldsymbol{\omega})$. We say that $\mathcal{D}$ is either a 'wet' or 'dry' discretionary equilibrium, if $\lim\limits_{\boldsymbol{\omega}\to\boldsymbol{\omega}_0} \mathcal{D}(\boldsymbol{\omega})$ is either a 'wet' or 'dry' equilibrium under benevolent discretion. Finally. we only report equilibria which are IE-stable under joint learning.

### 4.2 Interest rate smoothing, speed limits and the conservative central bank

Panel II in Figure 2 demonstrates that the large region of multiplicity is preserved when we include the interest rate smoothing term. We plot these figures assuming policy weights $\omega_i = \omega_i^* = 0.01$ such that $\omega_i^*$ maximizes social welfare in the best equilibrium, see also Table 3. Equilibria that we find for these delegation regimes are the three equilibria, which include IE-stable under joint learning 'dry' and 'wet' equilibria discussed in the previous section, as $\lim\limits_{\omega_i\to 0} \mathcal{D}(1, 1, 0, \omega_i) = \mathcal{D}(1, 1, 0, 0)$ for all three equilibria which we find.[7]

It is apparent that the area of a unique 'wet' equilibrium shrinks. The explicit requirement to move interest rates more smoothly lessen the conflict between the control of inflation and ensuring stability of the capital stock. As a result, the baseline calibration places the economy further away from the area of a unique 'wet' equilibrium.

The results are very similar for speed limits, where again we find 'dry' and 'wet' equilibria. Panel III in Figure 2 plots areas of multiplicity and uniqueness for the strict speed limit case, with $\omega_{\Delta y} = 1$. (We can also show that replacing the output target with a term in the change in the output gap maximizes social welfare.) A similar requirement of smoother policy, avoiding

[7] All our results are numeric. Here and below we look at the following data in order to argue that these equilibria are the same. For $\omega_i =$ 1e-6 and for each of the three equilibria $j = 1, 2, 3$ $\|\mathcal{D}_i - \mathcal{D}\|^j = \max(abs(\mathcal{D}_i^j - \mathcal{D}^j)) \approx$ 1e-6. With smaller $\omega_i$ the distance $\|\mathcal{D}_i - \mathcal{D}\|$ is nearly proportionally reduced for each equilibrium.

Table 3: Social loss of delegation regimes relative to the social loss loss under commitment of benevolent policymaker

| Delegation Regime | | Regime-defining parameter | 'Seemingly Dry' | 'Seemingly Wet' | 'Passive' |
|---|---|---|---|---|---|
| (1) | Benevolent Discretion | – | 1.2966 | 2.8046 | – |
| (2) | Conservative Central Bank | $\omega_y = 0.95$ | 1.2955 | 2.8060 | – |
| (3) | Interest Rate Smoothing | $\omega_i = 0.01$ | 1.2200 | 2.7863 | – |
| (4) | Strict Speed-Limit Policy | $\omega_{\Delta y} = 1$ | 1.0523 | 2.7894 | – |
| (5) | Strict Price Level Targeting | $\omega_\pi = 0$ | 1.0092 | – | 6.4055 |

Note: The figures express *social* losses as a ratio to the social loss obtained under the commitment solution of a benevolent policymaker (9).

large changes in output and thus all interrelated economic variables, lessens the conflict between the control of inflation and ensuring stability of the capital stock. The area of a unique 'wet' equilibrium does not exist for the values of parameters chosen for the graphs. Actually it does not completely disappear, but is reduced by more than for the interest rate smoothing scheme. One very notable outcome is that strict speed limit targeting results in large welfare gain in the 'dry' equilibrium: Table 3 suggests that the initial loss which without delegation is 30% higher than under commitment is reduced to become only 5% higher. However, both interest rate smoothing and speed limits result in slightly higher social loss in the 'wet' equilibrium.

Policy delegation to a conservative central bank does not result in any very noticeable changes to the areas of multiplicity although they do shrink slightly, so we do not show the corresponding charts. Table 3 reports that the optimal degree of conservatism is achieved with only small reduction of the relative weight on the output gap stabilization term, and the resulting welfare gain is relatively small. Moreover, implementation of such a scheme leads to a (slight) increase of the welfare loss in the 'wet' equilibrium.

To summarize, all three delegation schemes reduce the area of unique 'wet' equilibrium and increase the area of the unique 'dry' equilibrium. However, parameters of the model which result in the unique dry equilibrium are still in the unrealistic range. All three delegation schemes preserve all three equilibria, including 'dry' and 'wet' equilibria, i.e. when the regime-defining parameter tends to its value under benevolent discretion, the equilibria tend to the corresponding equilibria found there. The behavior of the economy in any of these equilibria is qualitatively similar to the one in the corresponding equilibrium under benevolent discretion, therefore we do not plot impulse responses.

Crucially, none of these delegation schemes change the stability properties of the 'dry' and 'wet' equilibria: for all considered parameter values both equilibria remain IE-stable under joint learning, and only the 'wet' equilibrium is IE-stable under private sector learning. We conclude, therefore, that these delegation schemes are unlikely to change the likelihood of successful coordination of the economic agents on the Pareto-preferred equilibrium.

### 4.3 Price level targeting

Under price level targeting the multiplicity is also widespread. The second row of charts in the top part of Figure 2 shows that the economy is still in the large area of multiplicity under the strict price level targeting, which is the limiting case of the hybrid price level targeting. However, there are important differences in properties of discretionary equilibria we find here. We only find two 'dry' equilibria, namely the two discovered equilibria tend to equilibria $A_1$ and $A_2$ under benevolent discretion if we take the limit: $\lim_{\boldsymbol{\omega}\to\boldsymbol{\omega}_0} \mathcal{D}(\boldsymbol{\omega})$. The third equilibrium, which is discovered under the hybrid price level targeting, does not exist for benevolent discretion ($\omega_\pi = 1$). We term this third equilibrium as 'passive', as it is characterized by a fall in interest rates in response to a positive cost-push shock, see impulse responses in the left side of Figure 3.[8]

This result shows, importantly, that *new* equilibria can arise under delegation policies. In order to understand this result recall that the stationarity of the price level under any degree of price-level targeting ($\omega_\pi < 1$) requires inflation overshooting when it converges back to the steady state.

It is possible to achieve inflation overshooting in two ways. A reputational policymaker raises interest rate and *keeps it high* for longer to ensure negative marginal cost, i.e. below its steady state level. Negative marginal cost means inflation should *rise* while converging to the steady state zero level. Following a cost push shock and an interest rate rise inflation falls in the first period, overshoots the zero level and then converges to the steady state from below, rising. In the 'dry' equilibrium policy maker tries to repeat this policy, but under a time-consistency constraint. Similar to the commitment case, marginal cost is kept below zero for most of the periods, inflation overshoots the zero steady state level and the price level is stationary.

However, the policymaker can keep marginal cost below zero by keeping capital stock suffi-

[8] Although we compute all limits numerically, and it is difficult to argue discontinuity because of this, we shall see that there are striking differences between the properties of 'wet' equilibrium with $\omega_\pi = 1$ and the 'passive' equilibrium with $\omega_\pi = 0.999$.

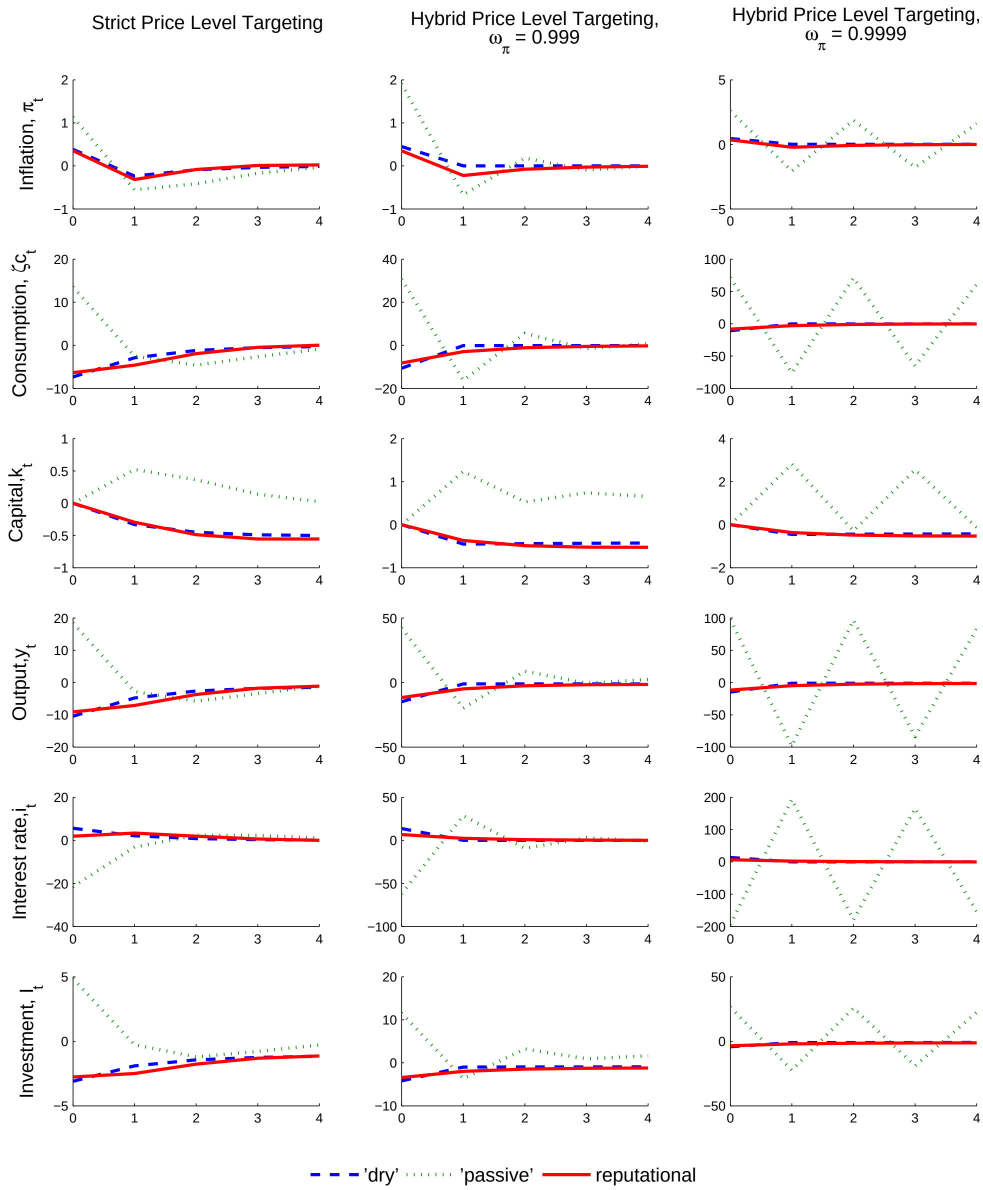


Figure 3: Impulse responses to a unit cost-push shock under the price-level targeting

ciently high *for some time.* This is achieved by lowering interest rate sharply in the response to a cost-push shock, see the dotted line scenario in the first column of plots in Figure 3. Following a sharp fall in interest rate and thus a negative real interest rate, future consumption falls below its steady state level. An initial rise in investment leads to higher stock of capital one period later. As all its components fall, the marginal cost also falls below the steady state level one period after the shock; it also stays below for several consequent periods and this ensures inflation overshooting and stationarity of the price level. Stabilization of the economy and, thus, the capital stock back to the steady state level requires small disinvestment within long period of time. Consumption also stays below the steady state level but rises as interest rates remain low. All this ensures inflation remains negative for a long period of time while the economy adjusts towards the steady state.

The second column of charts demonstrates that as $\omega_\pi$ becomes closer to one then there is a smaller need to bring the price level back to the steady state. Additionally, controlling inflation variability gains the priority (we consider the hybrid price level targeting). Therefore, monetary policy wants to bring price level down *slowly.* This, however, is impossible to do *smoothly,* under a time-consistent policy. For any given $\omega_\pi < 1$ monetary policy still has to ensure inflation overshooting. But if inflation stays below zero *for a long time,* then the price level falls *too* quickly for a small penalty $1 - \omega_\pi$ and the inflation cost dominates the loss. So, inflation might need to rise quicker and even to return to the positive area again. To achieve this, capital cannot *stay* high, it should go down to increase marginal cost and this would allow inflation to rise. Interest rate has to go *down* to allow this increase in marginal cost. The second column of plots in Figure 3 suggests that when $\omega_\pi$ becomes bigger ($\omega_\pi = 0.999$) all variables have to change direction of movements while converging to the steady state. Further reduction in penalty $1-\omega_\pi$ requires a clear 'zig-zag' dynamics for all economic variables, this ensures slow convergence of price level and (relatively) small inflation cost, see the third column of charts in Figure 3.[9]

For the base line calibration the 'passive' solution does not exist if $\omega_\pi$ is equal to one. With unit-root dynamic process for the price level, the inflation 'zig-zags' should be symmetric with respect to the zero inflation line, but this cannot be achieved in an economy with investment and both a positive depreciation rate and adjustment costs.

This explains the existence of the 'dry' and 'passive' equilibria under price level targeting. The 'wet' equilibrium, however, cannot exist for any $1 - \omega_\pi > 0$ (the numerical threshold is $10^{-14}$). The reason for this is again the need of inflation overshooting. If $\omega_\pi = 1$ the seemingly

[9] Neither Batini and Yates (2003) nor Roisland (2006) study this conflict of targets.

wet policymaker initially raises the interest rate in order to lower it sharply in the next period so that the resulting higher investment move capital to the steady state quickly. Moreover, it lowers the interest rate by more than a 'dry' policymaker does. Under this regime there is no extra requirement to control the additional stock variable, the price level. When stabilizing the price level is added to the objective, the second-period reduction in interest rate is not helpful: it does not generate inflation overshooting and so does not lead to the stationarity of the price level. That is why any small $\omega_\pi < 1$ that would, by continuity, lead to smaller *fall* in interest rate in the second and consequent periods, would not correspond to any *price-stationary* equilibrium.

In Figure 2 we only demonstrate the areas of multiplicity for strict price level targeting, where we reduce the relative weight on inflation, $\omega_\pi$, to zero as originally proposed by Vestin (2006).

Unlike the other delegation schemes discussed in Section 4.2, price level targeting *changes the stability properties* of the equilibria: all three equilibria, including the 'dry' equilibrium, are now IE-stable under private sector learning. Only the 'dry' and 'passive' equilibria are IE-stable under joint learning. IE-stability under private sector learning implies that the Pareto-preferred equilibrium is self-enforceable and a coalition of policymakers can ensure it realizes.

The above results suggests – in contrast to the other delegation schemes considered – adopting a price lever targeting scheme does affect the likelihood of successful coordination of the economic agents on the best equilibrium. Although our model is very simple, driven by very few parameters and in particular that numerical computations show that the required coalition is extremely and unrealistically long (about 200 quarters), the mere existence of such a coalition moves price level targeting to a different class of policies where the Pareto-preferred equilibrium is made self-enforceable. This result is likely to remain valid if we enrich the model with other features to make it more empirically relevant.

# 5 Summary of Results and Conclusions

This paper revisited the idea that speed limit, interest rate smoothing, price level targeting delegation schemes and straight conservatism can reduce the stabilization bias in monetary policy models. All of these delegation schemes were previously studied for economies with a unique time-consistent equilibrium. In this paper we work with a more general class of LQ RE models with expectations traps under discretionary policy, specifically, a New Keynesian model with capital accumulation.

We show that multiplicity survives under all studied delegation schemes. Not only that, we show that different delegation schemes can result in new equilibria arising, and these new equilibria can be worse than the worst equilibrium under the benevolent discretion. Although we confirm the previous results that all these delegation schemes *can* improve social welfare, we show that they only improve welfare if the economy remains in the best equilibrium. Thus the welfare consequences of delegation are ambiguous, since in order to quantify the benefit one needs to know from which equilibrium the economy starts and to which it will move under delegation.

However, delegation clearly still has advantages. It appears to shrink the parameter space for which only the worst equilibrium survives for all of the regimes we consider. Rather than being condemned to the bad equilibrium, the multiplicity under delegation at least offers a chance that the economy could end up in the good equilibrium. We also demonstrate that a delegation scheme can change the way agents can coordinate on an equilibrium. In particular, we show that price level targeting increases the likelihood that the best equilibrium manifests. Under this scheme the Pareto-preferred equilibrium is self-enforceable, so there is a coalition of subsequent policymakers which can coordinate and enforce it. This, however, is impossible in the other delegation schemes considered.

Despite demonstrating these results using a particular model, this model is at the core of more general and empirically relevant DSGE models widely used in policy analysis. Our results are likely to remain valid for this wide class of models.

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