Aggregate and welfare effects of long run inflation risk under inflation and price-level targeting[1]

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Abstract

This paper presents a DSGE model in which long run inflation risk matters for social welfare. Aggregate and welfare effects of long run inflation risk are assessed under two monetary regimes: inflation targeting (IT) and price-level targeting (PT). These effects differ because IT implies base-level drift in the price level, while PT makes the price level stationary around a target price path. Under IT, the welfare cost of long run inflation risk is equal to 0.35 per cent of aggregate consumption. Under PT, where long run inflation risk is largely eliminated, it is lowered to only 0.01 per cent. There are welfare gains from PT because it raises average consumption for the young and lowers consumption risk substantially for the old. These results are strongly robust to changes in the PT target horizon and fairly robust to imperfect credibility, fiscal policy, and model calibration. While the distributional effects of an unexpected transition to PT are sizeable, they are short-lived and not welfare-reducing.

Keywords: inflation targeting, price-level targeting, inflation risk, monetary policy.

JEL Classification: E52

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1. Introduction

The payoffs on many long-term contracts are fixed in nominal terms. Consequently, unanticipated changes in inflation that are not reversed lead to fluctuations in real wealth. Such fluctuations are important for retirees, since they must fund their consumption in old age using wealth accumulated in long-term nominal assets like pensions and bonds. Inflation risk also has implications for younger generations because it raises the average cost of issuing nominal government debt by the 'inflation risk premium',[2] increasing the level of taxes needed to finance a given level of government expenditure, or lowering the amount of government expenditure that can be funded from a given tax take. These aggregate effects could be important for two reasons. First, the amount of wealth accumulated in long-term nominal assets in developed economies is substantial (Doepke and Schneider, 2006; Meh and Terajima, 2008). Second, since many central banks have adopted formal inflation targets, inflation risk over a long horizon can be non-trivial even if yearly deviations from the inflation target are small, because there is base-level drift in the price level.[3] Assessing these aggregate effects speaks to the need for a careful general equilibrium analysis that takes into account the major effects of long run inflation risk, including those for government finances.

In this paper, an analysis of this kind is conducted using a dynamic stochastic general equilibrium (DSGE) model. In particular, aggregate and welfare effects of long run inflation risk are assessed under two different monetary policy regimes: inflation targeting (IT) and price-level targeting (PT). The main difference between these two regimes is that IT ignores unanticipated shocks to the price level, while PT offsets them. Specifically, a PT regime aims to return the price level to a target path for prices that is known *ex ante*. The price level is thus trend-stationary and can be predicted with much greater certainty at long horizons, so that long run inflation risk is largely eliminated by a PT regime. Given the prevalence of long-term nominal contracts in developed economies, assessing the aggregate and welfare effects of long run inflation risk is important for comparing IT and PT regimes. In recent years, both academics and policymakers have become interested in this comparison.

By contrast to IT, a PT regime has never been implemented in practice.[4] However, several papers have shown that PT offers short-term stabilisation benefits over IT when agents are forward-looking. Vestin (2006), for example, shows that in the standard New Keynesian model, PT reduces inflation variability for a given level of output gap variability when policy is discretionary. In the same model, the optimal commitment policy implies a stationary price level (Clarida, Gali and Gertler, 1999) and is therefore consistent with a PT mandate.[5] In addition to this literature, the Bank of Canada recently conducted a detailed review of PT (see Bank of Canada, 2011) and has contributed a large body of research to the literature, mainly from forward-looking models calibrated to the Canadian economy.[6] To date, however, no

[2] For a recent survey of the inflation risk premium, see Bekaert and Wang (2010).

[3] That is, the price level follows a random walk. Inflation risk increases with the forecast horizon in this case because inflation between period t and $t+k$ depends on the ratio of the price level in $t+k$ to that in period t.

[4] Berg and Jonung (1999) argue that PT was adopted in Sweden during the Great Depression, but this is disputed by Straumann and Woitek (2009).

[5] The issue of whether optimal policy in the New Keynesian model implies base drift is controversial. Negative results include Steinsson (2003), Levin *et al.* (2010) and Amano, Ambler and Shukayev (forthcoming).

[6] This work is surveyed in Ambler (2009), Crawford, Meh and Terajima (2009) and Bank of Canada (2011).

paper has assessed the aggregate and welfare effects of IT and PT in a DSGE model in which long run inflation risk matters for social welfare. The main contribution of this paper is to provide a first assessment of this kind.

An overlapping generations (OG) model with money is calibrated to roughly match the portfolios of UK retirees. The model has three features that make it useful for assessing aggregate and welfare effects of long run inflation risk. First, long run inflation risk matters for social welfare because young generations hold long-term nominal bonds – whose payoff is revalued by unanticipated inflation – in order to provide for old age when they are retired. By contrast, only short-term inflation risk matters for social welfare in the standard New Keynesian model (Woodford, 2003). Second, in an OG framework the effects of long run inflation risk on young and old generations can be assessed directly, hence providing useful information on the distributional effects of monetary policy. Third, the life-cycle setup of the OG model, where each period lasts between 20 and 30 years, allows one to parsimoniously model inflation risk and equilibrium asset prices over a long horizon without introducing a large number of additional state variables.[7] In sensitivity analysis, the model is calibrated for nominal portfolios in Canada as a check on the importance of portfolio structure and, subsequently, for Canadian price level shocks in order to estimate the aggregate and welfare effects of long run inflation risk in Canada.

Key findings are as follows. Under IT, the welfare cost of long run inflation risk is substantial at 0.35 per cent of aggregate consumption. Under PT, where long run inflation risk is largely eliminated, the welfare cost is reduced to only 0.01 per cent, implying a permanent welfare gain from PT of 0.34 per cent. PT increases social welfare because it lowers consumption risk substantially for old generations and raises average consumption by the young. Consumption risk for old generations is lower under PT because reducing long run inflation risk stabilises the real payoff on nominal retirement assets held by consumers. This effect accounts for around one-quarter of the total welfare gain. At the same time, PT raises average consumption by the young since a fall in long run inflation risk lowers the inflation risk premium on nominal government debt,[8] so that the same long run level of government spending as under IT can be maintained with lower taxes, hence raising disposable income. Since this latter effect accounts for three-quarters of the total welfare gain from PT, the analysis clearly highlights the importance of a general equilibrium approach that takes into account higher-order effects of inflation risk, including those for government finances.[9]

Although PT raises social welfare, its aggregate effects are not unambiguously positive. The old benefit from a reduction in consumption risk of more than one-tenth under PT, but consumption risk rises for the young, albeit only marginally. Moreover, average consumption by old generations falls non-trivially under PT, although the magnitude of this reduction is lower than the rise in average consumption for the young, so that each unborn generation can expect higher lifetime consumption under PT. These results are of interest from a distributional perspective since they suggest that the current old generation might lose out in

[7] In the New Keynesian model, for example, it would be necessary build up the term-structures of real and nominal interest rates over many quarters to match a 30-year holding horizon (as in the model here). To do so would require many additional state variables, making a large number of stochastic simulations infeasible.

[8] Rudebusch and Swanson (2012) report that a PT regime essentially eliminates the term premium on 10-year nominal bonds in a DSGE model, due to the reduction in long run inflation risk it engenders.

[9] Failure to account for higher-order effects of risk can lead to spurious welfare reversals (Kim and Kim, 2003).

a transition from IT to PT. However, formal analysis of unexpected transitions from IT to PT indicates that all generations alive at the time of the transition – including the current old – can expect to gain in utility terms from a switch in regime to PT.

Returning to long run impacts, several extensions of the baseline model were made to test robustness. First, if the price level is returned to its target path gradually over several years, most of the long run inflation risk present under IT is still eliminated, so that both the welfare gain and aggregate effects of PT are strongly robust. Second, the welfare gain from PT is reduced, but still quantitatively non-trivial, if there is a moderate or high degree of imperfect credibility. The reason is that the belief that policy may revert to IT raises the inflation risk premium on nominal government debt, so that taxes must be raised to maintain the same long run level of government spending. In turn, this leads to lower consumption by the young than in the perfect credibility case. Third, the assumption that the government sets taxes to meet a long run government spending target was relaxed in favour of a specification where fiscal policy equalises taxes across regimes but provides public goods valued by households. In this case the welfare gain from PT was somewhat lower, though still non-trivial, at around one-third of the baseline estimate, or 0.1 per cent of aggregate consumption. The extensions with respect to imperfect credibility and fiscal policy thus suggest that the baseline quantitative estimates should be treated with some caution.

Formal sensitivity analysis was split into two parts. First, sensitivity of the baseline results was tested by varying individual calibrated parameters. This analysis shows that the welfare gain from PT is quite sensitive to household risk aversion, the share of indexed government debt, and the extent of nominal risk, being higher in economies with greater risk aversion, less indexed debt and more nominal risk. Second, the model was calibrated for nominal portfolios in Canada, where nominal assets play a more important role in retirement income than in the UK. The results suggest that a shift towards Canadian portfolios in the UK would raise the welfare gains from PT substantially, from 0.34 to 0.64 per cent of aggregate consumption. However, when this model is additionally calibrated to match the lower magnitude of Canadian price level shocks over the IT period, the aggregate and welfare effects of long run inflation risk are similar to those in the UK. Consequently, the potential long run welfare gains from PT are likely to be broadly similar in the UK and Canada.

The analysis in this paper is also related to literature on the aggregate effects of unanticipated inflation. In a seminal paper, Doepke and Schneider (2006) document nominal portfolios in the US and show that an unanticipated increase in inflation has substantial redistributional effects through revaluations of nominal assets and liabilities. Subsequently, Meh and Terajima (2008) examined nominal portfolios in Canada. Building on these two papers, Meh, Rios-Rull and Terajima (2010) simulate aggregate and welfare effects from *one-off* episodes of unanticipated inflation in Canada under IT and PT. They find that unanticipated inflation has greater redistribution effects under IT because the initial change in inflation is not reversed, so that long-term nominal contracts undergo substantial revaluations. Consequently, induced welfare losses are somewhat larger under IT. However, an important limitation with their welfare analysis is that they consider only one-off shocks to inflation, not all possible realizations.[10] Since the current paper rectifies this limitation, it should provide additional insight into the welfare effects of PT in Canada through the nominal portfolio channel.

[10] That is, they consider individual draws from the distribution of shocks and not the entire distribution. As noted by Meh *et al.*, the latter is necessary to account for the impact of higher-order moments on social welfare.

The paper proceeds as follows. Section 2 presents the model and explains how it is used to assess the welfare cost of long run inflation risk. Section 3 describes monetary policy. In Section 4 the model is calibrated, and in Section 5 baseline simulation results are reported. Section 6 presents extensions relating to the PT target horizon, imperfect credibility, and fiscal policy. Section 7 conducts a sensitivity analysis. Finally, Section 8 concludes.

2. Model

The model has three sectors: a household sector, a government sector, and a sector devoted to production of a single output good. Each sector is explained in detail below, beginning with the household sector. This section also describes the aggregate resource constraint and the relationship between the preferences of individual generations and social welfare.

2.1 Consumers

A simple overlapping generations (OG) model with two generations is considered. Each generation is modelled as a representative consumer. These generations inelastically supply a unit of labour when young and retire in the second period when old, leaving no bequests to future generations. Let the subscripts $\{Y,O\}$ denote, respectively, the young and the old. Each period in the model lasts 30 years in order to match a plausible holding horizon for long-term assets such as pensions. There is no population growth, and the number of generations per period is constant and normalized to 1. The real wage income of each young generation is taxed by the government at a constant rate, τ. Young agents have access to four assets: indexed government bonds, b^i; nominal government bonds, b^n; capital, k; and money, m. Indexed government bonds and capital are real assets whose payoff is not affected by unanticipated inflation. By contrast, the payoffs on nominal government bonds and money – the two nominal assets in the model – are revalued by unanticipated changes in inflation.

When young, each generation consumes and chooses a portfolio of assets, $z \equiv (k, b^i, b^n, m)$, to enable them to consume in old age when retired. Capital earns a real gross rate of return r^k, which is taxed by the government at rate τ^k. Government bonds are in positive net supply. Indexed bonds pay a riskless real return of r^f, and nominal bonds a riskless nominal return R, both of which are endogenous. These returns ensure that, for each type of bond, demand is equated to the supply set by the government. Indexed bonds are a riskless real asset, whereas nominal bonds are riskless but for unanticipated inflation over the 30-year holding horizon from youth to old age. Money pays zero interest, implying a real gross return $r^m = 1/(1+\pi)$, where π is the rate of inflation between youth and old age (i.e. over a 30-year horizon). A positive demand for money results from a cash-in-advance (CIA) constraint which requires young agents to hold real money balances of at least $\delta > 0$, so that $m_t \geq \delta$ as in Champ and Freeman (1990). The main advantage of this constraint is that it provides a role for money without requiring that it offer explicit transactions services, thereby ensuring that differences in welfare under IT and PT are attributable to long run inflation risk and not transactions services derived from money. The CIA constraint binds with equality if $R_t > 1$ for all t, which was comfortably satisfied in all numerical simulations reported in this paper.[11] Let $r^n \equiv R/(1+\pi)$ denote the real return on nominal bonds.

[11] This condition is derived in the Technical Appendix (see section B).

Mathematically, the budget constraints faced by generations alive in period t are given by

$$c_{t,Y} = (1-\tau)w_t - k_{t+1} - b_{t+1}^i - b_{t+1}^n - m_t \tag{1}$$

$$c_{t,O} = (1-\tau^k)r_t^k k_t + r_{t-1}^f b_t^i + r_t^n b_t^n + r_t^m m_{t-1} \tag{2}$$

where (1) is the budget constraint of the young, and (2) is the budget constraint of the old.[12]

Given the focus in this paper, it is crucial to specify consumer preferences that can potentially match some of the main features of household attitudes to risk highlighted in empirical research. As is well known, standard CRRA preferences cannot match risk premia and the risk-free rate, because they imply that the elasticity of intertemporal substitution is the reciprocal of the coefficient of relative risk aversion. Consequently, Epstein and Zin (1989, 1991) and Weil (1989) preferences are used here. These preferences have the advantage that the elasticity of intertemporal substitution and the coefficient of relative risk aversion can be calibrated separately. In a recent paper, Rudebusch and Swanson (2012) show that this feature enables an otherwise standard New Keynesian model to match the term premium on nominal bonds without compromising its ability to fit key macroeconomic variables.

Consumers solve a maximisation problem of the form

$$\max_{\{c_{t,Y},\, z_t,\, c_{t+1,O}\}} u_t(c_{t,Y}, c_{t+1,O}) = \frac{1}{1-\gamma}\left[c_{t,Y}^{\varepsilon} + \beta\left[E_t c_{t+1,O}^{1-\gamma}\right]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma}{\varepsilon}} \quad \text{s.t. (1) and (2)} \tag{3}$$

where $0 < \beta < 1$ is households' discount factor, γ is the coefficient of relative risk aversion, and $1/(1-\varepsilon)$ is the elasticity of intertemporal substitution.

The first-order conditions are summarized by the following Euler equations:

$$1 = E_t(sdf_{t+1}(1-\tau^k)r_{t+1}^k) \qquad \text{for capital, } k \tag{4}$$

$$1 = E_t(sdf_{t+1} r_{t+1}^n) \qquad \text{for nominal bonds, } b^n \tag{5}$$

$$1 = r_t^f E_t(sdf_{t+1}) \qquad \text{for indexed bonds, } b^i \tag{6}$$

$$1 = E_t(sdf_{t+1} r_{t+1}^m) + \tilde{\mu}_t \qquad \text{for money, } m \tag{7}$$

where $sdf_{t+1} \equiv \beta\left(\frac{c_{t,Y}}{c_{t+1,O}}\right)^{1-\varepsilon}\left(\frac{c_{t+1,O}}{(E_t c_{t+1,O}^{1-\gamma})^{1/(1-\gamma)}}\right)^{1-\gamma-\varepsilon}$ and $r_{t+1}^n \equiv R_t/(1+\pi_{t+1})$.[13]

[12] The return r^f is dated in period t-1 because it is known by consumers when they save for old age. The real return on nominal bonds r^n is dated in period t because it is not known at the time consumers save for old age. Capital and bonds are dated t+1 but are decided at the end of period t, before shocks in period t+1 are realized.

[13] Here $\tilde{\mu}_t$ is the ratio of the Lagrange multiplier on the CIA constraint to that on the young's budget constraint. For a full derivation of consumers' first-order conditions, see section A of the Technical Appendix.

2.2 Firms

The production sector of the economy consists of a representative firm which produces output using a production function with constant returns to scale. The firm hires capital and labour in competitive markets to maximise current period profits, taking the wage rate and capital rental rate as given. Total factor productivity, A, is stochastic and follows an AR(1) process in logs. The production function is Cobb-Douglas, with the share of capital in output equal to α and the labour share equal to $1 - \alpha$.

The real wage and the return on capital are thus given by

$$w_t = y_t - r_t^k k_t = (1-\alpha) A_t k_t^{1-\alpha} \tag{8}$$

$$r_t^k = \alpha y_t / k_t = \alpha A_t k_t^{\alpha-1} \tag{9}$$

2.3 Government

The government performs three functions. First, to meet government spending commitments, it taxes wage income of the young at a constant rate $\tau > 0$, and capital income of the old at a constant rate $\tau^k > 0$. Second, it sets the total supply of government bonds. Third, the government conducts monetary policy by committing to an IT or PT money supply rule.

The government budget constraint is given by

$$g_t = \tau w_t + \tau^k r_t^k k_t + b_{t+1}^i - r_t^i b_t^i + b_{t+1}^n - r_t^n b_t^n + m_t - r_t^m m_{t-1} \tag{10}$$

The total supply of government bonds is $b = b^i + b^n$, and the shares of indexed and nominal government bonds in the total bond portfolio are constant and equal to v and $1–v$, respectively.[14] Since the tax rates on wage income and capital are constant, it follows that $\tau^k = a\tau$ for some finite constant $a > 0$.

The government budget constraint can therefore be rewritten as

$$g_t = \tau(w_t + a r_t^k k_t) + b_{t+1} - [v r_t^i + (1-v) r_t^n] b_t + m_t - r_t^m m_{t-1} \tag{11}$$

Since there are no social transfers in the model, it is assumed that the government sets the total supply of government bonds to facilitate consumption smoothing between youth and old age. In particular, it chooses the total supply of government debt so that $\beta^{-1} E_t(sdf_{t+1}) = 1$, which implies that the marginal utility of consumption when young is equated to the expected (undiscounted) marginal utility of consumption when old. This assumption implies perfect consumption smoothing in the deterministic steady-state and (hence) a steady-state real interest rate $1/\beta$. [15] Consequently, there is a degree of social insurance in the model without the burden of explicitly modelling a social security system.

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[14] The share of indexed bonds is calibrated to match data. Alternatively, the indexation share could be chosen optimally. This approach was not taken here because the aim is to assess the aggregate and welfare effects of long run inflation risk, not the way in which fiscal/debt policy could mitigate this risk.

[15] This assumption ensures that the gross real return on government bonds exceeds 1 and that the bond supply is stationary. Consequently, the standard transversality condition on government debt will be satisfied.

The government sets the nominal money supply according to an IT or PT rule. These policy rules are discussed in Section 3. Subject to the monetary rule implemented and the equilibrium conditions of the model, the government sets the constant tax rate τ to ensure that it achieves a given long run target ratio of government spending to output, or $E(g_t/y_t) = G^* > 0$, where E is the unconditional expectations operator.[16] Although τ is constant over time, it will generally differ under IT and PT because the level of long run inflation risk affects the average real return on money balances and the inflation risk premium on nominal government debt. It should thus be understood that the tax rate is regime-specific, though this dependence is suppressed in order to minimize notational burden.

2.4 Social welfare

Welfare is given by the socially-discounted sum of lifetime utilities across all generations:[17]

$$U = E\left[\sum_{t=0}^{\infty}\omega^t u_t\right] = \frac{E(u_t)}{1-\omega} \qquad (12)$$

where $0<\omega<1$ is the social discount factor, and E is the unconditional expectations operator.

The welfare gain from eliminating long run inflation risk is computed as the fractional increase in aggregate consumption, λ, necessary to equate social welfare under inflation risk with that when inflation is constant and equal to target (as denoted by the * superscript):

$$U(1+\lambda)^{1-\gamma} = U^* \qquad (13)$$

It is clear from (12) and (13) that the social discount factor will not affect λ. Consequently, uncertainty about the correct calibration is not an issue for the welfare results reported here.

2.5 Aggregate resource constraint

Capital depreciates fully within a period, an assumption which is empirically reasonable given that each period in the model lasts 30 years. It follows that investment in period t is $i_t = k_{t+1}$. The economy's aggregate resource constraint in period t is thus $y_t = c_{t,Y} + c_{t,O} + k_{t+1} + g_t$, where the sum of the first two terms on the right hand side is aggregate consumption.

3. Monetary Policy

The government conducts monetary policy via money supply rules, which are set yearly with annual inflation in mind. The government can commit to these rules but cannot control the money supply perfectly and so has imperfect control over inflation. In order to obtain IT and PT money supply rules consistent with the 30-year horizon of the model, the long run implications of these annual rules are traced out over a 30-year horizon.

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[16] Since there is a strong theoretical case for smoothing taxes (see Barro (1979)) but not government spending, a risk-neutral attitude to government spending (i.e. a concern for the mean but not volatility) seems justified.

[17] The social welfare function ignores the utility of the initial old, but the impact of a transition from IT to PT on the current old generation is investigated in section 5.2.

3.1 Inflation targeting (IT)

Under IT, the yearly nominal money supply grows at the annual target inflation rate, π^*, plus any deviation due to an exogenous money supply shock ε:

$$M_n = M_{n-1}(1+\pi^*)(1+\varepsilon_n) \tag{14}$$

where M_n is the nominal money stock at the end of year n, ε_n is an IID-normal random variable with mean zero and variance σ^2.

By substituting repeatedly for the previous year's money supply, the money supply rule in (14) can be expressed as follows:

$$M_n = M_{n-30}(1+\pi^*)^{30}\prod_{j=n-29}^{n}(1+\varepsilon_j) \tag{15}$$

It is clear from this equation that the IT money rule in (14) aims at a constant inflation target and does not attempt to offset past money supply shocks – i.e. 'bygones are bygones'. Given that each period in the model lasts 30 years and the nominal money supply is the end-of-period stock of money, the implied money supply rule in any period t is

$$M_t = M_{t-1}(1+\pi^*)^{30}\prod_{j=1}^{30}(1+\varepsilon_{j,t}) \tag{16}$$

where the money supply innovations are indexed by the year j = 1,2,…,30 of period t in which they occur, and $M_t \equiv P_t m_t$ is the nominal stock of money in period t.

Since $m_t = \delta$ by the CIA constraint, the money supply rule in (16) implies that inflation is period t is given by[18]

$$1+\pi_t = (1+\pi^*)^{30}\prod_{j=1}^{30}(1+\varepsilon_{j,t}) \tag{17}$$

It is clear from (17) that there is base-level drift in the price level under IT: each yearly money supply shock has a permanent impact on the price level. As a result, inflation risk accumulates over a 30-year horizon. Note that in the absence of money supply innovations (i.e. $\varepsilon_{j,t}$ = 0 for all j and t), this money supply rule would stabilise inflation perfectly at the long-term inflation target, $(1+\pi^*)^{30}$, consistent with annual inflation of π^* every year.

Finally, note that inflation expectations are anchored at the inflation target under IT:

$$1+E_{t-1}\pi_t = (1+\pi^*)^{30} \tag{18}$$

where E_{t-1} is the expectation conditional on information at the end of period t-1.

[18] In particular, $M_t = P_t m_t$ implies that if $m_t = \delta$, then $M_t/M_{t-1} = P_t/P_{t-1} \equiv 1+\pi_t$, by money market equilibrium.

3.2 Price-level targeting (PT)

Under PT, policy aims to stabilize the price level around a long run target price path that is known *ex ante* and whose slope is consistent with an annual inflation target of π^*. The crucial difference relative to IT is that past deviations from the yearly inflation target are offset in order to return the price level to its target path. The yearly money supply rule thus includes a correction for the previous year's shock:[19]

$$M_n = M_{n-1}(1+\pi^*)\frac{(1+\varepsilon_n)}{(1+\varepsilon_{n-1})} \tag{19}$$

where M_n is the nominal money stock at the end of year n, ε_n is an IID-normal random variable with mean zero and variance σ^2.

By substituting repeatedly for the previous year's money supply, the money supply rule in (19) can be expressed as follows:

$$M_n = M_{n-30}(1+\pi^*)^{30}\,\frac{(1+\varepsilon_n)}{(1+\varepsilon_{n-30})} \tag{20}$$

This equation implies a money supply rule in period t of the form

$$M_t = M_{t-1}(1+\pi^*)^{30}\,\frac{(1+\varepsilon_{30,t})}{(1+\varepsilon_{30,t-1})} \tag{21}$$

where $\varepsilon_{30,t}$ is the money supply innovation in year 30 of period t.

By the binding CIA constraint, the implied period-t inflation rate is

$$1+\pi_t = (1+\pi^*)^{30}\,\frac{(1+\varepsilon_{30,t})}{(1+\varepsilon_{30,t-1})} \tag{22}$$

In contrast to IT, the PT money supply rule prevents base-level drift: past money supply shocks have only a temporary impact on the price level.[20] Intuitively, inflation in period t depends on the money supply shock in year 30 of period t because policy offsets money supply shocks with a one-year lag and so cannot offset the shock in year 30 until the first year of the next period. Inflation in period t also depends on the money supply shock in year 30 of period t–1, because this shock must be offset in year 1 of period t to correct for the previous deviation from the target price path (i.e. the deviation in the final year of period t-1).

Since rational agents expect past deviations from the target price path to be offset, inflation expectations vary around the inflation target with the past money supply innovation:

[19] The economic intuition can be seen more easily by taking logs: $M_n \approx M_{n-1} + \pi^* + \varepsilon_n - \varepsilon_{n-1}$.

[20] It should be clear from a comparison of (17) and (22) that long run inflation risk is substantially higher under IT. The Technical Appendix (section C) derives analytical approximations for the two variance expressions which imply that the inflation variance is approximately 15 times higher under IT.

$$1+E_{t-1}\pi_t = (1+\pi^*)^{30}\frac{1}{(1+\varepsilon_{30,t-1})} \tag{23}$$

4. Model calibration

The main assets in UK retirement portfolios are classified as nominal or real assets, with nominal assets defined as those that are denominated in UK pounds and less than fully indexed to inflation. The model is calibrated in order to roughly match the shares of real and nominal assets in retirement portfolios over the IT period.[21] Where specific data is drawn upon, 2005 is used as the reference year. Free parameters in the model are calibrated to match standard values in the literature.

4.1 Retirement portfolios in the UK

Evidence on UK retirement portfolios is provided by the Pensioners' Incomes Series (PIS), published by the Department for Work and Pensions (DWP). The PIS provides information on the sources of pensioner incomes from the Family Resources Survey. In the PIS, pensioners are defined as those of state pension age.

The primary source of pensioner incomes is state benefits, consisting primarily of the Basic and Additional state pension. In the 2004/5 PIS, state benefits accounted for around 45 per cent of pensioner incomes (DWP 2009, Table 2.1). Both the Basic and Additional state pensions are currently indexed under a triple-lock system (i.e. the highest of earnings, inflation or 2.5 per cent), and prior to this they were indexed to inflation with a 2.5 per cent floor. State pensions are therefore classified as nominal assets, except that one-fifth of state pension income is assumed to be fully indexed. The second most important source of pensioner incomes is occupational pensions, including those 'contracted out' into private pension schemes and public sector pensions. Occupational pensions accounted for around 26 per cent of pensioner incomes in 2004/5 PIS. For calibration purposes, it is assumed that occupational pensions are split in the ratio 50:50 between the public and private sector.[22] Private sector pensions are classified as real assets since equities are the most popular asset class in which UK pension funds invest, with a portfolio allocation share of around 40 per cent in 2005, compared to a share of government bonds and bills of less than 12 per cent (see OECD 2009, Table 2.10).[23] Public sector pensions are classified as nominal since they are primarily defined benefit, but it is assumed that one-fifth of public sector pension income is fully indexed to reflect indexation provisions in practice.[24]

The remainder of pensioner incomes in 2004/5 is accounted for by investment income of around 10 per cent, personal pensions of around 3 per cent, and earnings from employment and self-employment of around 15 per cent (DWP, 2009). Personal pensions and investment income are treated as real assets. Since the closest counterpart to earnings from employment and self-employment in the model is the return on capital, earnings are treated as a real asset

[21] The model's nominal assets are nominal bonds and money, whereas indexed bonds and capital are real assets.

[22] The actual UK ratio is around 60:40, but a long-standing government objective is to change this to 40:60.

[23] Equities had the highest share at 37 per cent, followed by Mutual Funds at 19.8 per cent. The corporate bonds share is around 8 per cent, but inflation uncertainty is unlikely to be the main source of risk on such bonds.

[24] Public sector pensions are currently indexed to the Consumer Prices Index (CPI) with an 8-month lag.

for the purpose of calibration.[25] Overall, the data above suggest a target ratio of nominal assets to real assets of around 1. Calibration is discussed in the next two subsections.

4.2 Aggregate uncertainty

The model contains two aggregate shocks: a money supply disturbance and a total factor productivity shock. These disturbances are calibrated in this subsection. First, calibrating the money supply rules requires a standard deviation for the annual money supply shock. This standard deviation was set at $\sigma = 0.0105$, which is close to the standard deviation of annual CPI inflation from 1997 to 2011 in data from the Office for National Statistics (ONS). Calibrating the model standard deviation in this way should enable the model to match the amount of long run inflation risk that would be observed with typical price level shocks under an IT regime that permits base-level drift, consistent the mandates of IT central banks.[26, 27]

The productivity shock was also calibrated for a generational horizon of 30 years. Its calibration was based on the 30-year properties implied by a standard annual productivity process. In particular, if the log of annual productivity follows an AR(1) with a correlation coefficient ρ and IID-normal innovation e_n with standard deviation σ_n, then (by repeated substitution) log productivity at a generational frequency of 30 years will equal

$$\ln A_t = (1-\rho_A)\ln A_{mean} + \rho_A \ln A_{t-1} + e_t \tag{24}$$

where $\rho_A \equiv \rho^{30}$ and $e_t \equiv \sum_{j=1}^{30} \rho^{j-1} e_{n-(j-1)}$.

The mean productivity parameter A_{mean} was normalised to 1. The AR(1) coefficient ρ_A was set equal to 0.20, based on an annual serial correlation coefficient $\rho = 0.9478$, which is similar to standard calibrations for annual productivity in the business cycle literature.[28] Based on the expression for e_t and the IID assumption, the implied innovation standard deviation at a generational frequency is $\sigma_e = [(1 - \rho^{60})/(1 - \rho^2)]^{1/2}\sigma_n$. Accordingly, σ_e was set equal to 0.0553, consistent with an annual productivity innovation standard deviation of $\sigma_n = 0.018$. The latter is similar to the standard calibrations in the literature and matches the standard deviation of annual UK TFP growth from 1998-2010 based on data from the ONS.[29]

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[25] Recall that old generations are retired and so receive no income from employment.

[26] Note that annualised inflation, i.e. $(1+\pi)^{1/30}$, has a standard deviation that is approximately equal to σ under an IT regime (provided that π^* is sufficiently small).

[27] The variance of actual inflation at a 30-year horizon cannot be used since IT was not adopted as part of an independent regime in the UK until 1997. Some economists have argued that monetary policy in some IT economies appears to have been closer to PT, but the dynamics of the price level in these economies could be due to 'good luck' – i.e. offsetting disturbances. Ultimately, there is not enough data to make a conclusive judgement at the current time.

[28] This calibration implies only a small amount of persistence in productivity at a generational frequency, but there does not seem to be any strong evidence of positive serial correlation at long horizons.

[29] If the log of annual productivity follows an AR(1) process, then the variance of annual productivity growth will be approximately equal to the innovation variance provided that $(\rho - 1)^2 \approx 0$. Section D of the Technical Appendix provides a formal demonstration of this result.

4.3 Model parameter calibration

Given that this paper concentrates on the implications of long run inflation risk for households, it is important that preference parameters reflect real-world attitudes toward risk and intertemporal substitution. It is also important that the model give sensible portfolio shares to nominal and real assets and realistic GDP shares to key macroeconomic variables.

Preference parameters

The preference parameters in the model are ε (which determines the elasticity of intertemporal substitution $(1-\varepsilon)^{-1}$), the discount factor β, and the risk aversion coefficient γ. The parameter ε was set equal to –0.35, implying an elasticity of intertemporal substitution of 0.74. This calibrated value is close to that in the OG model of Olovsson (2010) and is consistent with micro studies that estimate an elasticity of intertemporal substitution below 1. The discount factor β was set at 0.70, implying an annual discount factor of 0.988, and hence an annual risk-free real rate of 1.2 per cent per annum. The risk-free real rate was deliberately set below the average UK estimate of 2.9 per cent per annum for the period 1965 to 2005 (see Mills, 2008). The reason is that matching a real rate this high gives an investment to GDP ratio that is somewhat lower than in the data. Finally, taking the calibrated values of ε, β and other model parameters as given (see below), γ was set in order to match the Sharpe ratio on capital, or $E[r^k-r^f]/\text{std}(r^k-r^f)$.[30] The target value in calibration was set at 0.43 based on the results in Constantinides, Donaldson and Mehra (2002), who estimate the Sharpe ratio using 20-year holding period real returns on equity and bonds in the US. Accordingly, the risk aversion coefficient γ was set at 15, which gives a Sharpe ratio of 0.42.

Other model parameters

The production function parameter α was set at 0.263, implying that the share of capital income in GDP is 26.3 per cent. This value is slightly on the low side of standard calibrations but helps the model to match a target ratio of long-term government bonds to GDP of around 10 per cent, which roughly matches the share of long-term government bonds in UK GDP over the past decade.[31] The tax rate on capital was set at 2.3 times the income tax rate, that is, a = 2.3. A substantially higher tax rate on capital is consistent with UK data over the period 1970-2005: Angelopoulos, Malley and Phillippopoulos (2012) calculate that the average tax rate on capital was 0.44, compared to an average tax rate on labour of 0.27. These figures imply that capital taxes should be roughly 1.6 times as high as labour taxes, but the higher calibrated ratio of 2.3 enables the model to get closer to target ratio for nominal to real assets. Consistent with UK data, the tax and production function calibrations imply that income tax is the main source of tax revenue.

There are four additional coefficients that need to be calibrated. First, an annual inflation target enters as a constant in both the IT and PT money supply rules. This target was set at 0.02, consistent with the 2 per cent UK inflation target for the Consumer Prices Index (CPI). Second, real money balances are equal to δ by the CIA constraint. The calibration sets δ = 0.015 so that money balances are around 3 per cent of GDP, consistent with UK data on notes and coins (ONS 2011, Table 1). Third, the long run government spending to GDP target, G^*,

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[30] Returns are annualised in order to calculate the Sharpe ratio. Since capital is taxed, the ratio was computed using the after-tax real return on capital.

[31] See ONS (2011) and historical data available on the Debt Management Office (DMO) website.

was set at 0.11 since this implies a tax rate τ in the model solution such that long-term government bonds and investment have plausible GDP shares. Finally, the share of indexed bonds in the total government bond portfolio, v, was set at 0.20, which is similar to the actual UK share of index-linked gilts in 2005 of around one-quarter (DMO, 2005).

4.4 Model solution and key ratios

It is crucial that the model give sensible real and nominal asset ratios, since these assets are important for the transmission of real and nominal risks. Furthermore, since the risk aversion coefficient – an important parameter for welfare – is calibrated to match the Sharpe ratio, it is vital that the relative importance of capital income is reasonable. This section discusses the performance of the calibrated model against key ratios.

Table 1 – Target versus model ratios

Ratio	Target	Model	Definition
$(b^n + m)/(k + b^i)$	1.00	0.86	Nominal/real assets
b/y	0.10	0.11	Long-term bonds/GDP
b^i/b	0.25	0.20	Indexation share
i/y $(=k/y)$	0.15	0.14	Investment/GDP
$(c_{Y} + c_{O})/y$	0.65	0.75	Consumption/GDP
$E[r^k – r^f]/\text{std}(r^k – r^f)$	0.43	0.42	Sharpe ratio
m/y	0.03	0.03	Notes and Coins/GDP

In the model, investment equals the capital stock since there is full depreciation. The ratio of UK investment to GDP has been close to 15 per cent over the past decade (see ONS 2012, Table 1.2). On this basis, the target capital to GDP ratio was set at 0.15. Over the same period the consumption share was around 65 per cent. This value is also taken as a target ratio. Turning to government debt, bonds have fluctuated somewhat as a percentage of GDP over the past decade but have averaged around one-third (see ONS 2011, Table 1.1D). Together with a 2005 share of long-term government debt in total government debt of just over 30 per cent,[32] this figure implies a target long-term government bonds to GDP ratio of around 10 per cent. Table 1 shows the performance of the calibrated model against these target values.[33] Overall, the model does quite well against target ratios.

[32] This figure is based on historical data available on the Debt Management Office (DMO) website. The DMO classifies gilts as 'long-term' if maturity exceeds 15 years. In the model, government bonds are held by households. Although households hold only a small fraction of government debt directly, they indirectly own a large fraction since pension funds and insurance companies are the main holders of government debt. In 2005, such companies held almost 60 per cent of the total stock of government debt (DMO 2010, Chart 11).

[33] All ratios were calculated using deterministic IT steady-state values, with the exception of the Sharpe ratio. The Sharpe ratio was calculated using simulated moments, as described at the start of section 5.

5. Baseline results

This section is split into two parts. The first part investigates the aggregate and welfare effects of long run inflation risk under IT and PT. The focus here is on long run regimes, as the analysis abstracts entirely from the IT-to-PT transition. The second part then investigates an unexpected transition from IT to PT, with a focus on distributional and welfare effects. The model was solved using a second-order perturbation approximation in Dynare++ (Julliard, 2001). For long run analysis, 500 random simulations were run, each with a simulation length of 1100 periods, and with the first 100 periods disregarded to randomise initial conditions, leaving a total of 500,000 simulated values to calculate unconditional moments and social welfare. The transition to PT was also analysed using a second-order approximation. Details of the simulation procedure in this case are provided in section 5.2.

5.1 Aggregate and welfare effects of long run inflation risk

The baseline results are reported in Table 2. Panel A concentrates on first and second moments under IT and PT, with the zero inflation risk case reported for completeness. Panel B reports the welfare cost of long run inflation risk in per cent of aggregate consumption, as well as the implied welfare gain from PT.

Panel A indicates that the impact of PT is not uniform across young and old generations. PT raises average consumption by the young by 0.4 per cent relative to IT but lowers average consumption by the old by 0.2 per cent. Comparing consumption risk across regimes, old generations' consumption risk (as measured by the unconditional variance) falls by more than one-tenth as a result of the reduction in long run inflation risk under PT, but there is an increase in consumption volatility for the young of just over 1 per cent. Given the magnitude of these impacts, PT has the net effect of raising aggregate (and hence lifetime) consumption and lowering aggregate consumption risk. Consumption by the young rises because the tax rate is lower under PT. That is to say, the government meets its long run government spending target with lower taxes than under IT, the reason being that PT essentially eliminates the inflation risk premium on nominal bonds, so that the average cost of issuing government debt is lower. Since taxes are lower under PT, the capital stock is higher on average by 0.8 per cent, and GDP by 0.2 per cent. Bond holdings are also higher, but only marginally. Average consumption by the old is higher under IT because the increase in the inflation risk premium raises the average real return on nominal government debt, and this increase dominates the negative income effect from the higher tax on capital income.[34]

Turning to Panel B, the welfare cost of long run inflation risk is substantial at 0.35 per cent of aggregate consumption. Since PT largely eliminates long run inflation risk, the welfare cost falls to only 0.01 per cent, implying a permanent welfare gain of 0.34 per cent.[35] There is an increase in social welfare under PT because it increases average consumption by the young and lowers consumption risk for the old. To investigate the relative importance of these two effects, the welfare gain was calculated in a log-linearized version of the model where risk-premia are zero. In such a model PT has an impact on consumption risk but not on average consumption levels, since the average cost of issuing government debt is equalised across

[34] The annualised inflation risk premium is equal to 6 basis points under IT but only 0.2 basis points under PT, due to the dramatic reduction in long run inflation risk under a PT regime.

[35] The standard deviation of inflation is equal to 0.104, or 10.4 per cent, under IT. Under PT, this figure is reduced to 0.027, or 2.7 per cent.

regimes, implying equal taxes. In the log-linearized economy the welfare gain from PT is equal to 0.08 per cent of aggregate consumption, indicating that around three-quarters of the total welfare gain is a result of lower taxes and only one-quarter the result of lower consumption risk (for any given level of taxes). Hence while the effects of PT on both average consumption levels and consumption risk are non-trivial from a welfare perspective, the impact on average consumption levels is more significant. These results clearly speak to the importance of non-linear approximations in models where inflation risk has important higher-order effects, as well as the potential importance accounting for the fiscal implications of alternative monetary policy regimes via the government budget constraint.

Table 2 – Aggregate and welfare effects of long run inflation risk

Panel A: Aggregate effects under IT and PT				
	Zero Inflation Risk	**IT**	**PT**	**Percent change from IT to PT**
$E(c_Y)$	0.1838	0.1831	0.1838	+0.4%
$E(c_O)$	0.1858	0.1863	0.1859	–0.2%
$\mathrm{var}(c_Y) \times 1000$	0.117	0.117	0.118	+1.2%
$\mathrm{var}(c_O) \times 1000$	0.146	0.165	0.147	–11.2%
τ	0.1093	0.1112	0.1093	–1.7%
Panel B: Welfare cost of long run inflation risk				
IT : 0.346		**PT:** 0.009		**PT welfare gain:** 0.337

Notes: Panel A: units of the consumption good. Panel B: percent of aggregate consumption.

5.2 The transition from IT to PT

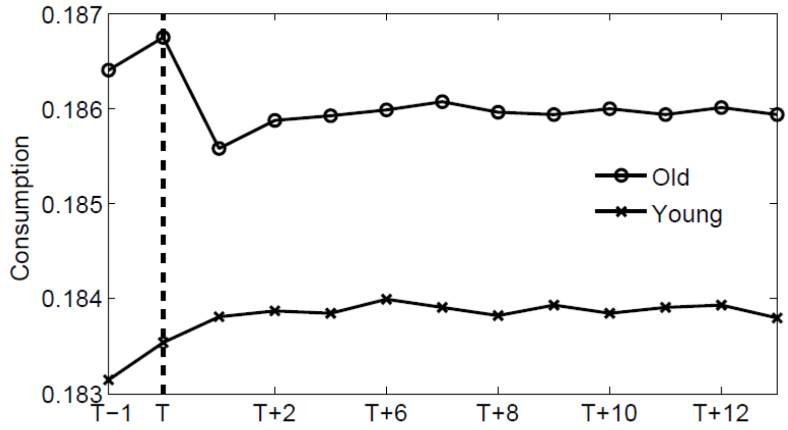
The long run analysis above abstracts entirely from the IT-to-PT transition and therefore ignores potentially important distributional and welfare effects of a change in regime. In this section, these effects are investigated. The analysis focuses on the impact of an unexpected transition to PT in period T on consumption levels and utility.[36]

As can been seen from Figure 1, the effects of an unexpected transition from IT to PT in period T are short-lived: generations born in period T+2 or later are essentially unaffected by the transition, with consumption levels settling down at their long run average levels under PT (see Table 2). Consumption rises for the old alive in period T, since they benefit from a lower tax rate on capital income under PT, while the yield on nominal government debt is unchanged relative to the previous period because it reflects the expectation (held in period T–1) that an a IT regime would be in place next period, and therefore incorporates the same inflation risk premium as in a long run IT regime. Given that the current old generation consumes more under PT and is exposed to less consumption risk, their utility is somewhat

[36] 20,000 separate transition paths were simulated using the second-order solution of the model given by Dynare++. The results were then averaged across stochastic simulations. Period T–1 is given by the final period of previous IT regime, conditional on the expectation that IT would continue with probability one in period T.

higher than under continuation of the IT, with a utility gain equivalent to 0.56 per cent of old age consumption.[37]

Fig 1 – An unexpected transition to PT in period *T*



Consumption also increases for the young alive in period *T*, since they benefit from a positive income effect from the lower tax on labour income under PT.[38] However, this generation consumes less than young generations in subsequent periods and also loses out relative to the old generations in period *T*+2 or later. To this extent, the transition to PT has sizeable distributional consequences that would not be present under continuation of IT (where average consumption levels would equal their period *T*–1 values). The loss in lifetime utility for the young generation born in period *T*, relative to those young generations born in period *T*+1 or later, is equivalent to 0.13 per cent of their lifetime consumption. Crucially, however, the young of period *T* still have significantly higher lifetime utility than they would under continuation of IT, with an implied welfare gain of 0.22 per cent of lifetime consumption.

In short, whilst an unexpected transition to PT has non-trivial implications for the distribution of consumption between current and future generations, these distributional effects are short-lived and all generations directly affected by the transition can expect to gain utility as compared to continuation of IT. Consequently, accounting for the transition from IT to PT does not alter in any substantive way the conclusions of the long run analysis in section 5.1. The next section returns to long run issues in order to investigate whether PT is robust to extensions that relax crucial assumptions in the baseline case.

6. Extensions

This section investigates the robustness of the baseline results by assessing the implications of (i) a PT target horizon that exceeds one year, (ii) imperfect credibility of PT, and (iii) a government that holds taxes constant across monetary regimes but provides public goods that are valued by private agents. Each of these extensions is considered in turn.

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[37] The utility of the old was calculated as $u_{T,O} = (1-\gamma)^{-1} c_{T,O}^{1-\gamma}$.

[38] Since young generations both consume and save, some of the increase in disposable income is used to increase consumption and the remainder to purchase extra bonds and capital for old age.

6.1 A flexible target horizon under PT

The case for a flexible regime that gradually restores the price level to its target path is made by Gaspar, Smets and Vestin (2007), who argue that the (short-term) costs of undoing price level shocks could be reduced in this way.[39] To assess the impact of returning the price level to target gradually, the aggregate and welfare effects of PT are reconsidered in this section, with the target horizon in the PT money rule varying from one to eight years.

For the case where the price level is returned to target in uniform steps over H years,

$$M_n = M_{n-1}(1+\pi^*)\frac{(1+\varepsilon_n)}{\prod_{j=1}^{H}(1+\varepsilon_{n-j})^{1/H}} \tag{25}$$

where, as previously, M_n is the end-of-period nominal money supply in year n.

Note that innovations up to H years old enter in the denominator of this money supply rule because each is offset only after H years in total, with a fraction $1/H$ offset each period.

By substitution, (25) implies the following money supply rule in period t:

$$M_t = M_{t-1}(1+\pi^*)^{30}\prod_{k=0}^{H}\left(\frac{1+\varepsilon_{30-k,t}}{1+\varepsilon_{30-k,t-1}}\right)^{(H-k)/H} \tag{26}$$

where it has been assumed that the policy horizon H is less than 30 years.[40]

Since the CIA constraint is binding, this equation implies that period t inflation is given by

$$1+\pi_t = (1+\pi^*)^{30}\prod_{k=0}^{H}\left(\frac{1+\varepsilon_{30-k,t}}{1+\varepsilon_{30-k,t-1}}\right)^{(H-k)/H} \tag{27}$$

The intuition for this expression can be seen by setting $H = 2$. In this case, only money supply shocks in years 29 and 30 of period t matter for inflation because shocks in years 1 to 28 will have been offset fully by the end of the period t (i.e. by year 30), given that the price level is returned to target in 2 years. Shocks in years 29 and 30 from the previous period enter in the denominator because these shocks will not have been offset before the end of period t–1 and must therefore be offset in period t in order to return the price level to its target path.

Returning the price level to target gradually over H years raises long run inflation risk, because the price level is allowed to deviate from its target path for longer. In fact, the simulated unconditional standard deviation of inflation rises steadily with the target horizon, from a standard deviation of 0.027 (or 2.7 per cent) when $H = 1$, to 0.037 when $H = 4$, and 0.048 when $H = 8$. By comparison, the unconditional standard deviation of inflation under IT

[39] Consistent with this argument, Smets (2003) finds that the optimal horizon for returning the price level to target is roughly double the optimal horizon for returning inflation to target in a New Keynesian model.

[40] Note that the original money supply rule arises as a special case of this equation when $H = 1$.

is around 0.104 (or 10.4 per cent) in the baseline case.[41] Hence, although long run inflation risk under PT rises steadily with the target horizon, it remains well below that under IT.

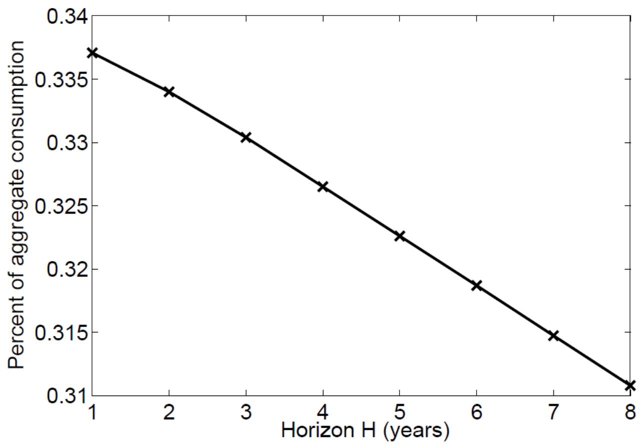
Robustness of the aggregate effects of PT is investigated in Table 3. Each entry shows the net change relative to the baseline IT analysis for a given target horizon *H*, and the baseline case is reported in the first column. For instance, in the baseline case of *H* = 1 the consumption variance for old generations is reduced by 11.2 per cent under PT, compared to 10.8 per cent when *H* = 4, and 10.3 per cent when *H* = 8. The other results reported in Table 4 are even more robust. Overall, then, the aggregate effects of PT are strongly robust to changes in the target horizon for the price level target.

Table 3 – Aggregate effects of PT as the target horizon *H* is varied

	Target Horizon *H* (years)							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
$E(c_Y)$	+0.42	+0.41	+0.41	+0.40	+0.40	+0.39	+0.39	+0.38
$E(c_O)$	-0.22	-0.22	-0.21	-0.21	-0.21	-0.21	-0.21	-0.20
$\mathrm{var}(c_Y)$	+1.2	+1.2	+1.1	+1.1	+1.1	+1.1	+1.1	+1.1
$\mathrm{var}(c_O)$	-11.2	-11.1	-11.0	-10.8	-10.7	-10.6	-10.4	-10.3
τ	-1.70	-1.69	-1.67	-1.65	-1.63	-1.61	-1.60	-1.58

Note: entries are net impacts, expressed as percent changes relative to IT. Baseline case: *H* = 1.

Fig 2 – The welfare gain from PT and the target horizon *H*



The welfare gain from PT is also rather robust to changes in the target horizon (see Figure 2). As the target horizon is increased, the welfare gain from PT falls from 0.337 per cent of aggregate consumption when *H* = 1 (the baseline case), to 0.327 per cent when *H* = 4, and 0.311 per cent when *H* = 8. The welfare gain falls because PT permits more long run inflation risk as the target horizon increases, so that the extent to which old generations benefit from reduced consumption risk is dampened. A rise in long run inflation risk also raises the inflation risk premium, so that taxes have to rise in order to meet the long run government

[41] These standard deviations refer to inflation over a 30-year horizon, not average annual inflation.

spending target. In turn, this increase in taxes lowers average consumption by young generations, albeit only marginally. The relationship between taxes and consumption by the young is evident from the first and final rows of Table 3.

6.2 Imperfect credibility of PT

The argument that a PT regime would be imperfectly credible is appealing given that a regime of this kind has never been adopted in practice. Consequently, imperfect credibility was an important factor in the Bank of Canada's deliberations about whether to switch from IT to PT (see Bank of Canada, 2011). In the literature, the impact of imperfect credibility of PT has been assessed by Gaspar, Smets and Vestin (2007) and Masson and Shukayev (2011).

Gaspar *et al.* (2007) argue that PT is likely to experience an initial period of imperfect credibility when agents would learn about the workings of the new regime. They set up a New Keynesian model with learning and find that an initial period of imperfect credibility is sufficient to turn the net welfare gains from PT negative, if agents are slow to learn. The reason is that inflation expectations act as automatic stabilisers under a credible PT regime, but this link is weakened somewhat under learning, because expectations become backward-looking. Masson and Shukayev (2011) build a New Keynesian model where PT operates with an 'escape clause', such that sufficiently large shocks lead to rebasing of the target price path. They show that there are two stable equilibria: one with a low probability of rebasing, and one with a high probability. They interpret the former as a PT regime with high credibility, and the latter as a PT regime with low credibility. In this environment, imperfect credibility reduces stabilization of supply shocks and is therefore costly for welfare.

In contrast to these two papers, the analysis in this section concentrates on the impact of imperfect credibility through the long run inflation risk channel. The analysis also differs in that the model is non-linear, so that imperfect credibility influences aggregate outcomes through the inflation risk premium on nominal bonds, as well as via the inflation expectations channel. In order to model imperfect credibility, it is assumed that young agents believe there is a constant probability p_{IT} that monetary policy will switch back to IT in the next period. Accordingly, agents believe with probability $1 - p_{\mathrm{IT}}$ that the current PT regime will remain in place. This probability can be taken as a measure of credibility, with $p_{IT} = 0$ corresponding to the baseline case of perfect credibility.

Given these beliefs over regimes $s = \{IT, PT\}$, agents' lifetime utility function is given by

$$u_t = \frac{1}{1-\gamma}\left[c_{t,Y}^{\varepsilon} + \beta\left[p_{IT} E[c_{t+1,O(IT)}^{1-\gamma} \mid \Omega_t] + (1 - p_{IT}) E[c_{t+1,O(PT)}^{1-\gamma} \mid \Omega_t] \right]^{\frac{\varepsilon}{1-\gamma}} \right]^{\frac{1-\gamma}{\varepsilon}} \tag{28}$$

where $E[c_{t+1,O(s)}^{1-\gamma} \mid \Omega_t]$ is the expectation in regime *s*, conditional on period-*t* information Ω_t.[42]

The first-order conditions are given by the following Euler equations:[43]

$$1 = R_t \left(p_{IT} E[SDF_{t+1(IT)} \mid \Omega_t] + (1 - p_{IT}) E[SDF_{t+1(PT)} \mid \Omega_t] \right) \tag{29}$$

[42] See Ferman (2011) for a Markov-switching application in a model with Epstein-Zin preferences.

[43] These first-order conditions are derived in full in section E of the Technical Appendix. The government is assumed to set the total bond supply so that the expected stochastic discount factor across regimes is equal to β.

$$1 = r_t^f \left(p_{IT} E[sdf_{t+1(IT)} \mid \Omega_t] + (1 - p_{IT}) E[sdf_{t+1(PT)} \mid \Omega_t] \right) \tag{30}$$

$$1 = \alpha k_{t+1}^{\alpha-1} \left([p_{IT}(1-\tau_{IT}^k) E[sdf_{t+1(IT)} A_{t+1} \mid \Omega_t] + (1-p_{IT})(1-\tau_{PT}^k) E[sdf_{t+1(PT)} A_{t+1} \mid \Omega_t] \right) \tag{31}$$

$$1 = \left(p_{IT} E[SDF_{t+1(IT)} \mid \Omega_t] + (1 - p_{IT}) E[SDF_{t+1(PT)} \mid \Omega_t] \right) + \tilde{\mu}_t \tag{32}$$

where $SDF_{t+1(s)} \equiv sdf_{t+1(s)}/(1+\pi_{t+1(s)})$, τ_s^k is the tax rate on capital in regime s, and

$$sdf_{t+1(s)} \equiv \beta \left(\frac{c_{t,Y}}{c_{t+1,O(s)}} \right)^{1-\varepsilon} \left[\frac{c_{t+1,O(s)}}{(p_{IT} E[c_{t+1,O(IT)}^{1-\gamma} \mid \Omega_t] + (1-p_{IT}) E[c_{t+1,O(PT)}^{1-\gamma} \mid \Omega_t])^{1/(1-\gamma)}} \right]^{1-\gamma-\varepsilon} \tag{33}$$

To assess the impact of imperfect credibility, the model was solved for three different values of p_{IT}, namely 0.5, 0.3, and 0.1. These values represent fixed beliefs that policy will revert to IT next period with 50 per cent, 30 per cent and 10 per cent probability and were chosen to represent situations of low, medium and high credibility, respectively. Table 4 reports aggregate effects relative to IT in these three cases, along with the implied welfare gain from PT and the baseline results.

Table 4 – Aggregate effects of PT under imperfect credibility

	Credibility of PT			
	Perfect (Baseline)	**High (p_{IT} = 0.1)**	**Medium (p_{IT} = 0.3)**	**Low (p_{IT} = ½)**
$E(c_Y)$	+0.42	+0.35	+0.25	+0.17
$E(c_O)$	-0.22	–0.20	–0.16	–0.13
var(c_Y)	+1.2	+0.8	+0.6	+0.5
var(c_O)	–11.2	–11.4	–11.3	–11.1
τ	–1.70	–1.54	–1.22	–0.93
PT welfare gain	0.337	0.287	0.198	0.129

Note: rows 1 to 5 are net effects, expressed as percent changes relative to IT. The welfare gain in row 6 is measured in percent of aggregate consumption.

The main impact of imperfect credibility is on young generations. With low credibility, average consumption by the young is only 0.17 per cent higher than under IT, as compared to 0.42 per cent under perfect credibility. This reduction in average consumption for the young is driven by agents' belief that policy could switch to IT next period, which raises the inflation risk premium on nominal government debt. Consequently, higher taxes are necessary for the government to meet its long run spending target, so that average consumption by the young falls due to the negative income effect. Imperfect credibility has relatively little impact on consumption risk for young and old generations because agents' inflation expectation 'errors' are small given that expected inflation is similar under IT and

PT.[44]. Finally, under high credibility, aggregate effects are similar to under perfect credibility, as is the welfare gain from PT at 0.29 per cent of aggregate consumption. Hence a small degree of imperfect credibility does little to diminish the potential long run benefits of a PT regime. Overall, the welfare gain from PT is quite sensitive to imperfect credibility, but it remains non-trivial, reaching a minimum of 0.13 per cent in the low credibility case.

6.3 Government spending on public goods

A theme that has emerged from the analysis thus far is the importance of the accounting for fiscal effects of changes in monetary policy regime using general equilibrium analysis. The maintained assumption that the government sets taxes in order to meet a long run government spending target is, however, only one possible interpretation of fiscal neutrality, and the government is assumed to pursue this target despite government spending having no intrinsic value. In this section, an alternative type of fiscally neutrality is investigated: taxes are equalised across regimes but government spending has value because it enters private agents' utility functions via spending on public goods. The aim is to determine whether the long run aggregate and welfare effects PT are robust to this change in specification.

Consumer preferences are now given by

$$u_t(c_Y, c_O, h) = \frac{1}{1-\gamma}\left[(c_{t,Y} + \theta h_t)^{\varepsilon} + \beta \left[E_t (c_{t+1,O} + \theta h_{t+1})^{1-\gamma} \right]^{\frac{\varepsilon}{1-\gamma}} \right]^{\frac{1-\gamma}{\varepsilon}} \tag{34}$$

where $h_t \equiv \chi g_t + (1-\chi) g_{t-1}$ is consumption services from public goods in period t.[45]

The parameter θ is the relative weight given to public consumption in composite consumption, while $0 < \chi < 1$ indicates the extent to which current consumption services from public goods are dependent on past government spending. Lagged effects of public spending are included because the beneficial effects of spending on public goods are unlikely to be limited to current generations given that much spending of this kind is on durable goods such as museums, parks and hospitals. Each generation takes spending on public goods as given.

To provide a quantitative assessment, the parameters χ and θ need to be calibrated. The relative weight on public consumption services was set at θ = 1/2, so that consumption services from public goods are given half the weight of private consumption in utility. The weight χ was set also equal to 1/2, implying that half of public consumption services received come from current public spending, with the other half coming from public spending in the previous period. The tax rate τ was set equal to 0.1112, its implied value under IT in the baseline analysis. The welfare gain in per cent of *composite consumption* is calculated using the same expression as in (13) and then converted to a percentage of private consumption by multiplying by the steady-state ratio of composite consumption to private consumption.

Table 5 reports aggregate and welfare effects, including the implied welfare gain from PT. As taxes are equalised across regimes, mean consumption by the young is essentially identical

[44] Expected inflation is constant under IT and only varies under PT with the past yearly deviation from the target price path; see (18) and (23).

[45] The specification for composite consumption is similar to Angelopoulos *et al.* (2012), though they work with CRRA preferences. First-order conditions for this case are derived in section F of the Technical Appendix.

under IT and PT, as compared to 0.4 per cent higher under PT in the baseline analysis. Mean consumption by the old remains lower under PT than IT, but the difference is larger than in the baseline analysis at 0.4 per cent (as opposed to 0.2 per cent), because old generations no longer benefit from lower taxes under PT. Turning to volatility, PT leads to a 10.0 per cent reduction in consumption risk for old generations (as compared to 11.2 per cent in the baseline model) and consumption risk for the young is essentially identical under IT and PT. Consequently, PT reduces consumption risk substantially for old generations and leaves private consumption risk essentially unchanged for the young. The final row of the table reports consumption services from public goods, h. Public consumption is higher under PT by 1.4 per cent because the average cost of issuing government debt is higher under IT due to the inflation risk premium, so that less government spending can be funded with a given tax rate. In summary, private consumption levels fall modestly under PT, but this fall is offset by a rise in public goods consumption and a substantial fall in consumption risk for the old.

Table 5 – Aggregate and welfare effects when public goods enter utility

Panel A: Aggregate effects under IT and PT				
	Zero Inflation Risk	**IT**	**PT**	**Percent change from IT to PT**
$E(c_Y)$	0.1820	0.1821	0.1820	-0.06%
$E(c_O)$	0.1860	0.1867	0.1860	-0.38%
$\text{var}(c_Y) \times 1000$	0.099	0.100	0.100	-0.02%
$\text{var}(c_O) \times 1000$	0.179	0.200	0.180	-9.99%
$E(h)$	0.055	0.054	0.055	+1.44%
Panel B: Welfare cost of long run inflation risk				
IT : 0.111		**PT:** 0.018		**PT welfare gain:** 0.093

Notes: Panel A: units of consumption good. Panel B: percent of aggregate private consumption.

Turning to welfare, the estimated long run welfare gain from PT falls to 0.09 per cent, which is just over one-quarter of the baseline value, but still non-trivial.[46] Consequently, the main conclusion of the baseline analysis that there are non-trivial potential welfare gains from PT cannot be overturned, although the potential gains look somewhat lower. This result suggests that the baseline estimated welfare gain from PT should be treated with some caution.

7. Sensitivity analysis

This section consists of two parts. In the first, a parameter sensitivity analysis is conducted. The results assess the extent to which baseline results are sensitive to changes in individual calibrated parameters. In the second part, the model is roughly calibrated for Canadian

[46] The welfare gain from PT is quite sensitive to the calibration of the relative weight on consumption services from public goods, θ. For example, when $\theta = 3/4$ the estimated welfare gain is 0.14 per cent of aggregate consumption, while if $\theta = 1/4$ it falls to only 0.02 per cent.

nominal portfolios to assess the importance of the nominal portfolio channel. The model is then calibrated for Canadian price level shocks in order to provide an estimate of the welfare cost of long run inflation risk in Canada.

7.1 Parameter perturbations

This section investigates robustness of the baseline results to 'high' and 'low' calibrations for individual model parameters. The parameters tested are the risk aversion coefficient γ, the intertemporal substitution coefficient ε, the productivity innovation standard deviation σ_e, the productivity persistence parameter ρ_A, the share of indexed government bonds v, and the money supply innovation standard deviation σ. The calibrated values tested were $\gamma = \{10, 20\}$, $\varepsilon = \{-0.45, -0.25\}$, $\sigma_e = \{0.051, 0.059\}$, $\rho_A = \{0, 0.40\}$, $v = \{0.10, 0.30\}$ and $\sigma = \{0.0095, 0.0115\}$.[47] These alternative calibrations represent symmetric deviations from the baseline case. The high value for ε implies an elasticity of intertemporal substitution of 0.80, and the low value an elasticity of 0.69 (as compared to 0.74 in the baseline case).

Aggregate effects of PT are reported in Table 6. Overall, the results are robust, but there is notable sensitivity with respect to the indexation share and the standard deviations of the innovations to productivity and the money supply. The reason is that these parameters determine the relative importance of real versus nominal risks. For example, with the high indexation share of 0.3, PT reduces old generations' consumption risk by only 9.0 per cent, as compared to 11.2 per cent under the baseline calibration, because higher indexation reduces agents' exposure to unanticipated fluctuations in inflation and therefore substitutes for the effects of PT. The results are also sensitive to the risk aversion coefficient, because the inflation risk premium – and hence the general equilibrium effects of PT – depend crucially on this parameter. However, none of the aggregate effects of PT changes sign, nor are any of the main quantitative findings of the baseline analysis overturned.

Table 6 – Sensitivity of the aggregate effects of PT

Parameter	**$E(c_Y)$** High case	**$E(c_Y)$** Low case	**$E(c_O)$** High case	**$E(c_O)$** Low case	**var(c_Y)** High case	**var(c_Y)** Low case	**var(c_O)** High case	**var(c_O)** Low case
γ	+0.56	+0.27	–0.30	–0.15	+1.4	+0.9	–11.7	–10.6
ε	+0.43	+0.41	–0.23	–0.21	+1.2	+1.2	–11.2	–11.2
σ_e	+0.42	+0.42	–0.22	–0.22	+1.2	+1.2	–10.1	–12.7
ρ_A	+0.41	+0.42	–0.22	–0.22	+1.0	+1.2	–9.6	–11.9
v	+0.33	+0.52	–0.18	–0.27	+1.0	+1.4	–9.0	–13.6
σ	+0.50	+0.34	–0.26	–0.18	+1.3	+1.0	–13.2	–9.3
Baseline	**+0.42**		**–0.22**		**+1.2**		**–11.2**	

Note: entries are percentage changes relative to IT.

Turning to welfare, Table 7 reports the welfare gain from PT for each sensitivity calibration. The welfare gain is strongly robust to the elasticity of intertemporal substitution (EIS), productivity persistence, and productivity risk. It is quite sensitive, however, to risk aversion,

[47] The low indexation share of 0.1 is similar to the share of inflation-indexed bonds in marketable Treasury debt in the US (see Campbell *et al.*, 2009), and the high indexation share of 0.3 is equal to the highest share that index-linked gilts attained in the UK government bond portfolio over the 1999 to 2010 period (see DMO, 2010).

the indexation share and the extent of nominal risk, being higher when indexation is low, and when risk aversion and nominal risk are high. For instance, raising the indexation share from 0.2 to 0.3 lowers the welfare gain from PT from 0.34 per cent of aggregate consumption to 0.26 per cent, while increasing the money supply innovation standard deviation to 0.0115 raises the welfare gain from PT to 0.41 per cent and increasing the risk aversion coefficient to 20 raises the welfare gain to 0.48 per cent. Again, however, there is not sufficient sensitivity to overturn any substantive conclusions from the baseline analysis.

Table 7 – Sensitivity of the welfare gain from PT

Parameter	Role in model	High case	Low case
γ	Risk aversion	0.481	0.207
ε	EIS = 1/(1–ε)	0.340	0.334
σ_e	Std(prod. innov.)	0.341	0.333
ρ_A	Prod. AR(1) coef.	0.342	0.337
ν	Indexation share	0.262	0.421
σ	Std (money innov.)	0.407	0.273
Memo: Baseline welfare gain = 0.337			

Note: entries are in percent of aggregate consumption.

7.2 Canadian nominal portfolios

In this section, the model is calibrated to roughly match Canadian nominal portfolios and the real side of the economy in Canada. This analysis acts as a check on the importance of nominal portfolios, since nominal retirement assets play a greater role than in the UK. The model is then calibrated for Canadian price level shocks in order to provide an estimate of the aggregate and welfare effects of long run inflation risk in Canada.

Meh and Terajima (2008) document nominal portfolios in Canada, based on data in the National Balance Sheet Accounts (NBSA) from 1990:1 to 2007:4 and the 2005 Survey of Financial Security (SFS), a household microdata survey collected by Statistics Canada. Meh and Terajima define four broad categories of nominal financial instruments: short-term instruments, bonds, mortgages, and employer pension plans. Here 'bonds' include non-mortgage and non-pension nominal claims with a maturity exceeding one year, and 'pensions' include defined contribution and defined benefit pension plans that are not fully indexed to prices. These nominal claims are classified as assets or liabilities for three sectors: the household sector, the government sector, and non-residents. Business sector portfolios are allocated to these three sectors based on ownership.

Meh and Terajima find that the household sector's net nominal position is substantial at 40.1 per cent of GDP, due mainly to large net nominal positions in bonds and pensions of 22.1 per cent and 17.7 per cent of GDP, respectively. The distribution of government bonds by maturity in 2005 suggests that around three-tenths of government debt is held in bonds with maturities exceeding 15 years (see Fig 4 in the paper). Combining this share with the household net nominal position in 'bonds' gives an implied (upper bound) net nominal position in long-term government debt of about 6 per cent of GDP. Along with the net nominal position in pensions, the latter implies a net household position in long-term nominal assets of around 24 per cent of GDP.[48] The calibration below aims at a more conservative target ratio of nominal assets to GDP of one-fifth.

OECD (2012) reports retirement income data for over-65s in Canada in the mid-2000s. The data suggest that public transfers are of similar importance to in the UK with an income share of around 45 per cent, and both occupational and private pensions and income from work have similar shares to the UK at 40 per cent and 13 per cent respectively. However, since around 60 per cent of private pensions in Canada are non-indexed defined benefit pensions (see Meh and Terajima 2008, Table A14), occupational and private pensions will be primarily nominal. Assuming such pensions are three-quarters nominal and one-quarter real (as opposed to the 50:50 ratio in the UK case), and that four-tenths of income received from public transfers is real (i.e. fully-indexed) and six-tenths nominal, implies a ratio of nominal to real retirement assets approximately two-fifths higher than in the UK, or around 1.4.[49]

Turning to the real side of the economy, the investment-GDP ratio has been higher than in the UK over the IT period and was around 20 per cent in 2005. To strike a balance between the greater relative importance of nominal assets in Canada on the one hand and the higher investment-GDP ratio on the other, the calibration below aims at a target investment-GDP ratio of 0.175 and a target ratio of nominal to real assets of 1.2. The share of consumption in GDP in 2005 was slightly less than 60 per cent. On this basis, the target consumption-GDP ratio was set at 0.60. The target Sharpe ratio was left at 0.43.[50]

Calibration

Model calibration was based on a target ratio of nominal to real assets of 1.2, a share of long-term government bonds in GDP of 20 per cent, an investment-GDP share of 17.5 per cent, and a target consumption share of 60 per cent. First, since indexed pensions are less prevalent in Canada and indexed government bonds have a lower share than in the UK, the indexation share was set at 0.14 (or 14 per cent), which is similar to the average share of indexed government debt from 2008 to 2012 (see Department of Finance, 2012). In order to roughly match the target ratios discussed above (subject to the calibrated indexation share), the ratio between wage and capital tax rates, a, and the production function coefficient α were allowed to vary. All other parameters were held at their baseline values. As in the baseline analysis, the government sets taxes to meet a long run government spending-GDP ratio of 11 per cent. A calibration that fits target ratios reasonably well is α = 0.250 and $\tau^k = 3.3\tau$ (i.e. a = 3.3), implying a capital income share in GDP of one-quarter and a tax rate on capital 3.3 times as high as that on wage income. As can be seen from Table 8, the model performs reasonably well against target ratios.[51]

[48] By comparison, government debt was 43 per cent of GDP in 2005 (Meh, Rios-Rull and Terajima, 2010; 645).

[49] The two main public transfers to retirees in Canada, the Old Age Security pension and the Guaranteed Income Supplement, are indexed to prices (albeit with a lag).

[50] The GDP shares cited were calculated using time series data available from Statistics Canada.

[51] All ratios were calculated using deterministic IT steady-state values, with the exception of the Sharpe ratio.

Table 8 – Target versus model ratios: Canadian portfolios

Ratio	Target value	Model value	Definition
$(b^n + m)/(k + b^i)$	1.20	1.15	Nominal/real assets
$(b^n + m)/y$	0.20	0.15	Nominal assets/GDP
b^i/b	0.14	0.14	Indexation share
i/y $(=k/y)$	0.175	0.113	Investment/GDP
$(c_{Y+}\, c_O)\,/y$	0.60	0.77	Consumption/GDP
$E[r^k - r^f]/\text{std}(r^k - r^f)$	0.43	0.33	Sharpe ratio

Results

Panel A of Table 9 reports moments of key variables under IT, PT and in the zero inflation risk case. The welfare cost of long run inflation risk and welfare gain from PT are reported in Panel B. The table reports two values in each cell: the first refers to the model calibrated for Canadian nominal portfolios only, and the second (in parentheses) refers to the calibration for both Canadian nominal portfolios and Canadian price level shocks during the IT period.[52]

The aggregate effects of PT are strengthened somewhat by moving to Canadian nominal portfolios. Most notably, consumption risk for old generations is reduced by almost one-fifth due to the greater importance of long run inflation risk when retirees hold a greater proportion of nominal assets. The greater importance of nominal portfolios increases the welfare cost of long run inflation risk under IT to 0.66 per cent of aggregate consumption. Consequently, the welfare gain from PT almost doubles to 0.64 per cent, indicating that the potential long run welfare gains from PT are likely to be somewhat higher if UK retirement portfolios shift toward the Canadian model where nominal bonds and non-indexed pensions are more prevalent. These results suggest that the aggregate and welfare effects of long run inflation risk will depend crucially on the structure of a country's retirement portfolios – in particular the relative importance of nominal versus real assets.

The results from the calibration for both Canadian nominal portfolios and price level shocks (see the figures in parentheses in Table 9) indicate that the aggregate effects of PT in Canada are likely to be similar to those in the UK. The reason is that the greater importance of the nominal portfolio channel in Canada is roughly offset by the lower magnitude of price level shocks in Canada over the IT period. Consequently, the estimated welfare gain is similar to that in the UK at 0.34 per cent. The potential long run welfare gains from PT are therefore likely to be broadly similar in the UK and Canada.

[52] In particular, the money supply innovation standard deviation was set equal to the standard deviation of the annual CPI in Canada over the period 1992-2011, or 0.008 (i.e. 0.8 per cent). Source: Statistics Canada website.

Table 9 – Aggregate and welfare effects with Canadian nominal portfolios

Panel A: Aggregate effects of IT and PT				
	Zero Inflation Risk	**IT**	**PT**	**Percent change from IT to PT**
$E(c_Y)$	0.1879	0.1864 (0.1871)	0.1878 (0.1878)	+0.8 (+0.4)
$E(c_O)$	0.1893	0.1898 (0.1896)	0.1893 (0.1893)	–0.3 (–0.1)
$\text{var}(c_Y) \times 1000$	0.123	0.120 (0.121)	0.123 (0.123)	+1.9 (+1.2)
$\text{var}(c_O) \times 1000$	0.143	0.174 (0.159)	0.144 (0.143)	–17.5 (–10.1)
τ	0.1005	0.1034 (0.1021)	0.1006 (0.1006)	–2.7 (–1.5)
Panel B: Welfare cost of long run inflation risk				
IT: 0.661 (0.353)		**PT:** 0.022 (0.012)		**PT gain:** 0.639 (0.340)

Note: Entries in Panel A are net effects, expressed as percent changes relative to IT.
Entries in Panel B are expressed in percent of aggregate consumption.
Entries in brackets refer to the case of Canadian nominal portfolios and price level shocks.

8. Conclusion

This paper has studied the aggregate and welfare effects of long run inflation risk under inflation and price-level targeting regimes. These two regimes have very different long run implications. Under inflation targeting, past inflationary shocks are ignored, so that that inflation risk increases with the forecast horizon due to base-level drift. With a price level targeting regime, by contrast, past inflationary shocks are reversed, so that the purchasing power of nominal assets is maintained over long horizons. These two regimes were analysed in a simple overlapping generations model that was roughly calibrated to match UK retirement portfolios. The model is well-suited for this task because long run inflation risk matters for social welfare and the distributional effects of the two regimes on young and old generations can be assessed directly.

The aggregate and welfare effects of long run inflation risk are found to be substantial. Under IT, the welfare cost of long run inflation risk is equal to 0.35 per cent of aggregate consumption. Since a PT regime largely eliminates long run inflation risk, it reduces this figure to 0.01 per cent, implying a social welfare gain equivalent to a permanent increase in aggregate consumption of 0.34 per cent. Price-level targeting raises social welfare because it reduces old generations' consumption risk by around one-tenth, and raises average consumption by the young non-trivially. However, its aggregate effects are not unambiguously positive: the old consume less on average, and consumption risk rises marginally for young generations.

The above results are strongly robust to returning the price level to target gradually over several years, and formal analysis of unexpected transitions from inflation to price-level targeting does not overturn any substantive conclusions. However, an important finding is that a low degree of credibility of PT reduces the potential long run welfare gain from price-level targeting by almost two-thirds, because the belief that policy may revert to inflation targeting (where the inflation risk premium is much higher) makes issuance of nominal government debt somewhat more costly than under perfect credibility. Consequently, taxes have to rise to maintain the same long run level of government spending, with a negative impact on lifetime consumption. The welfare gain from price-level targeting is also somewhat lower than in the baseline case if fiscal policy equalises taxes across regimes but provides public goods that enter private agents' utility functions. These findings suggest that the estimates from the baseline analysis should be treated with some caution.

Sensitivity analysis suggests that the welfare cost of long run inflation risk is likely to be substantially higher in economies with relatively little indexation and where risk aversion and nominal shocks are relatively high. The importance of nominal portfolios was assessed separately by calibrating the baseline model for portfolios in Canada, where nominal assets play a more important role than in the UK. Consistent with intuition, the welfare cost of long run inflation risk increased substantially, with the potential welfare gain from price-level targeting almost doubling. This result speaks to the importance of nominal portfolios for the transmission of inflation risk, suggesting that it is crucial to take this channel into account when comparing monetary regimes that have important implications for long run inflation risk. Lastly, calibrating the model with Canadian portfolios for price level shocks in Canada gives an estimated welfare gain similar to that in the UK case at 0.34 per cent, suggesting that the long run benefits from price-level targeting are likely to be broadly similar in these economies.

Of course, the results of this study do not imply that price-level targeting will dominate inflation targeting overall, because the model abstracts from short-term stabilization issues. Rather, the paper provides formal evidence on the potential long run benefits of price-level targeting that should be considered in conjunction with previous work assessing stabilization at business cycle frequencies. However, to the extent that the aggregate and welfare effects reported are quantitatively significant, the analysis in this study clearly points to the potential advantages of accounting for long run effects of monetary policy regimes in future research.

TECHNICAL APPENDIX (Not for publication)

A – Derivation of first-order conditions in the baseline model

Consumers solve a maximisation problem of the form

$$\max_{\{c_{t,Y},\, z_t,\, c_{t+1,O}\}} u_t(c_{t,Y}, c_{t+1,O}) = \frac{1}{1-\gamma}\left[c_{t,Y}^{\varepsilon} + \beta\left[E_t c_{t+1,O}^{1-\gamma}\right]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma}{\varepsilon}} \text{ subject to} \tag{A1}$$

$$c_{t,Y} = (1-\tau)w_t - k_{t+1} - b_{t+1}^i - b_{t+1}^n - m_t \qquad \text{(Budget constraint of young)}$$
$$c_{t+1,O} = (1-\tau^k)r_{t+1}^k k_{t+1} + r_t^f b_{t+1}^i + r_{t+1}^n b_{t+1}^n + r_{t+1}^m m_t \qquad \text{(Budget constraint of old)}$$
$$m_t = \delta \qquad \text{(CIA constraint)}$$

where $z_t \equiv (k_{t+1}, b_{t+1}^i, b_{t+1}^n, m_t)$ is the vector of assets chosen by households.

The Lagrangian for this problem is as follows:

$$L_t = E_t\left\{\begin{array}{l} u_t(c_{t,Y}, c_{t+1,O}) + \lambda_{t,Y}\left((1-\tau)w_t - k_{t+1} - b_{t+1}^i - b_{t+1}^n - m_t - c_{t,Y}\right) \\ + \lambda_{t+1,O}\left((1-\tau^k)r_{t+1}^k k_{t+1} + r_t^f b_{t+1}^i + r_{t+1}^n b_{t+1}^n + r_{t+1}^m m_t - c_{t+1,O}\right) + \mu_t(m_t - \delta)\end{array}\right\} \tag{A2}$$

First-order conditions are as follows:

$$c_{t,Y}:\ \frac{\partial u_t}{\partial c_{t,Y}} = \lambda_{t,Y}, \qquad c_{t+1,O}:\ \frac{\partial u_t}{\partial c_{t+1,O}} = \lambda_{t+1,O}, \qquad k_{t+1}:\ \lambda_{t,Y} = E_t(\lambda_{t+1,O}(1-\tau^k)r_{t+1}^k)$$

$$b_{t+1}^i:\ \lambda_{t,Y} = r_t^f E_t(\lambda_{t+1,O}), \qquad b_{t+1}^n:\ \lambda_{t,Y} = E_t(\lambda_{t+1,O} r_{t+1}^n), \qquad m_t:\ \lambda_{t,Y} = E_t(\lambda_{t+1,O} r_{t+1}^m) + \mu_t$$

By substitution, this system can be reduced to four Euler equations:

$$\frac{\partial u_t}{\partial c_{t,Y}} = E_t\left(\frac{\partial u_t}{\partial c_{t+1,O}}(1-\tau^k)r_{t+1}^k\right), \qquad \frac{\partial u_t}{\partial c_{t,Y}} = E_t\left(\frac{\partial u_t}{\partial c_{t+1,O}} r_{t+1}^n\right), \qquad \frac{\partial u_t}{c_{t,Y}} = r_t^f E_t\left(\frac{\partial u_t}{\partial c_{t+1,O}}\right),$$
$$\frac{\partial u_t}{\partial c_{t,Y}} = E_t\left(\frac{\partial u_t}{\partial c_{t+1,O}} r_{t+1}^m\right) + \mu_t$$

The partial derivatives of the utility function are as follows:

$$\frac{\partial u_t}{\partial c_{t,Y}} = \left[c_{t,Y}^{\varepsilon} + \beta\left[E_t c_{t+1,O}^{1-\gamma}\right]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma-\varepsilon}{\varepsilon}} c_{t,Y}^{-(1-\varepsilon)} \tag{A3}$$

$$\frac{\partial u_t}{\partial c_{t+1,O}} = \left[c_{t,Y}^{\varepsilon} + \beta\left[E_t c_{t+1,O}^{1-\gamma}\right]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma-\varepsilon}{\varepsilon}} \beta\left[E_t(c_{t+1,O}^{1-\gamma})\right]^{\frac{\varepsilon-(1-\gamma)}{1-\gamma}} c_{t+1,O}^{-\gamma} \tag{A4}$$

Dividing (A4) by (A3) gives

$$\frac{\partial u_t / \partial c_{t+1,O}}{\partial u_t / \partial c_{t,Y}} = \frac{\beta\left[E_t(c_{t+1,O}^{1-\gamma})\right]^{\frac{\varepsilon-(1-\gamma)}{1-\gamma}} c_{t+1,O}^{-\gamma}}{c_{t,Y}^{-(1-\varepsilon)}} = \beta\left(\frac{c_{t,Y}}{c_{t+1,O}}\right)^{1-\varepsilon}\left(\frac{c_{t+1,O}}{(E_t c_{t+1,O}^{1-\gamma})^{1/(1-\gamma)}}\right)^{1-\gamma-\varepsilon} \tag{A5}$$

Defining $sdf_{t+1} \equiv \frac{\partial u_t / \partial c_{t+1,O}}{\partial u_t / \partial c_{t,Y}}$, the four Euler equations above can be written as follows:

$$1 = E_t(sdf_{t+1}(1-\tau^k)r_{t+1}^k) \tag{A6}$$

$$1 = E_t(sdf_{t+1} r_{t+1}^n) \tag{A7}$$

$$1 = r_t^f E_t(sdf_{t+1}) \tag{A8}$$

$$1 = E_t(sdf_{t+1} r_{t+1}^m) + \tilde{\mu}_t \tag{A9}$$

where $\tilde{\mu}_t \equiv \mu_t / \lambda_{t,Y}$.

B – The binding cash-in-advance (CIA) constraint

It is shown in this section that the CIA constraint binds with strict equality if the gross money return on a nominal bond exceeds 1.

Proposition: The CIA constraint binds with strict equality when $R_t > 1$

Proof.

By equations (A7) and (A9), the Lagrange multiplier on the CIA constraint is given by

$$\mu_t = E_t[sdf_{t+1}(r_{t+1}^n - r_{t+1}^m)]\lambda_{t,Y} \tag{B1}$$

Since the real return on a nominal bond is $r_{t+1}^n = R_t /(1+\pi_{t+1}) = R_t r_{t+1}^m$, we can also write

$$\mu_t = E_t[sdf_{t+1}(R_t - 1)r_{t+1}^m)]\lambda_{t,Y} = \lambda_{t,Y}(R_t - 1)E_t[sdf_{t+1} r_{t+1}^m] \tag{B2}$$

where R_t is known at the end of period t.

The Kuhn-Tucker conditions associated with μ_t are as follows:

$$\{\, \mu_t \geq 0 \quad \text{and} \quad \mu_t(m_t - \delta) = 0 \,\} \tag{B3}$$

The second condition in (B3) is the complementary slackness condition. It implies that the CIA constraint will be strictly binding iff $\mu_t > 0$ for all t.

Dividing (B2) by $1 = E_t[sdf_{t+1} r_{t+1}^n] = R_t E_t[sdf_{t+1} r_{t+1}^m]$, it follows that $\mu_t = \lambda_{t,Y}(R_t - 1)/R_t$.

Since $\lambda_{t,Y} > 0$ (as the budget constraint of the young will always hold with equality), it follows that $\mu_t > 0$ iff $R_t > 1$ for all t. **Q.E.D.**

C – Approximate analytical expressions for long run inflation risk under IT and PT

This appendix derives approximate expressions for the inflation variance under IT and PT.

Inflation Targeting (IT)

Under IT, inflation is period t is given by

$$1+\pi_t = (1+\pi^*)^{30}\prod_{j=1}^{30}(1+\varepsilon_{j,t}) \tag{C1}$$

where $\varepsilon_{j,t}$ are IID-normal innovations with mean zero and variance σ^2.

Since a general non-linear function $g(\varepsilon)$ (where ε is a vector of variables) can be approximated by $\mathrm{var}(g(\varepsilon)) \approx \sum[g_j'(\mu)]^2\mathrm{var}(\varepsilon_j)$ using the 'Delta method' (where μ is the unconditional mean of the vector ε, and g_j' is the first derivative of $g(\varepsilon)$ with respect to variable ε_j), the inflation variance under IT can be approximated as follows:

$$\mathrm{var}(\pi_t) \approx \sum_{j=1}^{30}(1+\pi^*)^{60}\sigma^2 = (1+\pi^*)^{60}30\sigma^2 \tag{C2}$$

Price-level targeting (PT)

Under PT, inflation in period t is given by

$$1+\pi_t = (1+\pi^*)^{30}\frac{(1+\varepsilon_{30,t})}{(1+\varepsilon_{30,t-1})} \tag{C3}$$

where $\varepsilon_{30,t}$ and $\varepsilon_{30,t-1}$ are IID-normal innovations with mean zero and variance σ^2.

Using the same approximation method as above, the inflation variance under PT is given by

$$\mathrm{var}(\pi_t) \approx [(1+\pi^*)^{30}]^2\sigma^2 + [-(1+\pi^*)^{30}]^2\sigma^2 = (1+\pi^*)^{60}2\sigma^2 \tag{C4}$$

Hence the unconditional variance of inflation under IT is (approx.) 15 times that under PT.

D – Estimating the productivity innovation std. using productivity growth data

Section 4.2 in the paper notes that the productivity innovation standard deviation is similar to the standard deviation of annual UK TFP growth from 1998-2010. This section provides a formal justification for calibrating in this way using growth data.

It is assumed that annual productivity A_n follows an AR(1) process in logs:

$$\ln A_n = (1-\rho)\ln A_{mean} + \rho\ln A_{n-1} + e_n \tag{D1}$$

where e_n is an IID-normal innovation with variance σ_n^2.

Annual productivity growth can be approximated by the first difference of (D1):

$$g_A \approx \ln A_n - \ln A_{n-1} = (1-\rho)(\ln A_{mean} - \ln A_{n-1}) + e_n \tag{D2}$$

If annual productivity growth data is available, then σ_n^2 can be estimated using

$$\hat{\sigma}_n^2 \approx \text{var}(g_A)(1-(\rho-1)^2) \qquad \text{(D3)}$$

Therefore, if $(\rho - 1)^2 \approx 0$, the standard deviation of annual productivity growth will provide a good estimate of the standard deviation of the annual innovation to productivity.

E – First-order conditions under imperfect credibility

In this case, consumers solve the following problem where $s = \{IT, PT\}$:

$$\max_{\{c_{t,Y}, z_t, c_{t+1,O}\}} u_t = \frac{1}{1-\gamma}\left[c_{t,Y}^{\varepsilon} + \beta\left[p_{IT}E[c_{t+1,O(IT)}^{1-\gamma} \mid \Omega_t] + (1-p_{IT})E[c_{t+1,O(PT)}^{1-\gamma} \mid \Omega_t]\right]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma}{\varepsilon}} \qquad \text{(E1)}$$

subject to

$c_{t,Y} = (1-\tau)w_t - k_{t+1} - b_{t+1}^i - b_{t+1}^n - m_t$ (Budget constraint of young)

$c_{t+1,O(IT)} = (1-\tau_{IT}^k)r_{t+1}^k k_{t+1} + r_t^f b_{t+1}^i + r_{t+1(IT)}^n b_{t+1}^n + r_{t+1(IT)}^m m_t$ (Budget constraint of old with IT)

$c_{t+1,O(PT)} = (1-\tau_{PT}^k)r_{t+1}^k k_{t+1} + r_t^f b_{t+1}^i + r_{t+1(PT)}^n b_{t+1}^n + r_{t+1(PT)}^m m_t$ (Budget constraint of old with PT)

$m_t = \delta$ (CIA constraint)

where $E[X_{t+1(s)} \mid \Omega_t]$ is the expectation of X_{t+1} in regime s, conditional upon period-t information, Ω_t.

The Lagrangian for this problem is as follows:

$$L_t = u_t + E\left\{\left(\begin{array}{l}\lambda_{t,Y}((1-\tau)w_t - k_{t+1} - b_{t+1}^i - b_{t+1}^n - m_t - c_{t,Y}) + \mu_t(m_t - \delta) \\ + \lambda_{t+1,O(IT)}((1-\tau_{IT}^k)r_{t+1}^k k_{t+1} + r_t^f b_{t+1}^i + r_{t+1(IT)}^n b_{t+1}^n + r_{t+1(IT)}^m m_t - c_{t+1,O(IT)}) \\ + \lambda_{t+1,O(PT)}((1-\tau_{PT}^k)r_{t+1}^k k_{t+1} + r_t^f b_{t+1}^i + r_{t+1(PT)}^n b_{t+1}^n + r_{t+1(PT)}^m m_t - c_{t+1,O(PT)})\end{array}\right)\middle| \Omega_t\right\} \qquad \text{(E2)}$$

First-order conditions are as follows:

$$c_{t,Y}: \frac{\partial u_t}{\partial c_{t,Y}} = \lambda_{t,Y}, \quad c_{t+1,O(IT)}: \frac{\partial u_t}{\partial c_{t+1,O(IT)}} = \lambda_{t+1,O(IT)}, \quad c_{t+1,O(PT)}: \frac{\partial u_t}{\partial c_{t+1,O(PT)}} = \lambda_{t+1,O(PT)}$$

$$k_{t+1}: \lambda_{t,Y} = E([\lambda_{t+1,O(IT)}(1-\tau_{IT}^k)r_{t+1}^k + \lambda_{t+1,O(PT)}(1-\tau_{PT}^k)r_{t+1}^k] \mid \Omega_t)$$

$$b_{t+1}^i: \lambda_{t,Y} = r_t^f E([\lambda_{t+1,O(IT)} + \lambda_{t+1,O(PT)}] \mid \Omega_t)$$

$$b_{t+1}^n: \lambda_{t,Y} = E([\lambda_{t+1,O(IT)} r_{t+1(IT)}^n + \lambda_{t+1,O(PT)} r_{t+1(PT)}^n] \mid \Omega_t)$$

$$m_t: \lambda_{t,Y} = E([\lambda_{t+1,O(IT)} r_{t+1(IT)}^m + \lambda_{t+1,O(PT)} r_{t+1(PT)}^m] \mid \Omega_t) + \mu_t$$

By substitution, this system can be reduced to four Euler equations:

$$\frac{\partial u_t}{\partial c_{t,Y}} = E\left(\left[\frac{\partial u_t}{\partial c_{t+1,O(IT)}}(1-\tau_{IT}^k)r_{t+1}^k + \frac{\partial u_t}{\partial c_{t+1,O(PT)}}(1-\tau_{PT}^k)r_{t+1}^k\right]\middle|\Omega_t\right) \tag{E3}$$

$$\frac{\partial u_t}{\partial c_{t,Y}} = E\left(\left[\frac{\partial u_t}{\partial c_{t+1,O(IT)}} r_{t+1(IT)}^n + \frac{\partial u_t}{\partial c_{t+1,O(PT)}} r_{t+1(PT)}^n\right]\middle|\Omega_t\right) \tag{E4}$$

$$\frac{\partial u_t}{c_{t,Y}} = r_t^f E\left(\left[\frac{\partial u_t}{\partial c_{t+1,O(IT)}} + \frac{\partial u_t}{\partial c_{t+1,O(PT)}}\right]\middle|\Omega_t\right) \tag{E5}$$

$$\frac{\partial u_t}{\partial c_{t,Y}} = E\left(\left[\frac{\partial u_t}{\partial c_{t+1,O(IT)}} r_{t+1(IT)}^m + \frac{\partial u_t}{\partial c_{t+1,O(PT)}} r_{t+1(PT)}^m\right]\middle|\Omega_t\right) + \mu_t \tag{E6}$$

The partial derivatives of the utility function are as follows:

$$\frac{\partial u_t}{\partial c_{t,Y}} = A c_{t,Y}^{-(1-\varepsilon)} \tag{E7}$$

$$\frac{\partial u_t}{\partial c_{t+1,O(s)}} = \beta A p_s \left(p_{IT} E[c_{t+1,O(IT)}^{1-\gamma} \mid \Omega_t] + (1-p_{IT}) E[c_{t+1,O(PT)}^{1-\gamma} \mid \Omega_t]\right)^{\frac{\varepsilon-(1-\gamma)}{1-\gamma}} c_{t+1,O(s)}^{-\gamma} \tag{E8}$$

where $A \equiv \left[c_{t,Y}^{\varepsilon} + \beta\left[p_{IT} E[c_{t+1,O(IT)}^{1-\gamma} \mid \Omega_t] + (1-p_{IT}) E[c_{t+1,O(PT)}^{1-\gamma} \mid \Omega_t]\right]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma-\varepsilon}{\varepsilon}}$ and $p_{PT} \equiv 1 - p_{IT}$.

Dividing (E8) by (E7) gives

$$\frac{\partial u_t / \partial c_{t+1,O(s)}}{\partial u_t / \partial c_{t,Y}} = \beta p_s \left(\frac{c_{t,Y}}{c_{t+1,O(s)}}\right)^{1-\varepsilon} \left[\frac{c_{t+1,O(s)}}{\left(p_{IT} E[c_{t+1,O(IT)}^{1-\gamma} \mid \Omega_t] + (1-p_{IT}) E[c_{t+1,O(PT)}^{1-\gamma} \mid \Omega_t]\right)^{1/(1-\gamma)}}\right]^{1-\gamma-\varepsilon} \tag{E9}$$

Defining $sdf_{t+1(s)} \equiv \frac{1}{p_s} \frac{\partial u_t / \partial c_{t+1,O(s)}}{\partial u_t / \partial c_{t,Y}}$, the four Euler equations above can be written as

$$1 = R_t \left(p_{IT} E[SDF_{t+1(IT)} \mid \Omega_t] + (1-p_{IT}) E[SDF_{t+1(PT)} \mid \Omega_t]\right) \tag{E10}$$

$$1 = r_t^f \left(p_{IT} E[sdf_{t+1(IT)} \mid \Omega_t] + (1-p_{IT}) E[sdf_{t+1(PT)} \mid \Omega_t]\right) \tag{E11}$$

$$1 = \alpha k_{t+1}^{\alpha-1} \left([p_{IT}(1-\tau_{IT}^k) E[sdf_{t+1(IT)} A_{t+1} \mid \Omega_t] + (1-p_{IT})(1-\tau_{PT}^k) E[sdf_{t+1(PT)} A_{t+1} \mid \Omega_t]\right) \tag{E12}$$

$$1 = \left(p_{IT} E[SDF_{t+1(IT)} \mid \Omega_t] + (1-p_{IT}) E[SDF_{t+1(PT)} \mid \Omega_t]\right) + \tilde{\mu}_t \tag{E13}$$

where $SDF_{t+1(s)} \equiv sdf_{t+1(s)}/(1+\pi_{t+1(s)})$ and $\tilde{\mu}_t \equiv \mu_t / \lambda_{t,Y}$.

F – First-order conditions when public goods consumption enters the utility function

In this case, consumers solve the following problem:

$$\max_{\{c_{t,Y},\,z_t,\,c_{t+1,O}\}} u_t = \frac{1}{1-\gamma}\left[(c_{t,Y} + \theta h_t)^{\varepsilon} + \beta\left[E_t(c_{t+1,O} + \theta h_{t+1})^{1-\gamma}\right]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma}{\varepsilon}} \qquad \text{(F1)}$$

subject to

$c_{t,Y} = (1-\tau)w_t - k_{t+1} - b^i_{t+1} - b^n_{t+1} - m_t$	(Budget constraint of young)
$c_{t+1,O} = (1-\tau^k)r^k_{t+1}k_{t+1} + r^f_t b^i_{t+1} + r^n_{t+1}b^n_{t+1} + r^m_{t+1}m_t$	(Budget constraint of old)
$m_t = \delta$	(CIA constraint)
$h_t = \chi g_t + (1-\chi)g_{t-1}$	(Public goods consumption)

where consumption services from public goods received in period t, h_t, is taken as given.

Defining $\tilde{c}_{t,i} \equiv c_{t,i} + \theta\, h_t$ for $i \in \{Y, O\}$, the Lagrangian for this problem is as follows:

$$L_t = E_t\left\{\begin{array}{l} u_t(\tilde{c}_{t,Y}, \tilde{c}_{t+1,O}) + \lambda_{t,Y}((1-\tau)w_t - k_{t+1} - b^i_{t+1} - b^n_{t+1} - m_t - c_{t,Y}) \\ + \lambda_{t+1,O}((1-\tau^k)r^k_{t+1}k_{t+1} + r^f_t b^i_{t+1} + r^n_{t+1}b^n_{t+1} + r^m_{t+1}m_t - c_{t+1,O}) + \mu_t(m_t - \delta) \end{array}\right\} \qquad \text{(F2)}$$

First-order conditions are as follows:

$$c_{t,Y}:\ \frac{\partial u_t}{\partial \tilde{c}_{t,Y}}\cdot\frac{\partial \tilde{c}_{t,Y}}{\partial c_{t,Y}} = \lambda_{t,Y}, \quad c_{t+1,O}:\ \frac{\partial u_t}{\partial \tilde{c}_{t+1,O}}\cdot\frac{\partial \tilde{c}_{t+1,O}}{\partial c_{t+1,O}} = \lambda_{t+1,O}, \quad k_{t+1}: \lambda_{t,Y} = E_t(\lambda_{t+1,O}(1-\tau^k)r^k_{t+1})$$

$$b^i_{t+1}:\ \lambda_{t,Y} = r^f_t E_t(\lambda_{t+1,O}), \qquad b^n_{t+1}:\ \lambda_{t,Y} = E_t(\lambda_{t+1,O}r^n_{t+1}), \qquad m_t:\ \lambda_{t,Y} = E_t(\lambda_{t+1,O}r^m_{t+1}) + \mu_t$$

By substitution, this system can be reduced to four Euler equations:

$$\frac{\partial u_t}{\partial \tilde{c}_{t,Y}}\cdot\frac{\partial \tilde{c}_{t,Y}}{\partial c_{t,Y}} = E_t\left(\frac{\partial u_t}{\partial \tilde{c}_{t+1,O}}\cdot\frac{\partial \tilde{c}_{t+1,O}}{\partial c_{t+1,O}}(1-\tau^k)r^k_{t+1}\right), \qquad \frac{\partial u_t}{\partial \tilde{c}_{t,Y}}\cdot\frac{\partial \tilde{c}_{t,Y}}{\partial c_{t,Y}} = E_t\left(\frac{\partial u_t}{\partial \tilde{c}_{t+1,O}}\cdot\frac{\partial \tilde{c}_{t+1,O}}{\partial c_{t+1,O}}r^n_{t+1}\right),$$

$$\frac{\partial u_t}{\partial \tilde{c}_{t,Y}}\cdot\frac{\partial \tilde{c}_{t,Y}}{\partial c_{t,Y}} = r^f_t E_t\left(\frac{\partial u_t}{\partial \tilde{c}_{t+1,O}}\cdot\frac{\partial \tilde{c}_{t+1,O}}{\partial c_{t+1,O}}\right), \qquad \frac{\partial u_t}{\partial \tilde{c}_{t,Y}}\cdot\frac{\partial \tilde{c}_{t,Y}}{\partial c_{t,Y}} = E_t\left(\frac{\partial u_t}{\partial \tilde{c}_{t+1,O}}\cdot\frac{\partial \tilde{c}_{t+1,O}}{\partial c_{t+1,O}}r^m_{t+1}\right) + \mu_t$$

Given that $\partial\tilde{c}_{t,i}/\partial c_{t,i} = 1$, the partial derivatives of the utility function are as follows:

$$\frac{\partial u_t}{\partial \tilde{c}_{t,Y}}\cdot\frac{\partial \tilde{c}_{t,Y}}{\partial c_{t,Y}} = \left[\tilde{c}^{\,\varepsilon}_{t,Y} + \beta\left[E_t\tilde{c}^{\,1-\gamma}_{t+1,O}\right]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma-\varepsilon}{\varepsilon}}\tilde{c}^{\,-(1-\varepsilon)}_{t,Y} \qquad \text{(F3)}$$

$$\frac{\partial u_t}{\partial \tilde{c}_{t+1,O}}\cdot\frac{\partial \tilde{c}_{t+1,O}}{\partial c_{t+1,O}} = \left[\tilde{c}^{\,\varepsilon}_{t,Y} + \beta\left[E_t\tilde{c}^{\,1-\gamma}_{t+1,O}\right]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma-\varepsilon}{\varepsilon}}\beta\left[E_t(\tilde{c}^{\,1-\gamma}_{t+1,O})\right]^{\frac{\varepsilon-(1-\gamma)}{1-\gamma}}\tilde{c}^{\,-\gamma}_{t+1,O} \qquad \text{(F4)}$$

Dividing (F4) by (F3),

$$\frac{\partial u_t / \partial \tilde{c}_{t+1,O}}{\partial u_t / \partial \tilde{c}_{t,Y}} = \frac{\beta \left[E_t (\tilde{c}_{t+1,O}^{1-\gamma}) \right]^{\frac{\varepsilon-(1-\gamma)}{1-\gamma}} \tilde{c}_{t+1,O}^{-\gamma}}{\tilde{c}_{t,Y}^{-(1-\varepsilon)}} = \beta \left(\frac{\tilde{c}_{t,Y}}{\tilde{c}_{t+1,O}} \right)^{1-\varepsilon} \left(\frac{\tilde{c}_{t+1,O}}{(E_t \tilde{c}_{t+1,O}^{1-\gamma})^{1/(1-\gamma)}} \right)^{1-\gamma-\varepsilon} \quad \text{(F5)}$$

Hence, with $sdf_{t+1} \equiv \dfrac{\partial u_t / \partial \tilde{c}_{t+1,O}}{\partial u_t / \partial \tilde{c}_{t,Y}}$, the four Euler equations above can be written as follows:

$$1 = E_t (sdf_{t+1} (1-\tau^k) r_{t+1}^k) \quad \text{(F6)}$$

$$1 = E_t (sdf_{t+1} r_{t+1}^n) \quad \text{(F7)}$$

$$1 = r_t^f E_t (sdf_{t+1}) \quad \text{(F8)}$$

$$1 = E_t (sdf_{t+1} r_{t+1}^m) + \tilde{\mu}_t \quad \text{(F9)}$$

where $\tilde{\mu}_t \equiv \mu_t / \lambda_{t,Y}$.

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