First Version: June 2000
This Version: October 2000

Central Bank Goals, Institutional Change and Monetary Policy: Evidence from the US and UK[+]

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Abstract

The theoretical literature on monetary policy design places great emphasis on the influence exercised by institutional reforms on the conduct of monetary policy. There is little empirical evidence on this point, and on the impact which changes in interest rate policy have on the transmission mechanism through the Lucas critique. This paper uses superexogeneity/invariance tests and time-varying VAR models to examine how the transmission mechanism and policy reactions have varied in the UK over the last decade. As a benchmark, we also estimate these models for the US, a country in which the formal institutional framework has not changed markedly over recent years. Our results show that institutional change has affected UK interest rate setting, and that this in turn has affected the transmission mechanism. Interestingly our estimates for the US also detect changes in monetary policy reactions to macroeconomic conditions.

Keywords: central bank goals and institutional change; policy reaction functions; invariance; the transmission mechanism; Bayesian VAR models;

JEL Codes: E58, E43, C32.

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(+) We are extremely grateful to our colleague Ulrich Woitek for providing us with the basic GAUSS code used to perform the Bayesian VAR estimates in Section 3. We are grateful for helpful comments from participants at the CCBS Workshop on the transmission mechanism held at the Bank of England's CCBS in June 2000, and at the Royal Economic Society Conference 2000, held in St. Andrews. In particular, we should mention (without implicating them) Larry Ball, Jon Faust, Lavan Mahadeva, and particularly Ed Nelson who acted as discussant on the paper.

1. Introduction

A considerable empirical literature has emerged on the estimation of policy reaction functions, and the identification of the underlying preferences of monetary authorities (see Groeneveld *et al.*, 1996, Muscatelli and Tirelli, 1996, Clarida and Gertler, 1997, Clarida *et al.*, 1998, Favero and Rovelli, 1999, Muscatelli *et al.*, 1999). Some of these contributions examine whether recent changes in institutional structure, such as the shift to inflation targeting, have had an impact on the conduct of monetary policy[1]. The evidence is mixed. For instance Muscatelli *et al.* (1999) show that there is only slight evidence that the introduction of inflation targeting affected forward-looking policy reaction functions in the UK, New Zealand, Sweden and Canada. Instead they find some evidence of policy instability in Japan and the USA in the 1980s and 1990s, even in the absence of institutional change.

Of course one would also expect significant shifts in monetary policy, which bring about a reduction in inflation expectations, to affect the transmission mechanism of monetary policy. The standard New Keynesian model of aggregate demand and supply[2] which has extensively been used for policy analysis (see Svensson, 1997, Rudebusch and Svensson, 1999, McCallum and Nelson 1999 [a,b], Rudebusch, 1999) suggests that forward-looking expectations are important both on the demand and supply side. A typical formulation of a New Keynesian model is:

$$y_t = E_t y_{t+1} - a_1(i_t - E_t p_{t+1}) + e_{1t} \qquad (1)$$

$$p_t = b_1 E_t p_{t+1} + (1 - b_1) p_{t-1} + b_2 y_t + e_{2t} \qquad (2)$$

[1] In addition to contributions which estimate policy reaction functions, a number of other authors have used a variety of methods to test whether the introduction of inflation targeting had an impact on inflation expectations (see Freeman and Willis, 1995, King, 1995) or on a range of monetary policy indicators (see Almeida and Goodhart, 1996).

[2] See, *inter alia*, Goodfriend and King (1997), Walsh (1998), Clarida *et al.* (1999), McCallum and Nelson (1999a, 1999b), Rotemberg and Woodford (1999).

where y is the output gap, i is the nominal interest rate, p is the inflation rate, and E is the expectations operator. In empirical applications, lags of output could be added to (1) to capture costly adjustment or habit persistence, and a more complex lagged adjustment of inflation could be considered in (2). The presence of forward-looking expectations in these models means that any changes in monetary policy should lead to structural breaks in the aggregate supply relationship (2), or in the intertemporal aggregate demand equation (1). This is of course the standard Lucas (1976) critique. Hutchison and Walsh (1998) find that in the case of New Zealand the short-run output-inflation trade-off in a relationship like (2) increased in the 1990s after the monetary reform. Rudebusch and Svensson (1999) find that for the US there is little evidence of structural breaks[3] in their (backward-looking) models of output and inflation estimated over a sample period 1961(1)-1996(4).

This paper makes two contributions. First, for the US and UK we check whether there is any evidence for shifts in the transmission mechanism, as represented by the aggregate demand and supply relationships, due to shifts in agents' expectations. We use a class of invariance tests developed by Engle and Hendry (1993) to test whether the Lucas critique has any force in this case. We find evidence of shifts in the output and inflation models in the last two decades.

Second, as the invariance tests are not designed to examine the timing of shifts in the transmission mechanism, we use an alternative method to detect contemporaneous shifts in interest rate policy reactions and in the output and inflation equations. We estimate a Bayesian VAR of the monetary transmission mechanism for the US and UK, using a simple trivariate specification with the interest rate, output gap and inflation rate. In contrast to standard full-sample VAR estimates we find the policy rules which emerge from our Bayesian VAR estimates to be more plausible and interpretable in terms of

[3] They use Andrews (1993)-type stability tests.

policy reactions. Our results show that the policy stance in the UK and the US has changed and evolved from the late 1980s onwards. We also detect some minor shifts in the way in which output and inflation have responded to policy, which might be indicative of expectations effects. There is therefore some evidence that the Lucas critique is important in understanding the transmission mechanism. Quantitatively it might be less important in the relatively benign macroeconomic environment of the 1990s, but it may become more significant in the future. This is especially the case at a time when the spectre of oil shocks has returned.

The rest of this paper is divided as follows. In Section 2 we report our invariance tests of conditional models for the aggregate demand and supply relationships. In Section 3 we present our Bayesian VAR results. Section 4 concludes.

2. Invariance Tests

A full account of the relevant concepts of weak exogeneity, constancy, invariance and superexogeneity is provided elsewhere in the literature and will not be repeated here (see Favero and Hendry, 1990, and Engle and Hendry, 1993). More details of the test procedure used here are provided in Appendix A. For our purposes, we wish to test whether forward-looking expectations enter equations (1) and (2), the output and inflation relationships. The approach followed here is to estimate backward-looking conditional models for output and inflation, and to test whether variations in the moments of the regressors in each model influence the parameters of the backward-looking conditional model. Thus, we first estimate the output and inflation equations as backward-looking models, using autoregressive distributed-lag formulations[4]:

[4] Although in the case of the UK some additional regressors are added to take account of the greater openness of the UK economy.

$$y_t = c_1 + \sum_{i=1}^{n} a_{yi} y_{t-i} + \sum_{i=0}^{n} a_{ri} \overline{r}_{t-i} + e_{1t} \qquad (3)$$
$$p_t = c_2 + \sum_{i=1}^{n} b_{pi} p_{t-i} + \sum_{i=0}^{n} b_{yi} y_{t-i} + e_{2t} \qquad (4)$$

where c_i are constants, and $\overline{r}$ is a measure of the real interest rate, calculated using the current inflation rate, $\overline{r}_t = (i - p)_t$. If forward-looking expectations of the interest rate and inflation are important in determining current output and inflation then these equations will not be invariant to changes in policy regime as they will be convolutions of the 'deep' parameters of the forward-looking models and the forecasting equations. This can be tested by checking if shifts in the first and second moments of the real interest rate or output affect the regressions in (3)-(4) (see Appendix A). To obtain measures of these moments, we fit marginal models for output and the real interest rate using simple autoregressive models[5].

Table 1 shows the most parsimonious version of (3) and (4) which we estimated. Our conditional models generally follow the specification reported in Rudebusch and Svensson (1999) for the US and in Hall *et al.* (1999) for the UK. Definitions of variables used can be found in the Data Appendix. In computing the output gap, we follow two alternative approaches. The first is to follow these earlier contributions in using the BEA estimate (y^{bea} for the US), and the OECD estimate (y^{oecd} for the UK), of potential output, and computing the output gap as the deviation of actual output from potential as a percentage of potential output. The second approach is to fit a univariate structural time series (STS) model (see Harvey, 1989) for output. This decomposes output into stochastic trend, cycle and irregular components, and we define potential output as the stochastic trend element. A convenient decomposition can be obtained by applying the Kalman filter on the trend component and using the one-step-ahead predictions of the

[5] As an alternative, one could fit marginal models for the interest rate which have a structural interpretation. We experimented with the use of estimated policy reaction functions such as those estimated in Muscatelli *et al.* (1999). However, the results obtained were not very different from those which used an autoregressive model for the interest rate.

state vector. The output gap measure obtained is labelled as y^{kal}. The difference between the two output gap series in the case of the two countries is shown in Figures 1 and 2. As can be readily seen, the BEA and OECD measures lead to much smoother measures of the output gap, but ones which show a greater variance over the sample period.

The real interest rate effect $\bar{r}$ is measured as an average effect[6]: the deviation of the 4-quarter average nominal i interest rate from the 4-quarter inflation rate in the relevant price index, i.e. $\bar{r}_t = (1/4)\sum_{i=0}^{3} i_t - (1/4)\sum_{i=0}^{3} p_t$

In the case of the UK we also add some additional regressors to the models as specified in (3) and (4), following[7] Hall *et al.* (1999). In the case of the aggregate demand equation, we add a measure of the real effective exchange rate, *q*. In the case of the inflation equation, we add as regressors a series for import prices, *pim*, and a series on unit labour costs, *ulc*. This reflects the fact that the UK has a more open economy.

The results obtained for the US are very similar to those reported in Rudebusch and Svensson (1999), which is not surprising, as the sample used is very similar. The use of y^{kal} instead of y^{bea} does not yield very different results. The fit using y^{bea} is slightly better, which is not surprising given that it is a smoother series and that it has a higher sample variance. The UK results are similar to those of the US, except that the real interest variable has the wrong sign (but is insignificant, if the y^{oecd} definition of the output gap is used). The additional regressors all have the right signs, and again the fit is generally better is the y^{oecd} measure is used. In what follows we shall conduct our invariance tests using the y^{oecd} and y^{bea} measures of the output gap.

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[6] This provided a better fit than the use of a distributed lag of individual interest rates.
[7] However, we do not follow Hall *et al.* in adding terms for foreign demand, fiscal policy, and real money balances in the aggregate demand equation. In any case our models are not strictly comparable because our dependent variable is the output gap rather than growth in real output.

The marginal models fitted for the real interest rate and output are reported in Table 2. It should be noted that these regression models are not designed to have a structural interpretation: as outlined in Appendix A they merely allow us to construct some series for the conditional mean of the interest rate and output, which can be used in the invariance tests. In order to obtain measures of the second moments of these variables, we fitted 4-th order ARCH models for the fitted residuals $(\hat{\hbar}_i^{\,2})$ from the regressions in Table 2. In Table 2 we also report the estimates of single-equation forward looking policy reaction functions[8], as a potential alternative to a naïve autoregressive model for the real interest rate. In practice, however, the best results from the invariance tests are obtained using the simple AR model. As stressed in Engle and Hendry (1993) the marginal models can include dummy variables to capture important policy shifts. For the interest rate models we find that a dummy to capture the 1992 ERM crisis improves the fit of the UK model, and a dummy to capture the change in the Fed's operations in 1979-82 improves the fit of the US model.

Our invariance tests are reported in Table 3. They are computed over two different sample periods: the full sample available to us, and a shorter sample, from 1984 onwards. The latter period is usually seen as one of greater monetary stability, after the disinflation following the second oil shock.

Our invariance tests show some evidence in favour of the Lucas critique over both the full sample and the post-1984 sub-sample. In the case of the US, the null of invariance is rejected for both the output and inflation equations over the full sample period. But if we focus only on the post-1984 period, we find some evidence of the failure of invariance only in the case of the inflation equation. In the case of the UK the results are very similar, but stronger. There is evidence of non-invariance both in the

[8] These have the same structure as the forward-looking policy reaction functions reported in Muscatelli *et al.* (1999) and Clarida *et al.* (1998).

whole sample (1978:1-1999:4), and in the post-1984 sample, for both the output and inflation equation.

Of course one has to be cautious in concluding that Lucas-critique type effects are present from invariance/superexogeneity tests alone. First, one could argue that the empirical size of these tests will depend critically on the correct specification of the marginal models[9]. Second, it should be noted that even when the null hypothesis of invariance is rejected, we cannot date the shift in policy regime using these tests.

As stated in the introduction, we have strong evidence from previous studies (e.g. Muscatelli *et al.*, 1999) that some policy regime shifts have occurred over the last two decades, even in the USA. We now turn to an alternative method of checking whether there are shifts in the transmission mechanism which correspond to interest rate policy regime shifts; i.e. whether shifts in the transmission mechanism are attributable to the Lucas critique.

3. The Monetary Transmission Mechanism: Bayesian VAR Estimates

3.1 Standard VAR Analysis

Before turning to our Bayesian VAR estimates, as a benchmark we estimate a standard traditional VAR of the transmission mechanism. Figures 3 and 4 show the impulse responses and 95% confidence bands for the US and UK from a trivariate VAR which includes the nominal policy interest rate, the quarter-on-quarter inflation rate, and the output gap (using the y^{oecd} and y^{bea}) definitions[10]. Four lags of the variables are included. For reasons of space we only show impulse responses for up to 8 quarters in

[9] However, these tests can also suffer from the problem of low power.

[10] For the UK we decided not to include additional variables such as the real exchange rate. Although these might be helpful in describing the transmission mechanism in an open economy, restricting our attention to the output gap and inflation allows us to more readily interpret the policy rules which emerge from the VAR. Including the real exchange rate would also raise some issues regarding the appropriate ordering of the variables in the Cholesky factorisation, given that the UK has experienced different exchange rate regimes over the sample period.

Figures 4 and 5. Although some of the impulse responses seem not to converge towards zero (especially for the UK), in fact the VAR is stationary, and convergence generally occurs for all the impulse responses after 8-16 quarters. The impulse responses have been derived using a Cholesky factorisation with a causal order as follows: the output gap, inflation, and the interest rate. The VAR estimates[11] have been conducted for the same sample as our invariance tests, namely 1961:3-2000:1 for the US and 1977:4-1999:4 for the UK.

The results for the US are, not suprisingly, very similar to those reported in Rudebusch and Svensson (1999) for an equivalent trivariate VAR for the US over a similar sample period. The impulse responses in Figure 4 show a positive response of the Federal Funds rate to shocks to the output gap and the inflation rate. There is a slight price-puzzle effect following an interest rate shock but generally the other impulse responses are as one would have expected them. An output gap shock leading to a significant inflation response after 3 quarters, and an inflation shock leading to a fall in output (significant after 8 quarters). Interest rate shocks lead to output falls after 2-3 quarters.

In the case of the UK, the prize puzzle is also present, but the impulse response is insignificant (except in quarters 3-4). The only other feature of note is that the response of the interest rate to an inflation rate shock is insignificant, reflecting the inclusion of the 1970s data in our full sample.

These impulse responses reveal the typical weaknesses of VAR analysis. One problem with unrestricted VARs is that they are overfitted, which explains why some of the responses are insignificant. Another problem is the interpretation of the implicit policy rule of the monetary authorities. As noted by Rudebusch (1998), the implicit policy rule which emerges from a just-identified VAR model is implausible, suggesting a

[11] All the estimates in the paper are produced using GAUSS routines.

response of the Funds Rate to a unit shock in the inflation rate which is less than one. But the problem with the policy rule is also that it covers periods during which the Fed's behaviour changed dramatically. The same applies to the UK where, as we have already noted, the response of the interest rate to an inflation shock has the right sign but is insignificant, reflecting the very different nature of monetary policy in the early part of the sample.

Despite their limitations, VARs can be useful as descriptors of the dynamic correlations of jointly endogenous variables. In our case, we are particularly interested not only in examining policy responses, but also in examining whether any changes in policy regime elicited a change in the transmission mechanism responses.

The alternative to VARs has been to estimate single-equation interest-rate reaction functions (as in Clarida *et al.*, 1998 and Muscatelli *et al.*, 1999). These single-equation interest rate reaction functions can be estimated using rolling or recursive estimation. However, typically single-equation approaches ignore the fact that we are dealing with jointly endogenous variables. It is therefore important to check that the results obtained from single-equation models are not due to the arbitrary imposition of a theoretical structure on the reaction function. VARs remain useful in this context, because they impose the minimum amount of theoretical structure. However, we have to allow explicitly for changes or evolution in policy monetary regimes. In order to do this, we have to allow for some time-variation in the parameters of the VAR.

Before turning to our models we should mention that there are some caveats in interpreting shifts in the response of interest rates to the output gap and inflation as shifts in policy preferences. These apply to both single-equation reaction functions and VARs. Setting interest rates usually involves a complex decision-making process, where a policy committee or council sets interest rates based on a host of different macroeconomic indicators and models (see Bank of England, 1999; Vickers, 1999).

Where interest rates are set by committee each member of the decision-making body might rely on a different sub-set of indicators and might have a preferred 'model' of the transmission mechanism. Estimated interest rate reaction functions clearly cannot, and do not set out to, model every aspect of this complex decision-making process. Instead, they focus on simple policy rules where interest rates respond to expected inflation and the output gap (and display some short-term inertia). If the estimated reaction functions are found to be stable, this can be seen as an indication that the actual information set used by the policy authorities displays a stable relationship with the final policy objectives (output and inflation) in the estimated rule *and* that the policy authorities' preferences did not change over the estimation sample. On the other hand, if one detects instability in a reaction function, this would indicate either a shift in policy preferences *or* a shift in the relationship between the final objectives of policy and the wider information set (including the many indicators) used by the policy committee.

Some observers see VAR reaction functions as particularly prone to problems of interpretation (see Rudebusch, 1998). However, part of the reason for this is that most monetary policy VARs include many indicator and intermediate objective variables. By focusing only on a trivariate VAR, our approach is closer to those contributions which estimate single-equation reaction functions (Clarida *et al.*, 1998, Muscatelli *et al.*, 1999, Nelson, 2000). However, we are able to consider not only potential shifts in interest rate policy, but also the consequences of these policy shifts for the transmission mechanism.

3.2 Bayesian VAR Analysis

The estimation of VAR models with time-varying coefficients was pioneered by Doan, Litterman and Sims (1984). In this paper we follow this Bayesian approach to VAR estimation, which allows the parameters of the VAR to evolve as more observations are added[12]. This has intuitive appeal in modelling a situation where monetary policy changes occurred, as policy regime changes are likely to involve a gradual evolution of responses. This is even the case where shifts in policy regime seem sudden. In the UK, after the exit from the ERM and the implementation of inflation targets, it arguably took some time before the new system of inflation targeting became fully functional. In 1997, after the Bank of England acquired independence, it took time for the new MPC to learn how to react optimally to new information. Even in the case of the US, where the underlying institutional structure has not changed, the 1990s have been a period in which the authorities' have had to learn gradually about the changing nature of the underlying macroeconomic relationships. For instance in the mid-1990s the Fed tightened monetary policy at a time when it was uncertain about the 'productivity miracle' and the inflationary consequences of a tightening labour market. Similarly, if the monetary authorities' policy stance changes over time, we would expect private sector expectations to evolve gradually as they learn about the changes in policy response. This gradual learning in the light of new information makes a Bayesian VAR approach particularly attractive in this context.

The estimation procedure can be outlined as follows. We begin with the definition of a standard VAR(p):

$$X_t = c + \sum_{j=1}^{p} A_j X_{t-j} + e_t \tag{5}$$

where X_t is a $(n´1)$ vector of endogenous variables, A_js are the $(n´n)$ matrices of parameter coefficients, and e_t is a $(n´1)$ vector of disturbances for which:

$$\begin{aligned} &E\{e_t\} = 0 \\ &E\{e_t e_t^{'}\} = \Sigma \\ &E\{e_t e_s^{'}\} = 0,\ \forall t \neq s \end{aligned} \tag{6}$$

[12] For a recent application of Bayesian VAR analysis to the US economy during the Great Depression,

In our case, $X_t^{'} = (y, p, i)_t$ and n = 3. Following Lutkepohl (1991), and Hamilton (1994), we can write the model in the following way[13]:

$$
\begin{aligned}
&X = AZ + U, \\
&X = \begin{pmatrix} X_{p+1} & X_{p+2} & \dots & X_T \end{pmatrix}; \\
&A = \begin{pmatrix} c & A_1 & \dots & A_p \end{pmatrix}; Z = \begin{pmatrix} Z_p & Z_{p+1} & \dots & Z_{T-1} \end{pmatrix} \\
&Z_t = \begin{pmatrix} 1 \\ X_{t-1} \\ X_{t-2} \\ \vdots \\ X_{t-p} \end{pmatrix}; U = \begin{pmatrix} e_{p+1} & e_{p+2} & \dots & e_T \end{pmatrix}
\end{aligned}
\qquad (6)
$$

where now we have only $T^* = T - p$ observations available in each equation.

Assuming that the model is stationary, the VAR has the following finite MA representation:

$$X_t = \sum_{j=0}^{\infty} B_j e_{t-j}, \qquad (7)$$

where the B_js are the (n´n) MA parameter matrices. Given the information set W, if the residual variance-covariance matrix S is diagonal, the impulse-response function will be defined as

$$IR_X(h, d, \Omega_{t-1}) = E\{X_{t+h} | e_t = d, \Omega_{t-1}\} - E\{X_{t+h} | \Omega_{t-1}\}, \qquad (8)$$

i.e. the difference between the expected value of X_t at horizon h, given that a shock d hits the system in time t, and the expected value of X_t in the absence of shocks. The MA parameters j can then be interpreted as responses of X_{t+h} to a shock in t on variable j:

$$j_{j,h} = B_h e_j, \qquad (9)$$

where e_j is a vector of zeroes with one as the jth element. If, on the other hand, S is not diagonal, contemporaneous interactions amongst the variables prevent any interpretation

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see Ritschl and Woitek (2000).

[13] Henceforth we assume that p is known and fixed.

of the VAR residuals as fundamental disturbances, and the system is not identified. The method of identification employed here orthogonalises the shocks according to a Choleski decomposition of the residual variance-covariance matrix: $PP' = S$, where P is a lower triangular matrix. This way, the orthogonalised responses are recovered as

$$j^{O}_{j,h} = B_h P e_j \tag{10}$$

This defines the standard VAR model.

Next we consider the possibility of time-varying parameters. If we assume the VAR coefficients as time-dependent, equation j from (5) can be written as:

$$x_{t,j} = Z' \begin{pmatrix} c_j \\ b^1_{j1} \\ \vdots \\ b^1_{jn} \\ \vdots \\ b^p_{j1} \\ \vdots \\ b^p_{jn} \end{pmatrix} + e_{t,j} = Z' b_t + e_{t,j}\,, \tag{11}$$

where the betas are the elements of the VAR parameter matrices. A state-space representation of such a model would have equation (11) as the measurement equation. The Doan *et al.* (1984) procedure assumes that the VAR coefficients follow an AR(1) process, and the transition equation of the system is therefore:

$$b_t = c + T b_{t-1} + u_t \tag{12}$$

Doan, Litterman and Sims (1984) suggest the use of a Bayesian prior distribution for the initial value of the coefficient vector, $b \sim N(\bar{b}, P_{1|0})$ and then allow the parameters to be updated according to some law of motion. In fact, we assume that the VAR parameters behave as follows[14]:

[14] The description here closely follows the notation used in Hamilton (1994), Ch. 13.

$$b_t = (1 - p_1)\bar{b} + p_1 I_{np+1} b_{t-1} + u_t \tag{13}$$

In (13), the parameter vector follows an autoregressive process, in which the weighting parameter p_1 determines the importance of the steady-state value for the coefficient vector $\bar{b}$. The disturbance term is uncorrelated with the disturbances in the original VAR: $Cov[e_t, u_t] = 0$, whereas the expected value $\bar{b}$ consists of a vector of zeroes with one as elements corresponding to the own variable $x_{t-1,j}$ at lag 1 for each equation. This prior distribution holds that changes in the endogenous variable modelled are so difficult to forecast that the coefficient on its lagged value is likely to be near unity, while all other coefficients are assumed to be near zero. This prior distribution is independent across coefficients, so that the MSE of the state vector is a diagonal matrix.

The matrix $P_{1|0}$ is given by:

$$P_{1|0} = \begin{pmatrix} g\hat{t}_1^2 & 0' \\ 0 & (B \otimes C) \end{pmatrix}, \tag{14}$$

where

$$B = \begin{pmatrix} g^2 & 0 & 0 & \cdots & 0 \\ 0 & g^2/2 & 0 & \cdots & 0 \\ 0 & 0 & g^2/3 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & g^2/p \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & w^2\hat{t}_1^2/\hat{t}_2^2 & 0 & \cdots & 0 \\ 0 & 0 & w^2\hat{t}_1^2/\hat{t}_3^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & w^2\hat{t}_1^2/\hat{t}_n^2 \end{pmatrix}$$

Also, Q the variance of u_t, is equal to

$$Q = p_2 P_{1|0}, \tag{15}$$

Doan et al. (1984) suggest the use of predefined values for the parameters in (14) and (15). The following assumptions are made:

$$w^2 = \frac{1}{74}; g = 360; p_1 = 0.999; p_2 = 10^{-7}.$$

Note that this assumes that the coefficient vector β converges only very slowly towards the mean. Finally, if g defines the analyst's confidence that the first-order autoregressive coefficient $f_{ii,1}^{(1)}$ relating y_{it} to $y_{i,t-1}$ attached to each series, $i=1,\ldots n$, is near unity, Doan et al. recommend $g^2 = 0.07$.

This general time-varying formulation relocates the estimation problem into one of forecasting in each period the optimal state vector based on information available up to the previous period. Under the normality and independence assumptions about the disturbances, the computation of the state vector is simply obtained by applying the Kalman filter (Harvey, 1989; Hamilton, 1994). This allows us to obtain filtered estimates of the VAR parameters and residual variance-covariance matrix, for each observation in the sample. Orthogonalised impulse responses are finally computed according to the standard Choleski decomposition, producing a set of different impulse responses (10) for each period of our sample.

Let us first examine the impulse responses for the UK. Figures 5 and 6 show the impulse responses[15] of the interest rate at the first, second, third, fifth and eighth-quarter horizon following unit shocks to the inflation rate and the output gap respectively over the sample period 1988-1999. Note that in Figure 5 PII*x* denotes the impulse response function for the interest rate (I), following an inflation shock (PI) after *x* periods. In Figure 6, YI*x* denotes the impulse response of the interest rate (I) following a shock to the output gap (Y) after *x* periods. Four points can be noted.

First, from Figure 5 we can clearly see that the interest rate response to the inflation rate is much greater than unity, at least for the first three quarters following an

[15] We do not report confidence bands, because for each period and each impulse response we would need to report a confidence interval, making the graph difficult to read. However, having checked the impulse responses and confidence intervals (constructed using bootstrap methods) for critical periods discussed in the text, when the policy regimes were subject to change, we can confirm that the impulse responses discussed are significantly different from zero at the 95% confidence. Detailed results are available on request from the authors.

inflation shock. This confirms that the some of the usual critique of VAR policy rules (Rudebusch, 1998, Rudebusch and Svensson, 1999) is directly attributable to the use of constant coefficient VARs.

The second point to note, from both Figures 5 and 6, is that the implicit policy rule seemed to change from the early 1990s. This confirms evidence from some earlier contributions (Muscatelli *et al.*, 1992, Nelson, 2000). As expected, there is a sharp shift after 1992. From Figure 5 it is apparent that the interest rate response to an inflation shock becomes more sustained, even after the 3rd quarter, suggesting a more concerted and persistent response of monetary policy to an inflation shock. Figure 6 shows that the response to the output gap has also changed: prior to 1992 there was a perverse response to the output gap initially, and a more consistent response over time is apparent in the 1990s.

Third, Bank of England independence does seem to have had some effect on policy responses, with a more decisive response of the interest rate to an inflation shock from late 1998. However, it is probably too early to say whether this represents a shift in the policy rule, or whether it was due to the need to tighten policy more dramatically following the 1997 election, following the failure of the Conservative administration to tighten policy sufficiently early in 1997.

Fourth, the evolution of the policy rule post-1992 explains the fact that the invariance tests pick up shifts in the output and inflation equations. It appears that the shift in policy stance has affected the transmission mechanism (the response of output and inflation to interest rates). This is shown in Figures 7 and 8[16]. The response of inflation to shocks in the output gap (YPI*x* in Figure 7) seems to have changed only little post-1992. Most of the evolution (essentially a more responsive inflationary response

[16] The response of inflation to an interest rate shock still shows a minor price puzzle (one which tends to be statistically insignificant), and is not shown here. It appears from the VAR impulse responses that the main channel of transmission from interest rates to inflation is through the output gap.

after 8 quarters to an output shock) seemed to take place during the ERM membership phase (1990-92). The UK result seems to contradict the Hutchinson-Walsh (1998) result for New Zealand, namely that the acquisition of credibility should increase the output-inflation trade-off[17]. Figure 8 shows that there has also been a marked decrease in the responsiveness of output to the interest rate in the UK (IY*x*) post-1992. This is a result which has implications for the degree of policy activism which is used. There are three interpretations for this result. The first is that in the early part of the 1988-98 period nominal interest rate shifts had a more dramatic impact on real interest rates as inflationary expectations adjusted downward. Once inflation expectations had adjusted downward in the mid-1990s, nominal interest rate shocks have had a smaller impact on real interest rates, and hence aggregate demand. The second possibility is that there is a non-linear relationship between interest rates and aggregate demand, so that the responsiveness of the output gap is different at different interest rate levels. The third interpretation is that the UK economy has become less sensitive to interest rate increases in the late 1990s as the problems with mortgage indebtedness of the early 1990s were gradually reduced.

Turning next to the US, we see from Figures 9 and 10 that, despite the absence of formal institutional change, the Fed's reaction to inflation and output gap shocks in the 1980s and 1990s has evolved markedly. However this change has not been monotonic, as was the case in the UK. From Figure 9, we see that the interest rate response to inflation (pii*x*) increased sharply between 1991-94, but has fallen back sharply since. This fits with general commentaries on Fed policy, which suggest that in 1994-95 the Fed re-considered its policy stance in the light of evidence on productivity growth. Again, in contrast to the points made in Rudebusch (1998) and Rudebusch and Svensson (1999), the impulse responses show a greater than unit response of the Federal

[17] See also Lucas (1973) on the slope of the short-run aggregate supply relationship.

Funds rate to a unit shock in the inflation rate, so that real interest rates are raised by the Fed following a shock. The pattern of impulse responses is similar to that for the UK in terms of timing (the peak is reached 2 quarters after the shock), although the responses are larger. From Figure 10, we see that, in contrast, the response of the Fed to the output gap (yi$x$) has tended to decrease from 1992 onwards, especially over the medium horizon (x=3,5)[18]. The final observations of our sample are of particular interest because, using the BEA potential output data, the output gap shows a major surge in 1999-2000. This is because the implicit assumption is that the rate of growth in potential output has not increased: i.e. there has been no 'productivity miracle'. The interpretation which emerges from the VAR is then that monetary policy has become less countercyclical in the late 1990s. Using our alternative series for the output gap, based on a Kalman Filter procedure (see Figure 1) would yield slightly different results for the late 1990s, as this interprets the recent increase in output growth as partly due to an increase in potential output.

Have recent changes in policy responses affected the nature of the transmission mechanism in the US? Figures 11 and 12 show the response of output and inflation to an interest rate shock and an output gap shock respectively[19]. Figure 11 shows that, unlike the UK, there is very little evidence of the US experiencing a shift in the response of output to interest rate shocks, despite the presence of apparent policy shifts. This is consistent with the evidence from the application of invariance tests to our estimated aggregate demand equation (see Table 3). The absence of expectations effects in the output equation might be attributable to the absence of formal institutional change in the US, which means that the transmission mechanism has remained more stable. Figure 12

[18] This is partly in line with the findings of Favero and Rovelli (1999), although their single-equation policy reaction function estimates seem to suggest that the output gap does not matter to the Fed throughout their sample period (1983-98).

shows again that, in contrast to the UK, the short-run output-inflation correlation has shifted. An output gap shock results less readily in an inflationary surge in the 1990s. This is in line with the results of Hutchison and Walsh (1998) for New Zealand, but whether this captures an expectations effect or the impact of more rapid productivity remains an open issue. This might also explain why the Fed's interest rate response to the output gap has declined over the same period. Although the impulse responses in Figure 12 seem to change, it is hard to ascribe this to an expectations effect, as the shifts do not match variations in the implicit VAR interest rate policy response to inflation.

4. Conclusions

In this paper we have examined both how interest rate policy responses to output and inflation shocks in the UK and US have evolved during the last two decades, and how the transmission mechanism has been affected by apparent shifts in the policy rules followed by the Fed and the UK monetary authorities. Tests for invariance/superexogeneity seem to show some evidence in support of the Lucas Critique, confirming the importance of forward-looking expectations in aggregate demand and supply. The evidence relates both to full sample estimates, which cover known policy shifts (the switch to a much tighter inflation control from the 1980s), and post-1984 evidence when formal institutional change has been limited to the UK. In the UK, there is evidence that the Lucas (1976) critique has some weight in both the output and inflation equations, and this might be linked to the presence of formal institutional change (the adoption of inflation targeting and the granting of instrument-independence to the Bank of England). However, even for the US we find evidence of expectations effects in the post-1984 sample estimates.

[19] Once again the impulse responses of inflation to the interest rate show a slight price puzzle, which disappears after 3 quarters, and eventually a negative response of inflation to an interest rate increase. But there is little evidence of a change in the transmission lag or size of interest rate shocks to inflation.

Our Bayesian VAR analysis shows a much richer picture. It confirms the finding that interest rate policy rules have shifted both in the US and the UK in the 1990s. In the UK this is linked to institutional change, particularly to the adoption of inflation targets, but also, in a minor way to the granting of independence to the Central Bank in 1997. In the US the changes in policy stance have been less systematic, with a more activist policy pursued until the mid-1990s, and a less responsive interest rate policy since 1995. This is probably attributable to the Fed's changing attitude to the 'productivity miracle' in the US. The shift in policy rule seems to coincide, at least in the UK, to shifts in the transmission mechanism, lending some weight to the notion that the Lucas critique may need to be taken into account in setting interest rates, if policymakers significantly change their policy stance. Although independent central banks may be more immune to problems of credibility, it would appear that forward-looking expectations are still an important dimension of monetary policy design[20]. This has important implications for the vast theoretical literature which has emerged on the optimal design of feedback rules for monetary policy (see Haldane and Batini, 1998, Rudebusch and Svensson, 1999, Rudebusch, 1999, Onatski and Stock 2000). This literature far too often ignores forward-looking expectations in the specification of the underlying structural model.

[20] It is perhaps also worthwhile reminding ourselves that time-inconsistency not only arises in static models where there is a conflict between the objectives of policy-makers and the monetary authorities. It is also present in models where the authorities target the natural rate of unemployment (so no Barro-Gordon type time-inconsistency problem exists), but forward-looking variables enter the structural model.

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Table 1: Backward-looking Conditional Models of Output and Inflation

USA – Conditional models for inflation and output

Sample: 1958 (4) to 2000 (1)

$$y_t^{bea} = \underset{(2.44)}{0.031} + \underset{(15.87)}{1.187} y_{t-1}^{bea} - \underset{(-3.81)}{0.279} y_{t-2}^{bea} - \underset{(-2.68)}{0.010} \overline{r}_{t-1}$$

$R^2 = 0.90 \quad s = 0.0997$

$$y_t^{kal} = \underset{(1.467)}{0.021} + \underset{(6.81)}{0.512} y_{t-1}^{kal} - \underset{(-3.88)}{0.290} y_{t-2}^{kal} - \underset{(-1.56)}{0.007} \overline{r}_{t-1}$$

$R^2 = 0.59 \quad s = 0.1187$

$$p_t = \underset{(0.82)}{0.119} + \underset{(7.98)}{0.619} p_{t-1} + \underset{(0.266)}{0.024} p_{t-2} + \underset{(2.09)}{0.188} p_{t-3} + \underset{(1.706)}{0.132} p_{t-4} + \underset{(4.46)}{1.20} y_{t-1}^{bea}$$

$R^2 = 0.85 \quad s = 0.972$

$$p_t = \underset{(0.74)}{0.117} + \underset{(9.32)}{0.711} p_{t-1} + \underset{(0.094)}{0.027} p_{t-2} + \underset{(2.91)}{0.228} p_{t-3} + \underset{(2.24)}{1.06} y_{t-1}^{kal}$$

$R^2 = 0.84 \quad s = 1.012$

UK – Conditional models for inflation and output

Sample: 1977 (1) to 1994 (4)

$$y_t^{oecd} = \underset{(-)}{0.001} + \underset{(24.95)}{1.711} y_{t-1}^{oecd} - \underset{(-11.713)}{0.766} y_{t-2}^{oecd} + \underset{(1.74)}{0.023} \overline{r}_{t-2} - \underset{(-1.90)}{0.893} q_{t-4}$$

$R^2 = 0.98 \quad s = 0.325$

$$y_t^{kal} = \underset{(0.246)}{0.038} + \underset{(4.47)}{0.454} y_{t-1}^{kal} + \underset{(4.55)}{0.497} y_{t-2}^{kal} + \underset{(1.88)}{0.199} y_{t-3}^{kal} - \underset{(-2.91)}{0.278} y_{t-4}^{kal} + \underset{(2.33)}{0.077} \overline{r}_{t-2} - \underset{(-1.96)}{2.195} q_{t-4}$$

$R^2 = 0.88 \quad s = 0.881$

$$p_t = \underset{(1.60)}{0.255} + \underset{(11.28)}{1.081} p_{t-1} - \underset{(-1.27)}{0.165} p_{t-2} - \underset{(-1.65)}{0.132} p_{t-3} + \underset{(3.14)}{0.513} y_{t-2}^{oecd} - \underset{(-2.35)}{0.381} y_{t-3}^{oecd}$$
$$+ \underset{(4.40)}{0.091} pim_{t-1} - \underset{(-3.30)}{0.146} ulc_{t-1} + \underset{(5.56)}{0.255} ulc_{t-2}$$

$R^2 = 0.97 \quad s = 0.777$

Table 2: Marginal Models of the Interest Rate, Output and Inflation

US – Marginal Models

Sample: 1958 (4) to 2000 (1), unless otherwise stated.

Simple Autoregressive Model for Interest Rate:

$$r_t = 0.130 \ + 1.478 r_{t-1} - 0.532 r_{t-2} + 0.383 D79/82$$
$$(2.73) \quad (22.12) \quad (-8.19) \quad (2.944)$$

$R^2 = 0.97 \quad s = 0.392$

Forward-Looking Interest-Rate Reaction Function (1958:4-1999:1):

$$i_t = 0.091 + 1.062 i_{t-1} - 0.150 i_{t-2} + 0.124 p^{e}_{t+4} + 1.21 y^{kal}_t$$
$$(0.49) \quad (13.75) \quad (-1.98) \quad (3.493) \quad (2.66)$$

$R^2 = 0.92 \quad s = 0.949$

Forward-Looking Interest-Rate Reaction Function (1983:1-1999:1):

$$i_t = 0.062 + 1.264 i_{t-1} - 0.390 i_{t-2} + 0.910 p^{e}_{t+4} + 0.277 y^{kal}_t$$
$$(0.33) \quad (12.19) \quad (-3.99) \quad (2.317) \quad (3.45)$$

$R^2 = 0.96 \quad s = 0.443$

Simple autoregressive model for output gap:

$$y^{bea}_t = 0.003 + 1.195 y^{bea}_{t-1} - 0.072 y^{bea}_{t-2} - 0.224 y^{bea}_{t-3}$$
$$(0.40) \quad (10.09) \quad (-0.38) \quad (-2.01)$$

$R^2 = 0.94 \quad s = 0.055$

Note: D79/82 is dummy variable to capture the impact on Federal Funds rates of the Fed's change in operating procedures between 1979 and 1982.

Table 2 (Contd.)
UK – Marginal Models

Sample: 1977 (1) to 1999 (4), unless otherwise stated.

Simple Autoregressive Models for Real Interest Rate:

$$r_t = 0.420 + 1.160 r_{t-1} - 0.234 r_{t-2} - 0.178 r_{t-3} + 0.167 r_{t-4}$$

(2.65) (11.24) (–1.57) (–1.22) (1.92)

$R^2 = 0.91 \quad s = 0.893$

1983(1)-1999(4):

$$r_t = 0.51 + 1.27 r_{t-1} - 0.36 r_{t-2} - 0.52 D9293$$

(2.24) (11.26) (–3.17) (–2.43)

$R^2 = 0.90 \quad s = 0.45$

Forward-Looking Interest-Rate Reaction Function (1977:1-1999:1):

$$i_t = 0.773 + 0.825 i_{t-1} + 0.163 p^e_{t+4} + 0.148 y^{kal}_t$$

(1.91) (17.45) (3.83) (3.03)

$R^2 = 0.88 \quad s = 1.12$

Forward-Looking Interest-Rate Reaction Function (1983:1-1999:1):

$$i_t = 0.99 + 0.629 i_{t-1} + 0.57 p^e_{t+1} + 0.220 y^{kal}_t$$

(2.53) (7.61) (3.9) (3.99)

$R^2 = 0.91 \quad s = 0.891$

Simple autoregressive model for Output Gap:

$$y^{bea}_t = -0.011 + 1.80 y^{oecd}_{t-1} - 1.034 y^{bea}_{t-2} + 0.389 y^{bea}_{t-3} - 0.2 y^{oecd}_{t-4}$$

(–0.034) (17.15) (–4.79) (–1.80) (–1.90)

$R^2 = 0.98 \quad s = 0.326$

Note: D9293 is a dummy variable to capture the impact on UK interest rates of the exit from the ERM, which allowed interest rates to fall rapidly between 1992(4) and 1993(4).

Table 3: Invariance / Superexogeneity Tests

Output Equation

Variable	US: 1959:4-1999:4	US: 1984:1-1999:4	UK: 1978:1-1999:	UK: 1984:1-1999:4
$\hat{h}_r$	**1.87***	**0.56**	**1.50**	**2.60****
$\hat{x}_r^2$	**-1.62***	**0.11**	**0.07**	**-0.58**
$\hat{s}_{rr}^2$	**0.57**	**-1.12**	**1.71***	**-0.75**
$(\hat{h}_r \hat{s}_{rr}^2)$	**-0.84**	**-0.14**	**-1.37**	**-2.40****
$(\hat{x}_r^2 \hat{s}_{rr}^2)$	**-0.09**	**-0.83**	**-1.24**	**0.27**

Inflation Equation

Variable	US: 1959:4-1999:4	US: 1984:1-1999:4	UK: 1978:1-1999:4	UK: 1984:1-1999:4
$\hat{h}_r$	**-5.18****	**-3.19****	**2.20****	**-3.24****
$\hat{x}_r^2$	**-0.38**	**-0.70**	**2.27****	**0.37**
$\hat{s}_{rr}^2$	**-1.54**	**-0.24**	**-0.69**	**0.49**
$(\hat{h}_r \hat{s}_{rr}^2)$	**-0.03**	**0.16**	**-0.83**	**-0.66**
$(\hat{x}_r^2 \hat{s}_{rr}^2)$	**-1.79***	**-0.32**	**1.86***	**0.68**
$\hat{h}_y$	**-1.54**	**1.10**	**0.34**	**0.7**
$\hat{x}_y^2$	**-0.46**	**-0.84**	**0.89**	**-0.63**
$\hat{s}_{yy}^2$	**-0.55**	**0.48**	**-0.96**	**0.29**
$(\hat{h}_y \hat{s}_{yy}^2)$	**2.02****	**-1.15**	**-0.19**	**-0.06**
$(\hat{x}_y^2 \hat{s}_{yy}^2)$	**0.07**	**0.53**	**0.36**	**2.01****

Note: Numbers reported in table are t-values for addition of variable to conditional models for output and inflation. The notation for the variables follows the description of the tests in Appendix A. (*) Indicates a test statistic significant at the 10% level, ** indicates a test statistic significant at the 5% level.

Appendix A: Invariance / Superexogeneity Tests

The following tests are developed fully in Engle and Hendry (1993). This appendix briefly summarises the approach taking to the statistical tests of the Lucas critique in Section 2.

Consider the following joint distribution of the variables x_t and y_t, which are assumed to be conditional normal, with conditional means:

$$E(x_t \mid \Omega_t) = m_t^x \quad \text{(A.1)}$$
$$E(y_t \mid \Omega_t) = m_t^y$$

and covariance matrix:

$$\Sigma_t = \begin{bmatrix} s^{yy} & s^{yx} \\ s^{xy} & s^{xx} \end{bmatrix} \quad \text{(A.2)}$$

where the means and covariances need not be constant, but may depend on the information set, Ω. This information set contains past values of the two variables and also current and past values of other valid conditioning variables, z_t.

Consider a model which relates the two conditional means of x_t and y_t:

$$m_t^y = bm_t^x + z_t'g \quad \text{(A.3)}$$

Given the expectation of y_t conditional on x_t, and its conditional variance, we can write the following conditional model for y_t:

$$y_t = b\,x_t + z_t'g + (d_t - b)(x_t - m_t^x) + w_t \quad \text{(A.4)}$$

where $d_t = (s_t^{yx} / s_t^{xx}), \;\; w_t = s_t^{yy} - ((s_t^{yx})^2 / s_t^{xx})$

The efficient estimation of (A.4) requires: (i) the weak exogeneity of x_t, which is satisfied if $d_t = b$; (ii) constancy of the regression coefficients, which requires the constancy of d_t; (iii) invariance, which requires the parameter β to be invariant to changes in the process generating x_t. Weak exogeneity and invariance implies that the variable x_t is superexogenous for the parameters of interest in our model.

To test for superexogeneity, Engle and Hendry examine the impact of changes in the moments of x_t on β. Allowing β to be a function of the moments of x_t, and using a linear expansion for β(.) yields:

$$b = b_0 + b_1 m_t^x + b_2 s_t^{xx} + b_3 (s_t^{xx} / m_t^x) \quad \text{(A.5)}$$

(where higher-order terms of the expansion can be considered). The test regression then becomes:

$$y_t = b_0 x_t + z_t' g + (d_t - b_0)(x_t - m_t^x) + b_1 (m_t^x)^2 + b_2 m_t^x s_t^{xx} + b_3 s_t^{xx} + e_t \qquad \text{(A.6)}$$

To implement this test regression, one has to fit marginal models for the variable(s) x_t, and use the residuals for this model to obtain a measure of $(x_t - m_t^x)$, the fitted values to obtain a measure of m_t^x, and one can model the variance s_t^{xx} through, say, an ARCH model fitted to the residuals of the marginal model for x_t.

Data Appendix

Data were obtained from IMF International Financial Statistics, OECD Main Economic Indicators, the UK Office of National Statistics and the US Bureau of Economic Analysis. Where available, seasonally adjusted series were employed. Below we illustrate data sources and definitions.

Time Series	United Kingdom	United States
Price Index	CPIX up to 1987Q1 (OECD), RPIX thereafter (ONS)	GDP chain-weighted price index (BEA)
Output	GDP	GDP
Output Gaps	Interpolated OECD estimates	Bureau of Economic Analysis estimates
Short-Term Interest Rate	London Clearing Banks Overnight Rate (IFS)	Federal Funds Rate (IFS)
Real Exchange Rate	Real effective exchange rate, Index, 1995=100 (OECD)	
Unit Labour Cost	Unit Labour Cost, in Manufacturing Industries, Sterling Pounds, Index, 1995=100 (OECD)	
Import Prices	Import price index, 1995=100 (IFS)	

Symbols Adopted	Variable Definition
Y^{bea}	Output gap, BEA series. Gap between actual and potential, as a percentage of potential output
Y^{kal}	Output gap, Kalman-filtered series. Gap between actual and potential, as a percentage of potential output
Y^{oecd}	Output gap, OECD series. Gap between actual and potential, as a percentage of potential output
p	Quarterly inflation in percentage points at an annual rate
i	Short-term interest rate, in percentage points at an annual rate
$\overline{r}$	Real Interest rate, difference between four quarter average i and p
q	Real effective exchange rate, in logs
pim	First difference in the (log of) Import Price Index
ulc	First difference in the (log of) Unit Labour Cost Index