Optimal Monetary Policy and Asset Price Misalignments*

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Abstract

This paper analyses the relationship between monetary policy and asset prices in the context of optimal policy rules. The transmission mechanism is represented by a linearized rational expectations model augmented for the effect of asset prices on aggregate demand. Stabilization objectives are represented by a discounted quadratic loss function penalizing inflation and output gap volatility. Asset prices are allowed to deviate from their intrinsic value due to momentum trading. We find that in the presence of wealth effects and inefficient markets, asset price misalignments from their fundamentals should be included in the optimal interest rate reaction function.

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JEL Classification: E52; E60; G1

1 Introduction

Over the last twenty years, significant changes have occurred in the institutional and macroeconomic framework that central banks operate. In particular, there has been a widespread move towards financial liberalization, both within and across national borders, especially after the 1980s, while inflation rates have become lower and less variable. The disinflation process of the 1990s has been a global phenomenon since it is observed both in countries where formal inflation targets are in use, and in non-targeting countries[1]. The decline in inflation has gone hand in hand with a similar decline in interest rates. In many countries, both short term and long term interest rates are close to, or even bellow, post-war lows. As Bean (2003) argues, price stability has not been achieved at the expense of the real economy, as unemployment has been decreasing in a number of countries, while growth has also been relatively stable. Despite the good macroeconomic record of the past decade, there has been a growing concern among academics and policymakers that the achievement of price stability may be associated with an increased risk of financial instability.

Some commentators claim that the lower cost of capital along with exuberant growth projections have boosted the late 1990s stock market bubble. For instance, Borio and Lowe (2002) argue that booms and busts in asset prices should be considered as part of a broader

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[1] See e.g. Johnson (2002) for international evidence.

set of symptoms that typically also include a build-up of debt and high rate of capital accumulation. Rising asset prices and debt accumulation lead to stretched household and corporate balance sheets, vulnerable to sharp corrections of the type recently witnessed in global equity markets. In a series of articles, Goodhart and Hofmann (2000, 2003) establish empirically the link between output growth, credit aggregates, and asset price movements in a number of major economies. Kiyotaki and Moore (1997) develop a theoretical model that exhibits a crucial interaction between collateral values, asset prices, credit and economic activity. During the period of boom, balance sheets may look healthy as the increase in asset prices, and consequently the value of the collateral, offsets the build-up of debt. However, when optimism about further increases in asset values turns to pessimism, leading to a decrease in the net worth of households and firms, then financial distress may be the result of financial imbalances unwinding. It has been argued that the widespread financial deregulation of asset markets may have contributed to an increase in the frequency of such boom-bust episodes (IMF, 2003).

An important issue related to the above concerns is the establishment of the appropriate monetary policy response to asset price movements. Should the central bank care about the financial instability associated with large asset price fluctuations? Nowadays, everyone recognizes price level stability as the primary objective of monetary policy. Indeed, as Issing (2003) emphasizes, price stability and financial stability tend to mutually reinforce each other in the long run. However, as the examples of the US in the 1920s and 1990s and Japan in the late 1980s demonstrate, financial imbalances may build up even in an environment of stable prices (Borio and Lowe, 2002). Exponents of the 'new environment' hypothesis argue that low and stable rates of inflation may even foster asset price bubbles, due e.g. to excessively optimistic expectations about future economic prospects. Thus, price stability is not a sufficient condition for financial stability. Among the exponents of the new environment hypothesis, Crocket (2003) claims that: "[...] if the monetary policy reaction function does not incorporate financial imbalances, the monetary anchor may fail to deliver financial stability". The current consensus however, stresses that monetary policy should be directed exclusively at achieving price stability, and its role in promoting financial stability should be restricted to restricted to minimising the negative effects from bubbles bursting and financial imbalances unwinding[2].

A number of studies tried to provide an answer to the question of whether monetary policy should respond to asset prices, by simulating macroeconomic models where aggregate demand is affected by consumption wealth effects and/or investment balance sheet effects. The simulation evidence of Bernanke and Gertler (1999, 2001) and Gilchrist and Leahy (2002) opts for a reactive monetary policy response since they show that a central bank dedicated to price stability should pay no attention to asset prices per se, except insofar as they signal changes to expected inflation. On the other hand, Cecchetti, Genberg, Lipsky and Wadhwani (2000), and Kontonikas and Ioannidis (2005) find that, in line with the new environment proactive view, overall macroeconomic volatility can be reduced with a (mild) reaction of

[2]For instance, Alan Greenspan (2002) argues that: "The notion that a well-timed incremental tightening could have been calibrated to prevent the late 1990s bubble is almost surely an illusion. Instead, we...need to focus on policies to mitigate the fallout when it occurs and, hopefully, ease the transition to the next expansion."

interest rates to asset price misalignments from fundamentals[3]. Also, recent econometric evidence by Kontonikas and Montagnoli (2004) for the UK, and Chadha, Sarno and Valente (2003) for UK, US and Japan, suggests that monetary policymakers may use asset prices not only as part of their information set for setting interest rates, but also as elements in their reaction function. Chadha *et al.* (2003) use various measures of asset market disequilibria, such as the dividend-price ratio and the log difference in stock prices and exchange rates in their extended monetary policy rule, while Kontonikas and Montagnoli employ the log difference in stock prices and house prices.

All the aforementioned papers use the assumption that monetary policy is characterised by an augmented Taylor rule, where the nominal interest rate responds positively to inflation, demand pressures, and asset prices. Following the seminal work by Taylor (1993), feedback rules conditioning the interest rate instrument on current or expected inflation and the output gap have been extensively analysed in the theoretical and empirical literature. Svensson (1997), and Clark, Goodhart and Huang (1999) among others, show that such a feedback rule is optimal in that it derives from the first order condition for the optimisation of the central bank's objectives[4]. In this paper, we try to shed some more light in the relationship between monetary policy and asset prices in the context of optimal policy rules. In essence, we will examine whether there is any underlying theoretical motivation for the increasingly frequent use of an augmented (for asset prices) Taylor rule. To do so, we start from a backward-looking structural macroeconomic model where asset prices affect future inflation indirectly, through wealth effects on aggregate demand. In our model, market inefficiency implies that asset prices may deviate from their fundamental value due to 'momentum' effects from past asset price changes. In the context of our model if there are wealth effects in aggregate demand, monetary policy already takes into account asset prices indirectly (and with lag) by responding to output movements. The question that then arises is whether an extra direct reaction to deviations from fundamentals can be derived in an optimal setting.

Our results suggest that monetary policy should respond to asset price misalignments from their fundamental value, with the aggressiveness of the response being a positive function of the impact of asset prices on aggregate demand. We show that the optimal response to asset price disequilibria depends on the role of asset prices in the monetary policy transmission mechanism and the source of the asset price movements. If asset price increases can be justified on the basis of improvements in fundamentals *only*, then monetary policy will accomodate the boom by not responding directly to asset prices. If however, asset price movements cannot be explained by fundamentals only, then optimal monetary policy will systematically respond to the non-fundamental component of asset prices. This result has

[3]The aforementioned studies employ a new keynesian framework with sticky prices in order to examine the potential stabilisation benefits from monetary policy actions. This work has, at time, received some critisism; for instance as Kiley (2003, p. 1114) points out "optimal policies in such work are likely to have trival benefits". Since economic fluctuations are mainly driven by exogenous shocks in productivity, they "are primarily an efficient response to shifts in the economic environment, and hence optimal stabilisation policy only improves welfare slightly" (p.1089).

[4]One should keep in mind though, that simple instrument rules like the Taylor rule and its variants may not correspond to fully optimal policy in the context of a particular economic model (see e.g. Woodford, 2001). Also, as Svensson, (2003) argues, no central bank has so far made a commitment to a simple instrument rule like the Taylor rule or variants thereof. In addition, neither has any central bank announced a particular instrument rule as a guideline.

important implications for the conduct of monetary policy and contributes crucially to the existing literature, as previous work on deriving optimal interest rate rules considering asset prices, either fails to find a role for asset prices (Bean, 2003), or obtains complex, non linear rules (Bordo and Jeanne, 2002), or doesn't explicitly model the non-fundamental component of asset prices (Smets, 1997).

The remainder of the paper is structured as follows. Section 2 describes the theoretical model. In Section 3 we derive the optimal interest rate rule based upon dynamic optimization of the central bank's objectives. Section 4 analyzes the results and Section 5 concludes.

2 The model

We use a structural backward-looking model of a closed economy that allows for the effect of asset prices on aggregate demand. The model augments the standard Ball (1999) and Svensson (1997) specification by taking into account asset prices, which themselves are assumed to stochastically evolve influenced by both fundamentals and momentum trading. The model is given by the following equations:

$$\pi_{t+1} = \pi_t + a y_t + \epsilon_{t+1} \tag{1}$$

$$y_{t+1} = \beta_1 y_t - \beta_2 \left(i_t - E_t\left[\pi_{t+1}\right]\right) + \beta_3 q_t + \eta_{t+1} \tag{2}$$

$$q_t = q_t^* + q_t^{NF} = q_t^* + b\Delta q_{t-1} \tag{3}$$

$$q_t^* = -\delta_1 \left(i_t - E_t\left[\pi_{t+1}\right]\right) + \delta_2 E\left[y_{t+1}\right] + u_t \tag{4}$$

where y_t is the deviation of (log) output from its steady-state level (output gap), π_t is the inflation rate (strictly, the deviation from target), p_t is (log) price level, it is the monetary policy instrument (one-period nominal interest rate), q_t denotes (log) real asset prices and q_t^* the fundamentals. Different interpretations of q_t are possible (e.g. house prices, stock prices or the value of a portfolio containing both housing and equity investment), in what follows though we mainly treat it is an equity index. ϵ_{t+1}, η_{t+1}, u_t represent exogenous random shocks to aggregate demand, inflation, and asset price fundamentals. For simplicity, we assume that they are mutually uncorrelated *i.i.d.* processes with zero means and constant variances. The structural parameters can be interpreted as partial elasticities with the following properties: $0 < \beta_1 < 1$; a, β_2, δ_1, $\delta_2 > 0$; $\beta_3 \geqslant 0$, $0 \leqslant b < 1$.

Eq.(1) is a standard accelerationist (or backward-looking NAIRU type) Phillips Curve where the change in inflation is a positive function of the lagged output gap and the inflation shock. Such a specification has also been adopted by Ball (1999), Svensson (1997) and Rudesbusch and Svensson (1999)[5]. The presence of inflation inertia in the inflation equation implies that disinflations will be costly in terms of output losses, thus there is a short-run

[5]Rudebusch (2002) considers the hybrid Phillips curve: $\pi_{t+1} = \mu_\pi \pi_t + (1-\mu_\pi)E_{t+1}[\pi_{t+2}] + a y_{t+1} + \epsilon_{t+1}$, and points out that the accelerationist Phillips curve ($\mu_\pi \approx 1$) can be derived from well-known models of price-setting behavior (see e.g. Roberts, 1995).

trade-off between inflation and the output. However, since lagged inflation enters Eq. (1) with unity coefficient, the model implies a vertical long-run Phillips curve. This process is also consistent with the empirical finding that inflation in the major industrialised countries is so highly persistent that it may indeed contain a unit root as some studies have shown (see e.g. Evans, 1991; Grier and Perry, 1998). Eq. (1) posits no role for expected future inflation in the inflation adjustment equation. The parameter a is a positive constant which measures the sensitivity of inflation to excess demand[6].

The demand side, as given by Eq. (2), is consistent with the specification employed by Walsh (1998), Ball (1999), and Svensson (1997) with one important difference: aggregate demand depends positively on the past level of asset prices via consumption wealth effects and investment balance sheet effects[7]. For example, a persistent increase in the level of asset prices decreases the perceived level of households' financial distress causing a boost in consumption spending. The balance sheet channel implies a positive relationship between the firms' ability to borrow and their net worth which in turn depends on asset valuations. There is a vast amount of empirical evidence indicating that stock and house price movements are strongly correlated with aggregate demand in most major economies[8]. Parameter β_3 in the aggregate demand equation is of crucial interest since it indicates the magnitude of the asset price movements' effects on output. If there are no wealth effects/balance sheet effects then $\beta_3 = 0$ and Eq. (2) resembles a traditional dynamic IS curve. In our model, the central bank takes into account the effect of asset prices on aggregate demand; that is, it is fully aware of the effect of q_t on y_{t+1} and its magnitude. In other words, we assume that symmetric information exists between financial market participants and the central bank. It should be pointed out that by conditioning y_{t+1} on q_t, we allow the output gap to be affected by both the fundamental (q_t^*) and the non-fundamental (q_t^{NF}) component of the asset price. This is in line with Filardo (2004) and in contrast to Smets (1997), Gruen, Plumb and Stone (2003) who condition the output gap only upon the fundamental, non-fundamental component, respectively. It may be the case that permanent income consumers would consume from q_t^* only[9], however the recent experience of late 1990s indicates that sustained increases/decreases in asset prices, originating from bubbles and/or changes in the fundamentals, do affect consumption and investment.

Eqs. (3) and (4), represent the dynamic evolution of asset prices, q_t, and their underlying fundamentals, q_t^*, respectively. As in Kontonikas and Ioannidis (2005), in an effort to depict actual financial market behavior we assume a partial adjustment mechanism that allows observed asset prices not to always being equal to their fundamental value. Contrary to

[6] As Clark, Goodhart, and Huang (1999) point out, there are good reasons to believe that a is not constant. However, the assumption of linearity in the Phillips curve helps to obtain a closed-form solution for the optimal feedback rule.

[7] Despite that in principle the quantity of wealth (wealth effects) and/or the value of capital stock (balance-sheet effects), rather than the level of asset prices, should enter the demand equation, it is standard in the theoretical (see e.g. Smets, 1997) and empirical (see e.g. Goodhart and Hoffman, 2000) literature to augment the IS using an asset price related variable. This is based on the assumption that wealth and the value of capital stock depend positively on the asset price. We thank a referee for raising this point.

[8] See among others, Kontonikas and Montagnoli (2004) for relevant empirical evidence considering the UK economy, and Goodhart and Hofmann (2000) for international evidence. A recent study by the IMF (2003) points out that equity price reducions are associated with heavy GDP losses.

[9] We thank a referee for pointing out this issue.

this, the Efficient Markets Hypothesis (EMH) postulates that all information required to determine the intrinsic asset value will, by actions of rational profit-maximizing agents, be reflected in the actual market price; hence $b = 0$ and $q_t = q_t^*$. In the context of the EMH, the asset price changes if and only if the market receives new information about the asset's underlying economic fundamentals, and the actions of speculators are stabilising, in that they drive the actual asset price towards its fundamental value rather than away from it (e.g. by buying underpriced assets and selling overpriced ones).

However, the central tenets of the EMH, that future prices are not affected by past asset price movements and that speculation can only have a stabilizing effect have never been quite accepted by market participants. As Kortian (1995) points out, several aspects of modern asset markets trading, support the existence of 'momentum' effects from past changes in asset prices. For example, the widespread use of technical analysis, that tries to predict future prices on the basis of past asset price movements. Also, stop-loss orders (selling orders which are activated once the asset price has fallen by a particular pre-determined amount) are frequently employed, and dynamic hedging strategies, such as portfolio insurance, imply that investors buy in a rising market and sell into a falling one. All the aformentioned investment strategies, are based upon past asset price movements and agree with the view that investors from time-to-time act in a destabilizing manner. Economic history also provides many examples of destabilizing investment behavior with significant implications for asset prices and aggregate economic activity beginning as early as the seventeenth century[10].

Taking these arguments into account, Eq. (3) decomposes actual asset prices into a fundamental component, q_t^*, and a non-fundamental component, $q_t^{NF} = b\Delta q_{t-1}$. The non-fundamental component indicates that, if asset prices have increased in the past ($\Delta q_{t-1} > 0$) there is a positive 'momentum' effect on their current level ($b > 0$). In essence, investors bid up the demand for asset holdings in expectation that past capital gains will persist in the future. The higher the value of b, the stronger the effect from past asset price changes and therefore q_t can diverge significantly from its fundamental value, q_t^*, albeit not permanently. But once asset prices revert, at an unknown future date, the downward effect on aggregate demand could be large. Eq. (3) is essentially a backward-looking version of the Frenkel and Mussa (1985) asset price model[11]. Stability of the asset price path requires that the parameter b satisfies: $0 \leqslant b < 1$.

Obviously, our preferred asset price specification depicts only one of the potential types of departures from the EMH, that is, momentum effects. An alternative model is the rational stochastic bubble (Blanchard, 1979; Blanchard and Watson, 1982). In the context of it, bubbles have the property of growing over a certain interval before suddenly collapsing. In the literature on asset prices and monetary policy, explicit bubble specifications have been employed by Filardo (2004) among others. Our model differs from the aformentioned studies, in the sense that we do not regard the divergence of q_t from q_t^* as an explicit bubble since we do not assign any probabilistic structure to its evolution.

Eq. (4) describes the fundamental component of the asset price in line with the standard dividend asset pricing model. Hence, a positive effect from expected future dividends

[10] See Garber (2000) for a discussion on the tulip mania in the early seventeenth century as well as other famous bubbles.

[11] Frenkel and Mussa (1985) argue that a wide range of structural models for exchange rate determination can be subsumed under the reduced form asset price expression: $q_t = q_t^* + bE_t[\Delta q_{t+1}]$.

(assumed to depend on expected output) and a negative effect from real interest rates is postulated. This specification is in line with the majority of empirical studies examining the effect of macroeconomic variables on the stock market[12]. Finally, we allow for uncertainty in the fundamentals' process by including the random disturbance term, u_t.

3 Optimal interest rate rule

The structure of the model implies that at time t, the central bank chooses the nominal interest rate, i_t, which affects concurrent real asset prices, next period's output gap, and two-periods' ahead inflation while contemporaneous inflation and output gap are predetermined by previous decisions and current exogenous shocks. Solving the model, the original system of Eqs. (1) – (4) can be re-written compactly as[13]:

$$y_{t+1} = \varphi_t + \upsilon_{t+1} \tag{5}$$

$$\pi_{t+1} = k_t + \epsilon_{t+1} \tag{6}$$

where $\varphi_t \equiv \lambda_1 y_t - \lambda_2(i_t - \pi_t) + \lambda_3 b \Delta q_{t-1}$ is the control variable of the central bank, since π_t, y_t are predetermined when i_t is chosen, and $k_t \equiv \pi_t + a y_t$ is the state variable at time t.

We assume that the central bank's intertemporal quadratic loss function, L, penalizes both inflation and output gap volatility:

$$L = \frac{1}{2} E_t \sum_{i=1}^{\infty} \tau^i \left[\pi_{t+i}^2 + \lambda y_{t+i}^2\right] \tag{7}$$

where $\lambda \geqslant 0$ is the relative weight attached by the central bank on output stabilisation. τ is the discount factor, $0 < \tau < 1$. In the absence of discounting, the postulated loss function is a weighted average of the conditional volatility of inflation and output. It is evident from Eqs. (5), (6) that at time t, when the interest rate (and consequently φ_t) is chosen the only state variable is k_t. Therefore, the value function is defined in terms of k_t only, $V(k_t)$. Applying Bellman's dynamic programming principle, and substituting for the two constraints, Eqs. (5) and (6), in the value function, we obtain:

$$V(k_t) = \min_{\varphi_t} E_t \left\{ \frac{1}{2} \left[(k_t + \epsilon_{t+1})^2 + \lambda (\varphi_t + \upsilon_{t+1})^2 \right] + \tau V(k_{t+1}) \right\} \tag{8}$$

The first order condition and the envelope theorem allow us to derive the optimal path for the control variable[14]:

$$\varphi_t = -\left(\frac{a\tau}{\lambda + a^2\tau}\right) k_t + \left(\frac{\tau\lambda}{\lambda + a^2\tau}\right) E_t[\varphi_{t+1}] \tag{9}$$

Given the linear-quadratic structure of the model, the solution will be of the form $\varphi_t = c k_t$. In terms of the interest rate actually set by the policy maker, we can use the definitions of φ_t, k_t, λ_1, λ_2, λ_3 and q_t to obtain the optimal rule for the nominal interest rate i_t:

$$i_t = f_\pi \pi_t + f_y y_t + f_{q-q^*}(q_t - q_t^*) \tag{10}$$

[12] See among others Fama (1981), Conover Jensen and Johnson (1999).
[13] See Appendix I for a detailed solution.
[14] See Appendix II for more details.

where $f_\pi = 1 - \frac{c(1-\beta_3\delta_2)}{\beta_2+\beta_3\delta_1}$, $f_y = a + \frac{\beta_1 - ca(1-\beta_3\delta_2)}{\beta_2+\beta_3\delta_1}$, $f_{q-q^*} = \frac{\beta_3}{\beta_2+\beta_3\delta_1}$ are the respective interest rate weights on inflation, output and asset price misalignments from fundamentals. The 'Taylor principle' implies that the inflation coefficient, f_π, should exceed the value of one, to ensure a real interest rate response that will lead to lower inflation[15].

4 Analysis of the results

The rule for adjusting nominal interest rates shown in Eq. (10) indicates that the central bank should not only take into consideration inflation and output, but also react to asset price disequilibria. In the presence of wealth effects ($\beta_3 > 0$) the central bank should raise interest rates in response to higher misalignments ($f_{q-q^*} > 0$). Hence, the optimal response depends on the role of asset prices in the monetary policy transmission mechanism and the source of the asset price movements. When asset prices increase due to improvements in fundamentals *only* (i.e. $\Delta q_t = \Delta q_t^*$), for example an increase in productivity, then monetary policy will accomodate the boom by not responding directly to asset prices[16]. On the other hand, when movements in financial markets cannot be justified solely on the basis of changing fundamentals, such as over-optimistic expectations about future developments, then optimal monetary policy will systematically respond to the non-fundamental component of asset prices. This finding is consistent with the simulation evidence provided by Filardo (2004), Kontonikas and Ioannidis (2005), and Cecchetti *et al.* (2000). Using a rational stochastic bubble model to describe non-fundamentals, Filardo (2004) showed that it is more efficient for the central bank to focus only on the non-fundamental, rather than the fundamental component of asset prices, when calibrating its monetary policy response. Kontonikas and Ioannidis (2005) simulate a forward-looking variant of the macroeconomic model presented here, and find that a mild response to asset price disequilibria ($f_{q-q^*} = 0.1$) promotes overall macroeconomic stability. Such a pro-active response has also been advocated by Cecchetti *et al.* (2000) using the Bernake and Gertler (1999) new keynesian sticky wages - financial accelerator model. A common feature in the aforementioned studies is that they assume, rather than derive, a rule for interest rate setting and then examine the effects on macroeconomic volatility from reacting or not reacting to asset prices. Our main focus however, was to show that in the context of optimal central bank behavior, asset price misalignments should be an element in the monetary authority's feedback rule. Hence, this paper extends the literature that obtains analytical expressions for interest rates based upon optimization of the central banks' objectives. The augmented Taylor rule depicted by Eq. (10) points out explicitly that the financial and real instability associated with growing financial imbalances should not be tolerated by the central bank.

Optimal monetary policy in our model is in line with the suggestions provided by Allen and Gale (2000), in the sense that it is desirable for the central bank to step in and provide liquidity and prevent asset prices falling below the level justified by the underlying fundamentals. Our results are related to the findings of Smets (1997) in the case of symmetric information between policymakers and financial market participants with one important dif-

[15]As we show in Appendix III, $f_\pi > 1$ and $f_y > 0$ if $0 \leqslant \beta_3\delta_2 < 1$.

[16]As previously mentioned, in the context of our model if there are wealth effects in aggregate demand, monetary policy already takes into account asset prices indirectly (and with lag) by responding to output movements.

ference, that is, in his model there is an optimal reaction of interest rates to the actual real asset price, rather than its non-fundamental component. Smets (1997) examines also the case of asymmetric information, i.e. when financial market participants have information not available to policymakers. He shows that with asymmetric information optimal monetary policy reacts to asset prices, even when there is no link between asset prices and aggregate demand ($\beta_3 = 0$), since current prices contain information about current supply shocks. Particularly, when rising asset prices are due to positive supply shocks (which in turn lower the inflation forecast) it is optimal to reduce interest rates. Our view however, is that, despite that asset market participants may have stronger financial incentives to acquire information about current market conditions, there is no reason to assume that their information set is superior to that of modern central banks. In another study related to ours, Bean (2003) also assumes a wealth effects augmented demand curve in his analysis, but the results that he obtains for optimal policy differ significantly from the ones presented here. In particular, Bean finds no role for asset prices in the commitment and discretionary equilibrium. Bean's optimality conditions contain neither the policy instrument, nor anything to do with the demand side of the economy. Finally, our results differ from Bordo and Jeanne (2002) who show that the monetary policy response to asset prices movements is highly non-linear and is dependent on the probability of the bubble emerging[17]. In our framework we derive a linear response to asset price disequilibria that does not depend upon assumptions about the probabilistic nature of the bubble. In essence, we obtain the optimal interest rate as a function of asset price misalignments ($q_t - q_t^*$) that can be approximated by $b\Delta q_{t-1}$. Hence, central bank policies can be based upon the observable level of (past) asset price inflation[18]. Indeed, Chadha *et al.* (2003) estimate an augmented Taylor rule using the past change in stock prices as a measure of stock market disequilibrium, as suggested by our optimal rule, and find a positive and statistically significant reaction.

It is easy to show that the standard Taylor rule (Taylor, 1993) can be obtained as a special case of the augmented rule in two cases. First, in the absence of a link between aggregate demand and asset prices, i.e. $\beta_3 = 0$, there is no scope for monetary policy to directly react to asset prices ($f_{q-q^*} = 0$), and the feedback rule which implements optimal policy takes the form of a Taylor rule with the interest rate being an increasing function of inflation and the output gap.

$$i_t = f_\pi \pi_t + f_y y_t \tag{11}$$

where the inflation and output gap weights are : $f_\pi = \left[1 - \frac{c}{\beta_2}\right] > 1, \;\; f_y = \left[a + \frac{\beta_1 - ca}{\beta_2}\right] > 0.$

Second, if markets are efficient and actual asset prices are always equal to their intrinsic value ($b = 0$), there is no direct monetary policy reaction to asset prices. In this case,

[17] Bordo and Jeanne (2002) use a three period model as in Kent and Lowe (1997).

[18] As we already argued, our 'momentum effects' measure of asset price disequibria is one of the possible specifications that have been employed in the literature. Allowing for alternative departures from the Efficient Markets Hypothesis, such as the rational stochastic bubble model (Blanchard and Watson, 1982), would not invalidate our main result. In other words, it is easy to show within our framework that in an optimal setting, interest rates react only to the non-fundamental component of asset prices, with the reaction being stronger as wealth effects grow. The drawback of such alternative specifications is that, as pointed out by Bordo and Jeanne (2002), the optimality of a monetary policy action depends upon subjective assesments about the probability of a bubble emerging and/or bursting. We thank a referee for raising this issue.

monetary policy takes into account asset prices, indirectly and with a lag, via their demand wealth effects.

In order to further examine the impact of asset prices on the interest rate setting behavior of the central bank, we calculate the elasticity of the extended Taylor rule reaction coefficients in Eq. (10) with respect to the magnitude of wealth effects, β_3. The results, presented in Table 1 below, lead to Propositions 1 to 3.

[TABLE 1]

Proposition 1 *The stronger the wealth effect, β_3, the smaller is the optimal interest rate weight on inflation.*

Proof: since β_2, β_3, δ_1, δ_2 are all positive and $c < 0$, $1 > \beta_3\delta_2$, it is implied that $\partial f_\pi / \partial \beta_3 < 0$.

Proposition 2 *The stronger the wealth effect, β_3, the smaller is the optimal interest rate weight on output gap.*

Proof: since β_2, β_3, δ_1, δ_2, a are all positive and $c < 0$, $1 > \beta_3\delta_2$, it is implied that $\partial f_y / \partial \beta_3 < 0$.

Thus, when the role of capital markets as creator of wealth and collateral is taken into account, the magnitude of the inflation related-interest rate adjustment should be smaller. This does not imply that the central bank intervenes less frequently. In fact, if the true data generation process for aggregate demand is given by the augmented IS, Eq. (2), then monetary policy may have to be more frequently adjusted. Proposition 1 suggests that as wealth effects build up, a too aggressive interest rate response to inflation may lead to recession and threaten the price stability objective. In addition, Proposition 2 calls for a less pronounced response to the output gap in the presence of a significant correlation between asset prices and aggregate demand.

Proposition 3 *The stronger the wealth effect, β_3, the larger is the optimal interest rate weight on asset price misalignments from fundamentals.*

Proof: since β_2, β_3 and δ_1 are all positive, it is implied that $\partial f_{q-q^*} / \partial \beta_3 > 0$.

The intuition and policy implications of Propositions 1 and 2 become clearer when considered in combination with Proposition 3. In essence, if aggregate demand is affected by the evolution of asset prices then monetary authorities should include asset price misalignments in their optimal feedback rule and there should be a change in the distribution of the relevant interest rate weights. Particularly, the interest rate weight on inflation and output decreases while the weight attached to asset price misalignments increases. This allows asset prices to be considered as an element of the authorities' reaction function without necessarily implying overall tighter, than before, policy since the response to inflation and output will be less aggressive. In other words, our results imply that first, asset price misalignments should have an independent role and not only be considered as instruments to help forecast output and inflation; and second, there should be a shift in the magnitude of reaction, away from the traditional variables (inflation, output gap) and towards a direct response to financial imbalances.

In order to gain some further insight in the properties of the model, we calculate the optimal interest rate coefficients (f_π, f_y, f_{q-q^*}) by calibrating the behavioral parameters in Eqs. (1) $-$ (4). For the coefficients (a, β_1, β_2) we employ the values used by Ball (1999). In the Phillips curve, Eq. (1), the sensitivity of inflation to the output gap, a, is set to 0.4. In the aggregate demand, Eq. (2), the interest rate slope, β_2, is 1; the autoregressive output coefficient, β_1, is 0.8; while a value of 0.1 is employed for the elasticity of aggregate demand with respect to asset price changes (β_3). This value is consistent with the Bank of England's model for consumption expenditure, where 1 percent rise in real net financial wealth and real gross housing wealth boosts aggregate spending by 0.12 percent in the long run (Gramlich, 2002). Following Kontonikas and Ioannidis (2005), in Eq. (4), the expected output effect on current fundamentals, δ_2, is assumed to be twice as large as the interest rate effect, δ_1, (0.8 as opposed to 0.4). Finally, in the asset price adjustment Eq. (3) we allow for momentum trading since $b \neq 0$ ($b = 0.5$). The lag structure of the model is more appropriate for annual data therefore we use a discount factor $\tau = 0.96$. For the parameter values shown in Table 2 and equal weight on inflation and output in the loss function ($\lambda = 1$) the optimal policy rule becomes:

$$i_t = 1.7\pi_t + 0.9y_t + 0.1(q_t - q_t^*)$$

The coefficient of the asset price disequilibrium term is 0.1, consistent with the empirical estimates in Chadha *et al.* (2003) and Kontonikas and Montagnoli (2004). It is also in line with the simulation evidence of Kontonikas and Ioannidis (2005), whereas such a mild response to asset price misalignments leads to lower overall macroeconomic volatility. Figure 1 plots the optimal reaction coefficients as a function of the magnitude of wealth effects (β_3). As implied from Propositions (1-3), the interest rate weights on inflation and the output gap are negatively related to β_3, while the response to asset price misalignments is increasing in β_3.

[TABLE 2]

5 Conclusions

Although there is still no widespread agreement among economists on whether central banks should explicitly target asset price inflation, in addition to conventional consumer price inflation targets, a vast consensus that emerges states that the financial-market channel plays an important role in the transmission of the monetary policy. Our aim in this paper is to examine how the conduct of monetary policy is affected by the dynamic evolution of asset prices. Starting from these considerations, we build a backward-looking structural macro model where asset price fluctuations have an impact on aggregate demand and consequently on inflation. A crucial property of our model is that the asset market is not necessarily efficient, thereby generating deviations between actual asset prices and their fundamental value. In order to construct the optimal interest rate rule, we assume that the central bank solves a stochastic control problem to minimise intertemporally the variance of the output gap and inflation.

The derived optimal policy rule conditions the monetary policy instrument not only on inflation and demand pressures, as standard in the Taylor rule literature, but also on financial imbalances, as represented by asset price misalignments from fundamentals. The magnitude of the interest rate reaction depends, among other factors, on the relative importance of wealth effects for aggregate demand. The response to asset market disequilibria becomes more aggressive as wealth effects build up, while the reaction to inflation and the output gap becomes less pronounced. The derived augmented Taylor rule, nests the standard Taylor rule as a special case. When there is no difference between actual and intrinsic asset value and/or when there are no aggregate demand wealth effects, then the interest rate should respond to inflation and demand pressures only.

Thus, our main contribution is to extend the optimal monetary policy literature towards recognizing that, in the presence of wealth effects and inefficient capital markets, monetary authorities should grant an independent role to asset prices and not only regard them as instruments to help forecast inflation and output. Future work should consider an open economy model, where the firms' financing and the households' capital gains derive not only from domestic but also from foreign capital markets.

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Tables, Figures

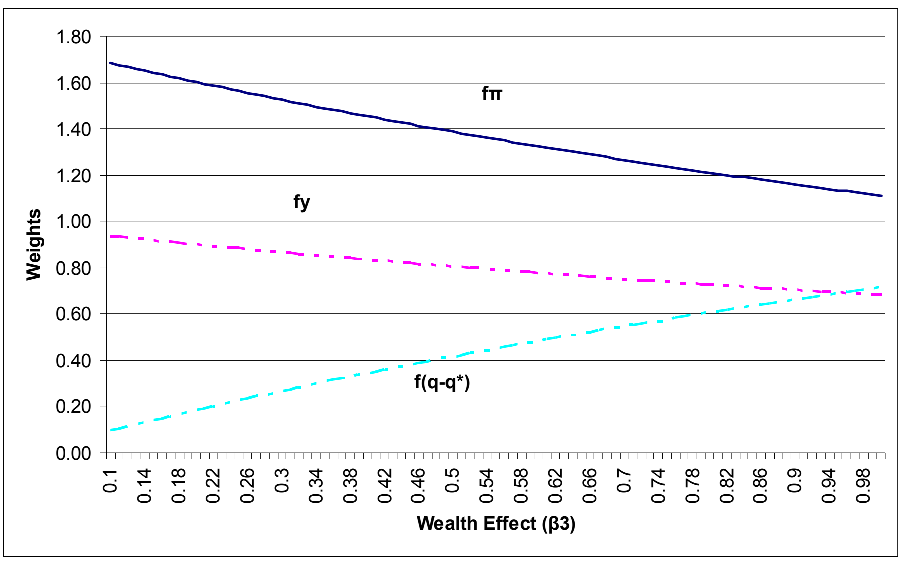
Table 1. Partial derivatives of reaction coefficients

f	$\partial f/\partial\beta_3$
f_π	$\frac{c\delta_2}{\beta_2+\beta_3\delta_1}+\frac{c(1-\beta_3\delta_2)\delta_1}{(\beta_2+\beta_3\delta_1)^2}<0$
f_y	$\frac{ca\delta_2}{\beta_2+\beta_3\delta_1}-\frac{[\beta_1-ca(1-\beta_3\delta_2)]\delta_1}{(\beta_2+\beta_3\delta_1)^2}<0$
f_{q-q^*}	$\frac{\beta_2}{(\beta_2+\beta_3\delta_1)^2}>0$

Table 2. Model calibration

Parameter	*Value*
α	0.4
β_1	0.8
β_2	1
β_3	0.1
δ_1	0.4
δ_2	0.8

Figure 1. Interest rate weights



Appendix I

Substituting for fundamentals, q_t^*, in Eq. (3) we get an alternative expression for real asset prices:

$$q_t = b\Delta q_{t-1} - \delta_1 \left(i_t - E_t \left[\pi_{t+1}\right]\right) + \delta_2 E \left[y_{t+1}\right] + u_t \tag{12}$$

We then use this expression to eliminate q_t from the aggregate demand Eq. (2):

$$\begin{aligned} y_{t+1} \quad = \quad & \beta_1 y_t - \beta_2 \left(i_t - E_t \left[\pi_{t+1}\right]\right) + \\ & \beta_3 \left[b\Delta q_{t-1} - \delta_1 \left(i_t - E_t \left[\pi_{t+1}\right]\right) + \delta_2 E \left[y_{t+1}\right] + u_t\right] + \eta_{t+1} \end{aligned} \tag{13}$$

Collecting $(i_t - E_t \pi_{t+1})$ terms:

$$\begin{aligned} y_{t+1} \quad = \quad & \beta_1 y_t - \left(\beta_2 + \beta_3 \delta_1\right) \left(i_t - E_t \left[\pi_{t+1}\right]\right) \\ & + \beta_3 (b\Delta q_{t-1} + \delta_2 E \left[y_{t+1}\right] + u_t) + \eta_{t+1} \end{aligned} \tag{14}$$

Using the expectational version of Eq. (1) to substitute for $E_t \left[\pi_{t+1}\right]$ and rearranging we obtain:

$$\begin{aligned} y_{t+1} \quad = \quad & [\beta_1 + a(\beta_2 + \beta_3 \delta_1)] y_t - \left(\beta_2 + \beta_3 \delta_1\right) \left(i_t - \pi_t\right) \\ & + \beta_3 (b\Delta q_{t-1} + \delta_2 E \left[y_{t+1}\right]) + \upsilon_{t+1} \end{aligned} \tag{15}$$

where $\upsilon_{t+1} = \beta_3 u_t + \eta_{t+1}$. Thus, the composite output shock (υ_{t+1}) can be decomposed into pure demand-shock (η_{t+1}) and shock to asset prices (u_t). The magnitude of the asset price shock on output depends on the magnitude of wealth effects (β_3).

Taking expectations on both sides of the above expression, conditional upon time t information, yields the following expression for $E_t \left[y_{t+1}\right]$:

$$E_t[y_{t+1}] = \lambda_1 y_t - \lambda_2 (i_t - \pi_t) + \lambda_3 b\Delta q_{t-1} \tag{16}$$

where $\lambda_1 = \frac{\beta_1 + a(\beta_2 + \beta_3 \delta_1)}{1 - \beta_3 \delta_2}$, $\lambda_2 = \frac{\beta_2 + \beta_3 \delta_1}{1 - \beta_3 \delta_2}$, $\lambda_3 = \frac{\beta_3}{1 - \beta_3 \delta_2}$

Using Eq. (16) to eliminate $E \left[y_{t+1}\right]$ from Eq. (15) yields:

$$y_{t+1} = \lambda_1 y_t - \lambda_2 (i_t - \pi_t) + \lambda_3 b\Delta q_{t-1} + \upsilon_{t+1} \tag{17}$$

We now define φ_t as the the control variable of the central bank, since π_t, y_t are predetermined when i_t is chosen:

$$\varphi_t \equiv \lambda_1 y_t - \lambda_2 (i_t - \pi_t) + \lambda_3 b\Delta q_{t-1} \tag{18}$$

Thus, the original system of Eqs. (1) – (4) can be re-written compactly as:

$$y_{t+1} = \varphi_t + \ \upsilon_{t+1} \tag{19}$$

$$\pi_{t+1} = k_t + \epsilon_{t+1} \tag{20}$$

where $k_t \equiv \pi_t + a y_t$ is the state variable at time t.

Appendix II

Using Eqs. (5), (6) the loss function in Eq. (7) can be rewritten as:

$$L=\frac{1}{2}E_t\sum_{i=1}^{\infty}\tau^i\left[\left(k_t+\epsilon_{t+1}\right)^2+\lambda\left(\varphi_t+\ \upsilon_{t+1}\right)^2\right] \tag{21}$$

The objective is to minimize L subject to the constraint that:

$$\begin{aligned}k_{t+1} &= \pi_{t+1}+ay_{t+1}\\ &= k_t+a\varphi_t+\varpi_{t+1}\end{aligned}$$

where $\varpi_{t+1}=\epsilon_{t+1}+a\ \upsilon_{t+1}$

The value function,$V\left(k_t\right)$, is the expected present value of the policymaker's loss function if φ_{t+i} is set optimally. The value function is defined in terms of the state variable, k_t:

$$V\left(k_t\right)=\min_{\varphi_t}E_t\left\{\frac{1}{2}\left[\left(k_t+\epsilon_{t+1}\right)^2+\lambda\left(\varphi_t+\ \upsilon_{t+1}\right)^2\right]+\tau V\left(k_t+a\varphi_t+\varpi_{t+1}\right)\right\} \tag{22}$$

The first order condition that yields the optimal response is:

$$\frac{\partial V(k_t)}{\partial\varphi_t}=0\Leftrightarrow\lambda\varphi_t+a\tau E_tV'\left(k_t+a\varphi_t+\varpi_{t+1}\right)=0 \tag{23}$$

We employ the envelope theorem in order to derive an expression for $E_tV'\left(k_{t+1}\right)$:

$$\begin{aligned}dV(k_t) &= E_t\frac{\partial}{\partial k_t}\left[\frac{1}{2}\left(k_t+\epsilon_{t+1}\right)^2+\lambda\frac{1}{2}\left(\varphi_t+\ \upsilon_{t+1}\right)^2+\tau V\left(k_t+a\varphi_t+\varpi_{t+1}\right)\right]dk_t\Longleftrightarrow\\ V^{'}(k_t)dk_t &= E_t\left[k_t+\tau E_tV'\left(k_t+a\varphi_t+\varpi_{t+1}\right)\right]dk_t\Longleftrightarrow\end{aligned}$$

$$V'(k_t)=k_t+\tau E_tV'\left(k_t+a\varphi_t+\varpi_{t+1}\right) \tag{24}$$

Multiplying Eq. (24) by a and adding it to Eq. (23) yields:

$$\lambda\varphi_t+aV'(k_t)=ak_t\Leftrightarrow V'(k_t)=k_t-\frac{\lambda}{a}\varphi_t$$

This implies that:

$$E_tV'(k_{t+1})=E_t[k_{t+1}]-\left(\frac{\lambda}{a}\right)E_t[\varphi_{t+1}]$$

Substituting for $E_t[k_{t+1}]$:

$$E_tV'(k_{t+1})=k_t+a\varphi_t-\left(\frac{\lambda}{a}\right)E_t\varphi_{t+1}$$

Substituting the derived expression for $E_t V'(k_{t+1})$ back into the first order condition yields:

$$\lambda\varphi_t + a\tau\left[k_t + a\varphi_t - \left(\frac{\lambda}{a}\right)E_t[\varphi_{t+1}]\right] = 0$$

or

$$\varphi_t = -\left(\frac{a\tau}{\lambda + a^2\tau}\right)k_t + \left(\frac{\tau\lambda}{\lambda + a^2\tau}\right)E_t[\varphi_{t+1}] \tag{25}$$

When policy is set at time t, k_t summaries the state, so optimal policy, given the linear-quadratic structure, will be of the form $\varphi_t = ck_t$. Using this proposed policy, and recalling that $E_t[\varphi_{t+1}] = cE_t[k_{t+1}] = c(1+ac)k_t$, Eq. (25) becomes:

$$ck_t = -\left(\frac{a\tau}{\lambda + a^2\tau}\right)k_t + \left(\frac{\tau\lambda}{\lambda + a^2\tau}\right)c(1+ac)k_t$$

This yields the following quadratic equation for c:

$$a\tau\lambda c^2 - \left(\lambda - \lambda\tau + a^2\tau\right)c - a\tau = 0 \tag{26}$$

The solutions are :

$c_1 = \frac{1}{2a\tau\lambda}\left(\lambda - \lambda\tau + a^2\tau + \sqrt{(\lambda - \tau\lambda + a^2\tau)^2 + 4a^2\tau^2\lambda}\right)$,

$c_2 = \frac{1}{2a\tau\lambda}\left(\lambda - \lambda\tau + a^2\tau - \sqrt{(\lambda - \tau\lambda + a^2\tau)^2 + 4a^2\tau^2\lambda}\right)$

To determine which of these solutions we accept, note that:

$$k_{t+1} = k_t + a\varphi_t = (1+ac)\,k_t$$

so that k_{t+1} is a stable process if and only if $c < 0$ so that $1 + ac < 1$. Now consider the product of the two solutions c_1 and c_2:

$$\begin{aligned} c_1c_2 &= \frac{\left(\lambda - \tau\lambda + a^2\tau\right)^2 - \left[\left(\lambda - \tau\lambda + a^2\tau\right)^2 + 4a^2\tau^2\lambda\right]}{(2a\tau\lambda)^2} \\ &= -\frac{4a^2\tau^2\lambda}{4a^2\tau^2\lambda^2} = -\frac{1}{\lambda} < 0 \end{aligned}$$

So one solution must be positive the other negative. We are looking for the negative solution, which is c_2, so that our optimal policy rule is

$$\varphi_t = c_2k_t$$

$$\varphi_t = \left\{\frac{1}{2a\tau\lambda}\left(\lambda - \lambda\tau + a^2\tau - \sqrt{(\lambda - \tau\lambda + a^2\tau)^2 + 4a^2\tau^2\lambda}\right)\right\}k_t \tag{27}$$

Appendix III

The inflation coefficient in the interest rate reaction function, f_π, has to be greater than one in order to satisfy the stability condition that real rates increase in response to inflation, with higher values implying a more aggressive response:

$$f_\pi = 1 - \frac{c\,(1-\beta_3\delta_2)}{\beta_2+\beta_3\delta_1} > 1 \Longleftrightarrow \frac{c\,(1-\beta_3\delta_2)}{\beta_2+\beta_3\delta_1} < 0 \tag{28}$$

Since $\beta_2+\beta_3\delta_1 > 0,\;\; c < 0,\, \beta_3 \geqslant 0$ the stability condition can be re-expressed as:

$$0 \leqslant \beta_3\delta_2 < 1 \tag{29}$$

Note also that if $0 \leqslant \beta_3\delta_2 < 1$, a countecyclical monetary policy response is ensured with interest rates increasing in response to higher output gap, since the output gap coefficient in the interest rate reaction function, $f_y = a + \frac{\beta_1 - ca(1-\beta_3\delta_2)}{\beta_2+\beta_3\delta_1}$, becomes unambigously positive.