Growth, Knowledge Structure, and Quality-Variety Innovations*

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Abstract

This paper aims to construct a model of endogenous technological change, incorporating variety and quality innovations. The technological frontier advances as a result of their interactions. The importance of this exercise lies not only in richer realism but also that it enlarges the set of possible equilibrium and the policy implications of the model with homogeneous R&D can be reversed in some cases. This is because knowledge created in variety and quality R&D differ in nature and the *structure* of the knowledge stock assumed determines the way its externality affects productivity of research activities.

JEL Classi cation: O10, O30, O40.

Key words: variety and quality innovations, growth, knowledge structure, multiple steady states, hysteresis.

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1 Introduction

The R&D-based growth model places profit-seeking research activities at the centre of endogenising technological progress. There are two types of models in this literature. In the first type, technological advance expands variety of products available (see Grossman and Helpman (1991, Ch.3) Romer (1990)), and the second type focuses on the improvement of the quality of products. (see Aghion and Howitt (1992) and Grossman and Helpman (1991, Ch.4)). These models capture many important aspects of technological progress, which are characterized by the horizontal form of increasing specialization of goods (e.g. the original invention of computer chips) and the vertical form of continual replacement of old goods with the state-of-the-art products (e.g. the dramatic increase of transistors contained in a single chip).

However, an important limitation in this literature is that all different forms of technological advance that the economy achieves is aggregated in the homogeneous form of *either* variety *or* quality innovations.[1] Although it is justified as a first approximation of a complicated process of technical change, it misses one of its essential aspects, i.e. the technological frontier advances as a result of interactions between *heterogeneous* research activities.

The literature on technical progress stresses that different types of innovation play a qualitatively different role in driving the technological frontier. For example, Dosi (1982) distinguishes between *technological paradigms* and *technological trajectories.* The former determines the broad directions of technical progress and the latter drive technological progress within a paradigm. The discovery of semiconductors and subsequent innovation

[1] See Jones (1995) for other limitations on empirical grounds..

in computer chips, etc. can be interpreted as technological paradigm and trajectory respectively. Similar concepts of technological paradigms are found in Freeman and Perez's (1986) "techno-economic paradigm", Rosenberg's (1976, Ch.6) "focusing devices" and Sahal's (1985) "technological guide-posts". Furthermore, from the standpoint of economic history, Mokyr (1993) distinguishes *macro*- and *micro*-inventions in explaining the Industrial Revolution. For example, the invention of the steam engine (*macro*-invention) generated a discontinuous leap in the technological frontier, and its subsequent improvement in designs (*micro*-invention) resulted in significant productivity gains.

A contribution of the present paper is to construct a more general model of endogenous technological change to capture the heterogenous nature of technological progress, which was described above. It gives a disaggregative view of how the technological frontier advances on the basis of interactions of different research activities. In our model, variety R&D increases the range of goods and creates the possibility of quality R&D. In return, quality R&D affects profits and productivity of variety R&D, and the latter also influence those of quality R&D. The resulting relative research productivity and profitability determines the allocation of resources, which in turn decides the course of technological progress. Thus, the present paper also makes a methodological contribution to integrating the two types of R&D-based growth model based on quality and variety innovations.

The importance of integrating variety and quality innovations lies not only in richer realism but also in that (i) policy implications of the benchmark model can be reversed in some cases and (ii) multiple steady states arise and transitional dynamics are characterised by histeresis, despite the fact that R&D — the engine of growth in this model — is a forward-looking activity. This arises due to the fact that knowledge created in variety R&D and knowledge generated in quality R&D differ in nature, and hence the *structure* of the knowledge stock accumulated in the *past* determines the way its externality affects the productivity of research activities.

We examine two different structures of knowledge stock which are consistent with constant long-run growth. The case of a unique steady-state in the present model is comparable with the benchmark homogeneous R&D model. The latter predicts that subsidizing (taxing) R&D unambiguously improves (worsens) the growth prospect. This straightforward implication does not hold in our quality-variety framework, because encouraging one type of R&D makes the other relatively less attractive. It will be demonstrated that whether a subsidy to R&D improves growth and welfare depends upon the knowledge structure, the strength of externality and which R&D is subsidized. This result indicates that the effect of industrial policy is not as predictable as the benchmark model suggests.

Furthermore, when two steady states exist, a low growth equilibrium is Pareto-inferior. There can also be a no-growth equilibrium. These equilibria are interpreted as the underdevelopment trap. It turns out that a subsidy to variety R&D is conducive to industrialization, but the situation gets worse if quality R&D is subsidized. Some recent studies (e.g. Bland and Francois (1996)) suggest that a research subsidy is a powerful instrument in bringing about take-off. Our result casts doubt on this straightforward implication.

When there exist multiple steady states, the selection of an equilibrium path arises as an important issue. It is typically determined by expectations in the homogeneous R&D-based growth models (e.g. Young (1993a)), since expectations are an essential element of forward-looking research activities. In our model, despite the fact that it shares the same feature, history (the initial condition) determines a unique equilibrium trajectory. Intuitively, the knowledge stock is the accumulation of innovations in the *past*, and heterogeneity in knowledge makes this aspect more prominent in the determination of an equilibrium. This result is notable, since the history versus expectation distinction is important in the context of policy implications. If expectations determine an equilibrium, the role of a government should be limited to encouraging entrepreneurial spirits to promote growth. If hysteresis arises, on the other hand, merely promoting optimism is not sufficient and a more active public intervention may be called for.

There were earlier attempts to combine the two strands of R&D-based models in somewhat different ways. In Young s (1995) discrete-time period model, quality innovations occur every period, and the number of varieties is determined in each period by equating one period pro ts to a xed cost. Thus, variety R&D technology is not explicitly speci- ed. In Helpman and Trajtenberg (1994,1996), variety R&D creates inputs compatible with a particular level of quality. That is, the roles of quality and variety innovations in our model are reversed in their studies. However, quality innovations are assumed to arrive exogenously. A similar model was developed by Amable (1995), in which both types of R&D are endogenized. But in this model knowledge is created only by quality innovations, i.e. knowledge is homogeneous despite the fact that research activities are heterogeneous.

Much closer to our study is Aghion and Howitt (1996), who distinguish between fundamental and secondary R&D. However, their model is of a variety-variety type. As a result, there is no complete obsolescence of goods. More importantly, Aghion and Howitt (1996) consider a unique steady state only, whereas we will pay close attention to transitional dynamics and the possibility of multiple steady states.

There are several other important contributions in the area where two endogenous activities drive long-run growth. Citing some representative studies, learning by doing and product innovation are modelled in Stokey (1988) and Young (1993b). Stokey (1991) combines human capital accumulation and quality innovation. Segerstrom (1991), Grossman and Helpman (1991, Chs.11,12) and Jovanovic and MacDonald (1994) analyze the interactions of innovative and imitative research. Bresnahan and Trajtenberg (1992) explore the implications of distinguishing between innovations and General Purpose Technologies .

The plan of the paper is as follows. The model is developed in Section 2, and Section 3 derives equilibrium conditions. This is followed by the analysis of equilibrium dynamics and steady state under different assumptions regarding the knowledge structure in Section 4. In addition, we examine comparative statics and explore their implications there. Section 5 conducts welfare analysis, and Section 6 concludes.

2 The Model

The present model is based on Grossman and Helpman (1991). Hence, we highlight only distinctive features of the model in what follows.

2.1 Consumers and Final Output Producers

The intertemporal utility function of consumers is time-separable, and their common instantaneous utility function is logarithmic in the final output, which is denoted by $y(t)$. They differ as suppliers of labor service: (i) L unskilled workers are employed to manufacture intermediate products, and (ii) H skilled workers are exclusively used for research purposes.[2] L and H are constant. Since the aggregate expenditure is normalized, the interest rate equals consumers rate of time preference ρ.

The aggregate production function in the perfectly competitive nal goods industry is

$$y(t) = \left[\int_0^{N(t)} X_i(t)^\alpha \, di \right]^{\frac{1}{\alpha}}, \quad X_i(t) = \sum_{n=0}^{\infty} x_{ni}(t) \, q_{ni}(t)^{(1-\alpha)/\alpha} \tag{1}$$

where $0 < \alpha < 1$, $x_{ni}(t)$ denotes the quantity of inputs, $q_{ni}(t)$ is their quality levels, and $N(t)$ is the number of varieties of intermediate goods.[3]

In the benchmark models, the initial quality of products is given. However, it is more realistic to assume that it is endogenously determined.[4] To capture this, we de ne the quality index as

$$q_{ni}(t) = \gamma^{n_i(t)} z(\tau)^\varepsilon, \quad \gamma > 1, \quad 1 > \varepsilon \geq 0, \quad n_i(t) = 0, 1, 2, ... \tag{2}$$

where τ is the time when the ith variety was invented and $z(\tau)$ denotes the economy-wide average quality at τ.[5] A parameter ε represents the strength of externality of the past innovations on the initial quality of the newest variety. We will de ne $z(\tau)$ more speci cally later.

2.2 Intermediate Goods Producers

Distinct quality products are perfect substitutes in the ith variety in the production function (1). Hence only the state-of-the-art intermediate good is used, and its demand

[2] We could assume homogenous labour force, following the literature. But it just complicates the presentation without affecting the results to be derived below except for welfare analysis.

[3] The exponent of $q_{ni}(t)$ in (1), i.e. $(1-\alpha)/\alpha$, is intended to facilitate the presentation without affecting the results.

[4] For example, the invention of transistors required a high standard of purity of semiconductors. The purity level was constrained by the then technology. If technological and scienti c knowledge at that time were higher, the purity standard would be higher and the initial quality of transistors may be higher as well. (Rosenberg, 1982, p.151).

[5] We suppress the notation τ on the left-hand side of (2).

is

$$x_{ni}(t) = \frac{q_{ni}(t)\, p_{ni}(t)^{-1/(1-\alpha)}}{\int_0^N q_{ni'}(t)\, p_{ni'}(t)^{-1/(1-\alpha)}\, di'}. \tag{3}$$

Since we assume that one unskilled worker is required to produce one unit of $x_{ni}(t)$ and (3) has the constant price elasticity of $-1/(1-\alpha)$, the pro t maximizing price is $p_{ni}(t) = w_L(t)/\alpha$ for $\gamma \geq \alpha^{-\frac{\alpha}{1-\alpha}}$ where $w_L(t)$ is wages of unskilled workers.[6] Thus the *total* pro ts arising from selling the ith input is

$$\pi_{ni}(t) = (1-\alpha)\frac{q_{ni}(t)}{\int_0^{N(t)} q_{ni'}(t) di'}. \tag{4}$$

A variety innovator earns pro ts equivalent to (4) with $q_{ni}(t) = z(\tau)^{\varepsilon}$, $t \geq \tau$. But his product becomes obsolete once it is improved upon by a quality innovator. However, it is assumed that the original variety technology is essential to the production of that variety irrespective of its quality levels. Thus, the quality innovator producing the state-of-the-art good pay a fraction $0 < \kappa < 1$ of pro ts as royalty to the original variety inventor. There are two possible interpretations of κ. It may be viewed as the breadth of patents. Alternatively, it may represent a relative bargaining power measure, such as legal enforcement of patent protection that in uences the magnitude of the royalty payment. In any case, pro ts of each type of innovators are[7]

$$\pi^v_{ni}(t) = \begin{cases} \pi_{ni}(t) & \text{for } n_i = 0, \\ \kappa\pi_{ni}(t) & \text{for } n_i \geq 1, \end{cases} \qquad \pi^q_{ni}(t) = \begin{cases} 0 \text{ for } n_i = 0, \\ (1-\kappa)\,\pi_{ni}(t) & \text{for } n_i \geq 1, \end{cases} \tag{5}$$

where the superscripts v and q are for $v$ariety and $q$uality.

Observe that (5) captures various types of the business-stealing effect. First, pro ts of a variety innovator are reduced by $1-\kappa$ when the rst quality innovation occurs. Second,

[6] This pricing rule arises due to drastic innovation . If the innovation is not drastic, $p_{ni}(t) = \gamma w_L(t)$. We concentrate on the case of drastic innovation, since the main results do not hinge on this.

[7] We could assume $\pi^v_{ni} = 0$ for $n_i = 0$, in which case a variety innovation does not directly lead to a blueprint of a commercial product and generates pro ts only after the rst quality innovation. But it does not make substantial differences in the following analysis.

profits of quality innovators are lost whenever quality improvement occurs in the same variety. Third, as the new variety is introduced, final goods producers spread their costs over wider range of goods, reducing profits of existing inputs. This is captured by an increase in $N(t)$ in (4). Fourth, as quality improvement occurs in the ith variety, final goods producers increase their demand for it at the expense of the other products, reducing their profits. This is represented by an increase in $q_{ni'}(t)$, $i' \neq i$, in the denominator of (4).

2.3 Quality and Variety R&D

Next we describe R&D technologies:[8]

$$\dot{N}(t) = \frac{H^v(t)\,K^v(t)}{a^v Z(t)}, \qquad Z(t) \equiv z(t)^{\varepsilon} \tag{6}$$

$$\xi^q_{ni}(t) = \frac{H^q_i(t)\,K^q(t)}{a^q q_{ni}(t)}, \qquad n_i = 1, 2, ... \tag{7}$$

where $\dot{N}(t)$ is an incremental increase in variety; $\xi^q_{ni}(t)$ is the Poisson arrival rate of the nth quality innovation;[9] $H^v(t)$ and $H^q_i(t)$ are the number of skilled workers used in variety and quality R&D; a^k, $k = v, q$, are positive constants; and $K^v(t)$ and $K^q(t)$ are knowledge conducive to each type of R&D. The presence of $Z(t)$ and $q_{ni}(t)$ in the denominators in (6) and (7) implies that both inventions become increasingly difficult as the technological frontier advances.

For simplicity, incumbent firms are assumed not to conduct research due to the so-

[8] A dot means a time derivative.

[9] At first look, variety R&D seems to involve no uncertainty, whereas quality R&D is stochastic. But we could modify the model to introduce uncertainty into variety R&D in the following way. Assume $y = \left[\sum_{i=0}^{N(t)} X_i(t)^{\alpha}\right]^{1/\alpha}$ instead of the first equation of (1) and that N takes integers and rises with a Poisson arrival rate of $\frac{H^v K^v}{a^v Z}$. The expected number of N up to t is $\int_0^t \frac{H^v K^v}{a^v Z} ds$, so that the expected change of N is equivalent to $\frac{H^v K^v}{a^v Z}$. Thus, the first equation of (6) should be viewed as a reduced form of a stochastic process.

called replacement effect. Successful entrepreneurs with new innovative inputs attain the stock market value $V_{ni}^{q}(t)$ or $V^{v}(t)$. In the frictionless stock market, $V_{ni}^{q}(t)$ and $V^{v}(t)$ must satisfy the following no-arbitrage conditions:

$$\rho = \frac{\dot{V}_{ni}^{q}(t)}{V_{ni}^{q}(t)} + \frac{\pi_{ni}^{q}(t)}{V_{ni}^{q}(t)} - \xi_{n+1i}^{q}(t), \tag{8}$$

$$\rho = \frac{\dot{V}^{v}(t)}{V^{v}(t)} + \frac{\pi_{0N}^{v}(t)}{V^{v}(t)} - \frac{V^{v}(t) - v^{v}(t)}{V^{v}(t)}\xi_{1N}^{q}(t). \tag{9}$$

The left-hand sides are the rate of return to safe bonds, and the right-hand sides are the rate of return to an equity of innovative rms (consist of the capital gain, the earning-price ratio and the risk of losing pro ts in future). In (9), $v^{v}(t)$ is the present value of the ow of pro ts which accrue to the variety innovator after the rst quality innovation. The presence of $1 > [V^{v}(t) - v^{v}(t)]/V^{v}(t) > 0$ re ects the fact that the variety innovator loses only a part of his pro ts following the rst quality innovation.[10]

Free entry in R&D races ensures

$$V^{v}(t) = \frac{w_H(t)\, a^{v} Z(t)}{K^{v}(t)}, \qquad V_{ni}^{q}(t) = \frac{w_H(t)\, a^{q} q_{ni}(t)}{K^{q}(t)} \tag{10}$$

for $H^{v}(t) > 0$ and $H_i^{q}(t) > 0$ where $w_H(t)$ is wages of skilled workers.

[10] The derivation of (9) goes as follows. The value function $V^{v}(t)$ must satisfy the following recursive equation:

$$V^{v}(t) = \pi_{0N}^{v}(t)\, dt + (1 - \rho dt)\left\{V^{v}(t + dt)\left[1 - \xi_{1N}^{q}(t)\, dt\right] + v^{v}(t + dt)\, \xi_{1N}^{q}(t)\, dt\right\}$$

which says that the variety innovator earns $\pi_{0N}^{v}(t)$ during the time interval dt and, at the end of this interval, attains $V^{v}(t+dt)$ if quality innovation does not occur and $v^{v}(t+dt)$ if it does. Rearranging this equation and letting $dt \to 0$ gives rise to (9).

3 Equilibrium Conditions

3.1 Research Arbitrage

The rst equilibrium condition concerns the choice of R&D by entrepreneurs. First we consider the choice among different quality R&D projects.

Lemma 1 *For $H_i^q(t) > 0$ for all i, we have*

$$\xi_{ni}^q(t) = \xi^q(t) \quad \textit{for all } i,\ n. \tag{11}$$

Proof. In order for quality R&D to be active in all existing varieties, entrepreneurs must be indifferent to any quality R&D projects. Then the second condition of (10) implies that $V_{ni}^q(t)K^q(t)/a^q q_{ni}(t) = w_H(t)$ must hold for any i and n. Equating this for any i and n and using (8) gives (11). ■

Given this lemma, one can verify that[11]

$$Z(\tau) \equiv z(\tau)^{\varepsilon} = e^{\varepsilon(\gamma-1)\int_0^{\tau}\xi^q(s)ds}. \tag{12}$$

Besides, using (7) and (11) and de ning $H^q(t) \equiv \int_0^{N(t)} H_i^q(t)\,di$, we have

$$H^q(t) = \frac{\gamma a^q \xi^q(t) A(t)}{K^q(t)}, \qquad A(t) = \int_0^{N(t)} q_{ni}(t)di, \quad \text{for } n_i(t) = 0, 1, 2... \tag{13}$$

where $A(t)$ is the level of technology that the economy has achieved at t. Rearranging the rst equation of (13), we obtain

$$\xi^q(t) = \frac{H^q(t) K^q(t)}{\gamma a^q A(t)}. \tag{14}$$

It implies that the Poisson arrival rate is affected not only by the number of researchers at t but also by the knowledge stock.

[11]Indeed, $z(\tau) = N(\tau)^{-1}\int_0^{N(\tau)} q_{ni}(\tau)di = \sum_{n=0}^{\infty} e^{-\int_0^{\tau}\xi^q(s)ds}(\int_0^{\tau}\xi^q(s)\,ds)^n/n! = e^{(\gamma-1)\int_0^{\tau}\xi^q(s)ds}$.

For later use, we de ne

$$\xi^{v}(t) \equiv \frac{\dot{N}(t)}{N(t)} = \frac{H^{v}(t)\,K^{v}(t)}{a^{v}Z(t)\,N(t)} \tag{15}$$

from (6). Observe that the values of ξ^q and ξ^v crucially depend on how the knowledge structure, i,e. K^q and K^v, is speci ed.

Next we turn to entrepreneurs choice between variety and quality R&D. Since they must be indifferent in equilibrium, the two equations in (10) are equalized in $w_H(t)$ to obtain

$$V^{v}(t) = \frac{V^{q}_{ni}(t)\,Z(t)}{aq_{ni}(t)\,K(t)} \tag{16}$$

where $a = a^q/a^v$ and $K = K^v/K^q$. For those engaging in variety R&D, their opportunity costs re ect the foregone value of quality innovation. Hence the right-hand side of (16) can be interpreted as the opportunity cost of variety R&D, which is equated to the value of variety innovation.

Lemma 2 *Research arbitrage between quality and variety R&D (16) can be rewritten as*

$$\rho = \frac{(1-\kappa)\,\dot{V}^{v}(t)/V^{v}(t) - aK(t)\,\dot{V}^{q}(t)/V^{q}(t) + \kappa aK(t)\,\xi^{q}(t)}{1-\kappa-aK(t)} \tag{R}$$

for $\xi^v(t) > 0$, $\xi^q(t) > 0$ *and* $1-\kappa-aK(t) > 0$.

Proof. See Appendix 6.

Equation (R) re-expresses (16) in terms of the two no-arbitrage conditions (8) and (9). The parameter restriction $1-\kappa-aK(t) > 0$ holds if quality research productivity is relatively more efficient and the pro t share of quality innovators is relatively large. This re ects the fact that the risk of losing pro ts is greater for quality innovators than variety innovators.[12]

[12] If this restriction does not hold, all entrepreneurs opt for variety R&D. As we noted earlier, the present model can be easily modi ed such workers are homogenous. Then if $1-\kappa-aK \leq 0$, the variety model of Grossman and Helpman (1991, Ch.3) emerges as a special case of the present model.

3.2 Financial Arbitrage

Consumers save in bonds or/and equities of research rms. For consumers to be indifferent between investing in any variety and quality research rms, the no-arbitrage conditions in (8) and (9) should be equalized in ρ, as their right hand sides give the returns from investing in those rms. Thus we obtain

$$\frac{1}{w_H(t)} = \frac{\dot{V}^v(t)/V^v(t) - \dot{V}^q(t)/V^q(t) + [1 - aK(t)]\,\xi^q(t)}{(1-\alpha)\,[1 - \kappa - aK(t)]} \cdot \frac{a^q A(t)}{K^q(t)} \tag{F}$$

for $\xi^v > 0$ and $\xi^q > 0$.

3.3 Factor Markets

The demand for unskilled workers comes only from manufacturing. Their full employment requires

$$\frac{\alpha}{w_L(t)} = L, \tag{U}$$

using (3). Skilled workers are used only for research:

$$H^v(t) + H^q(t) = H. \tag{S}$$

3.4 Growth Rate

The aggregate production function (1) can be reduced to $y(t) = LA(t)^{\frac{1-\alpha}{\alpha}}$. Appendix 6 shows that the growth rate of output $g(t) \equiv \dot{y}(t)/y(t)$ is

$$g(t) = \frac{1-\alpha}{\alpha}\left[\frac{\xi^v(t)}{Q(t)} + (\gamma - 1)\,\xi^q(t)\right], \qquad Q(t) = \frac{A(t)}{N(t)\,Z(t)}. \tag{17}$$

In (17), $Q(t)$ is interpreted as the average quality level achieved across intermediate goods industries through quality R&D. This interpretation may become clearer if one considers the case of $\varepsilon = 0$, i.e. $Z(t) = 1$. Appendix 6 also shows that $Q(t)$ changes off steady state according to

$$\dot{Q}(t) = (1-\varepsilon)(\gamma-1)\,\xi^q(t)\,Q(t) - [Q(t)-1]\,\xi^v(t)\,. \tag{18}$$

In steady state, equations (17) are reduced to

$$g = \frac{1-\alpha}{\alpha}\left[\xi^v + \varepsilon(\gamma-1)\,\xi^q\right], \qquad Q = \frac{\xi^v}{\xi^v - (1-\varepsilon)(\gamma-1)\,\xi^q} \tag{19}$$

where $\xi^v > (1-\varepsilon)(\gamma-1)\,\xi^q$ is required for a positive finite output. The intuition for (18) and the second equation of (19) is straightforward. First suppose $\varepsilon = 0$. The level of Q and its rate of change is positively affected by ξ^q, since a higher ξ^q means more frequent quality innovations. But a higher ξ^v increases the speed of introducing new varieties with the lowest quality level among existing goods (i.e. $Z(t) = 1$). This tends to reduce Q and $\dot{Q}$. This intuition also holds for $\varepsilon > 0$. Turning to the first equation of (19), recall that $\xi^v(t)$ and $\xi^q(t)$ are affected by how knowledge $K^v(t)$ and $K^q(t)$ are specified (see equations (14) and (15)). It follows that the growth rate crucially depends upon the structure of knowledge.

4 Knowledge Structure

There are four types of knowledge structure which are consistent with constant long-run growth: (i) $K^v = K^q = A$, (ii)$K^v = K^q = NZ$, (iii) $K^v = A$ and $K^q = NZ$, and (iv) $K^v = NZ$ and $K^q = A$. Due to limits of the space, we consider only the symmetric cases of (i) and (ii) in detail, as other asymmetric cases generate similar results.[13]Case (i) implies that the past variety and quality innovations are both beneficial to research. In contrast, case (ii) postulates that only past variety innovations make explicit contributions

[13]See Li (1997) for cases (iii) and (iv).

to R&D,[14]although quality innovations make variety-speci c knowledge spillovers which are implicit in the quality index.

4.1 Case I $K^v = K^q = A$

4.1.1 Equilibrium

Equation (15) and the second equation of (19) imply $H^v = a^v\xi^v - a^v(1-\varepsilon)(\gamma-1)\xi^v$ in steady state where the second term represents the positive effect of K^v.[15]Equation (14) also implies $H^q = \gamma a^q \xi^q$, so that the skilled labor market condition (S) can be written as $H = a^v\xi^v + [1-(1-\varepsilon)(1-1/\gamma)/a]\gamma a^q\xi^q$. A parameter restriction

$$\frac{1}{1-\varepsilon} > \frac{1-1/\gamma}{a} \tag{20}$$

is required if we rule out the case where the positive externality of knowledge on variety research more than offsets the factor demand of quality R&D. This assumption is maintained in this subsection.

Proposition 1 *For $K^v = K^q = A$,*

1 *the equilibrium conditions consist of*

$$\dot{H}^q = H^q\left(\frac{\Delta H^q}{a^q} - \frac{H}{a^v} - \rho\right), \tag{R_1}$$

$$\dot{Q} = Q\left[\left(1-\frac{1}{\gamma}\right)\frac{H^q}{a^q} - (Q-1)\frac{H-H^q}{a^v}\right], \tag{S_1}$$

where $\Delta = a[\varepsilon(\gamma-1)+\kappa]/[(1-\kappa-a)\gamma] + a - (1-\varepsilon)(1-1/\gamma) > 0$

a unique saddle path stable equilibrium with $H^v > 0$ and $H^q > 0$ e ists if $a^q\rho < (1-a/\Delta)H$;

[14]Note that the latest variety innovation generates intermediate goods with quality level of .

[15]In what follows, the time argument is dropped where it is obvious.

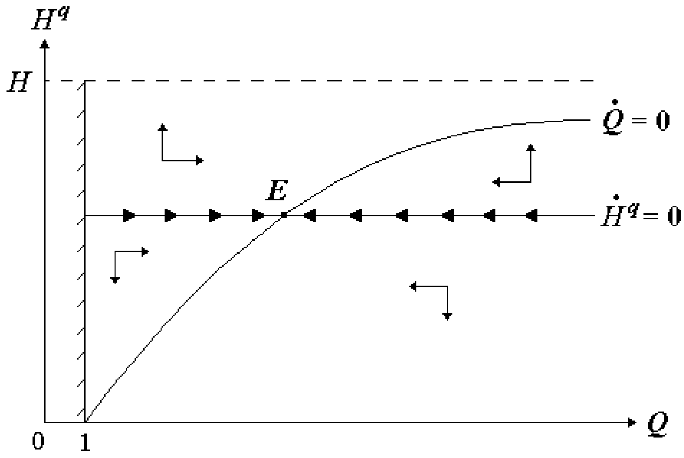

Figure 1: Equilibrium dynamics for $K^v = K^q = A$.

perfect foresight is contradicted in any other trajectories diverging from the equilib rium

Proof. see Appendix 6.

Equations (R_1) and (S_1) are interpreted as the research arbitrage and skilled-labor full-employment conditions respectively in (H^q, Q) space.[16]Figure 1 depicts the two schedules. It demonstrates that a unique equilibrium labelled E is saddle-path stable and that H^q and H^v are constant even in transition. What is changing along the convergent path is Q. In (17), Q is a function of A, N and Z, all of which depends upon the cumulation of the past innovations. Hence the economy does not immediately jump to the equilibrium, unlike in Grossman and Helpman (1991). Equation (15) can be rewritten as $\xi^v = H^v Q(t)/a^v$, implying that $\dot{\xi}^v > 0$ to the left of E, and $\dot{\xi}^v < 0$ to the right of E. But ξ^q is constant, as (14) gives $\xi^q = H^q/\gamma a^q$.

[16]To analyze the market allocation of skilled workers, we can ignore equation (   ), since it just determines w_L.

The growth rate is given by[17]

$$g = \frac{1-\alpha}{\alpha}\left[\frac{H}{a^v} + \left(\frac{1-1/\gamma}{a} - 1\right)\frac{H^q}{a^v}\right]. \tag{21}$$

Note that it is independent of Q, so that an iso-growth contour is a horizontal line in Figure 1. If $(1-1/\gamma)/a > 1$, the growth rate is increasing northward, but it is decreasing if the inequality is reversed.

4.1.2 Comparative Statics

First, we establish the following proposition.

Proposition 2 *he intertemporal utility function of a representative consumer is given by*

$$U = \int_0^\infty e^{-\rho t} \ln y(t)\, dt = \frac{1}{\rho}\left(\ln L + \frac{g}{\rho}\right) \tag{22}$$

in steady state, which is strictly increasing in the growth rate

Proof. Denote $\zeta = \dot{A}/A$, so that the reduced form of the production function (1) becomes $y(t) = Le^{\frac{1-\alpha}{\alpha}\zeta t}$ where $A_0 = 1$ without loss of generality and $g = \frac{1-\alpha}{\alpha}\zeta$. Using these equations, a consumer s intertemporal utility function can be rewritten as (22). ■

Note that due to Proposition 2, the comparative statics of g is equivalent to U apart from ρ and L. Results are summarized in Table 1 where ψ^k, $k = v, q$, denotes the proportion of R&D costs subsidized by a lump-sum transfer. Rows (i) and (ii) show that as ρ rises, the number of skilled workers used in variety R&D falls whereas quality R&D increases its employment. Using the intuition offered by Aghion and Howitt (1996), this is due to a more forward-looking nature of variety R&D than quality R&D, as the opportunity for the latter is opened up by the former.

[17] This is derived from equations (14), (15) and (19).

Table 1: *Comparative tatics*

	H	L	α	ρ	κ	ε	γ	a^v	a^q	ψ^v	ψ^q
(i) H^v	$+$	0	0	$-$	$+$	$+$	*	$-$	$+$	$+$	$-$
(ii) H^q	$+$	0	0	$+$	$-$	$-$	**	$+$	$-$	$-$	$+$
(iii) g for $\frac{1-1/\gamma}{a} < 1$	$+$	0	$-$	$-$	$+$	$+$	*	$-$	$+$	$+$	$-$
(iv) g for $\frac{1}{1-\varepsilon} > \frac{1-1/\gamma}{a} > 1$	$+$	0	$-$	$+$	$-$	$-$	**	$+$	$-$	$-$	$+$

$*$ $-$ if $1-a$ and $+$ if $1-a$.

$**$ $+$ if $1-a$ and $-$ if $1-a$.

If $\varepsilon = 0$, we must have $\frac{1-1/\gamma}{a} < 1$ from (20). In this case, g is an increasing function of H^v (see (21)). Thus, the intuition behind the results of row (iii) that g is higher as κ and a^q increase and γ and a^v decrease should be clear: they make variety R&D relatively more attractive to entrepreneurs. A higher κ raises the pro t share of variety innovators; a high a^q makes quality R&D less productive; a lower a^v increases variety research productivity; a smaller γ means that quality R&D productivity improves (see (7)), and this effect is only partially offset by a lower pro t (see (4)), tending to increase H^q. Given these results, it is clear that for $\varepsilon = 0$, subsidizing variety R&D and taxing quality R&D raises g.

However, some of the results concerning g can be reversed if the strength of the externality on the initial quality ε is sufficiently strong, as row (iv) shows. This case can be interpreted as the over-accumulation of knowledge, as A is an increasing function of ε. In this case, the government should subsidize quality R&D or tax variety R&D to raise the growth rate and welfare. Thus the effect of the industrial policy crucially depends on the parameter ε, indicating the difficulties facing governments. Such difficulties may be even greater, as it is often hard to distinguish between variety and quality innovations before and even after innovations are actually made. Thus, a more realistic case is the untargetted subsidy, i.e. both types of R&D are subsidised at the same rate. In thise case, nothing will change since both research activities become equally attractive to entrepreneurs.

Finally, we brie y touch upon a parameter κ. If it is interpreted as the breadth of patent, the government can maximize welfare by setting κ sufficiently high to the extent that $H^v = H$ or sufficiently low so that $H^q = H$, depending on the value of ε.[18] As we shall see, this second best outcome coincides with the rst best. However, this kind of intervention may not be as straightforward as it seems, because of the difficulty in distinguishing between variety and quality R&D, as mentioned above.

4.2 Case II $K^v = K^q = NZ$

4.2.1 Equilibrium

Under this knowledge structure, it is convenient to illustrate the prominent dynamic feature by specifying the system in terms of ξ^q and Q rather than H^q and Q.

Proposition 3 *For* $K^v = K^q = NZ$,

1 *the equilibrium conditions consist of*

$$\dot{\xi}^q = \xi^q \left(\phi \xi^q - \frac{H}{a^v Q} - \rho \right), \qquad (R_2)$$

$$\dot{Q} = \left[(1-\varepsilon)(\gamma-1) + \gamma a (Q-1) \right] \xi^q Q - \frac{H}{a^v} (Q-1), \qquad (S_2)$$

where $\phi = a[\varepsilon(\gamma-1)+\kappa]/(1-a-\kappa) + \gamma a - (1-\varepsilon)(\gamma-1) > 0$, *as non trivial equilibrium requires* $\gamma a > (1-\varepsilon)(\gamma-1)$

there are two steady states (i) the high growth steady state is saddle path stable, and (ii) the low growth equilibrium is unstable

the high growth steady state is areto superior to the low growth one

[18] If R&D technologies exhibit diminishing returns in research workers, the social optimum involves ^{v} 0 and ^{q} 0.

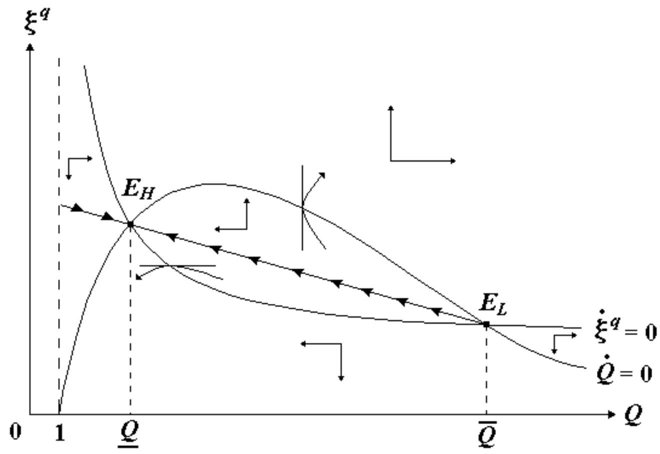

Figure 2: Equilibrium dynamics for $K^v = K^q = NZ$.

perfect foresight is contradicted in any other trajectories diverging from these steady states

Proof. see Appendix 6.

Equations (R_2) and (S_2) are the research arbitrage and skilled labor market conditions in (ξ^q, Q) space. They are depicted in Figure 2. There are three possibilities: (i) the two curves intersect at two steady states; (ii) they are tangential with a unique steady state; and (iii) no intersection exists. Implications of (ii) and (iii) will be explored later on, and we rst focus on case (i). Two equations in (19) can be rearranged into

$$g = \frac{1-\alpha}{\alpha} \cdot \frac{Q-\varepsilon}{Q-1} \cdot (\gamma - 1)\, \xi^q, \tag{23}$$

which gives a set of iso-growth contours in Figure 2. Since they are upward-sloping and g becomes higher as the economy moves leftward, the growth rate is higher at E_H than E_L. A salient feature of dynamics is that a saddle path is emanating directly from the low growth steady state to the high growth one.[19]Result (3) of Proposition 3 is obvious

[19]To show this, consider other trajectories starting from the $\xi^q = 0$ and $= 0$ curves between $_H$ and $_L$. The economy must move leftward horizontally if it is on the curve $\xi^q = 0$ and vertically upward if

from Proposition 2.

An intuition behind multiplicity of steady states is best provided by examining the rate of return to quality R&D in the ith variety, which is the right-hand side of (8). It is denoted by ^{q}.[20]Using (14) and (15) and the second equations of (10) and (19), steady state ^{q} can be rewritten in terms of H^q and H^v

$$(\rho =) \quad ^{q} \equiv -\frac{H^v}{a^q} - \varepsilon(\gamma-1)\frac{H^q}{\gamma a^q Q} + \frac{(1-\kappa)(1-\alpha)}{a^q Q}\frac{1}{w_H} - \frac{H^q}{\gamma a^q Q} \tag{24}$$

$$\equiv -\frac{H^v}{a^q} + \frac{[\kappa+\varepsilon(\gamma-1)]\,a}{1-\kappa-a}\frac{H^q}{\gamma a^q Q} \tag{25}$$

$$\text{where} \quad Q = \frac{(1-\varepsilon)(\gamma-1)}{a\gamma}\frac{H^q}{H^v} + 1 \tag{26}$$

The rst two terms on the right-hand side of (24) represents the rate of depreciation of the value of quality innovation as time elapses by dt. As newer varieties are introduced at $t+dt$, nal goods producers dissipate their expenditure, reducing their demand for individual inputs. This is captured by the rst term $-H^v/a^q$. Moreover, the newer varieties have the initial quality $Z(t+dt)$. This tends to make nal goods producers reduce demand for other existing intermediate goods. The second term $-\varepsilon(\gamma-1)H^q/\gamma a^q Q$ represents this effect. The third term is the earnings-price ratio (π_{ni}/V^q_{ni}), and the fourth term is the risk of losing pro ts due to future quality innovation in the same variety.

The second line (25) is obtained by using the nancial arbitrage condition (F) which relates w_H to H^q.[21]Note that H^v and H^q in (25) are the current number of skilled workers in respective R&D, whereas those in (26) represent employment in the research sector

it is on the curve $= 0$. It follows that those paths cannot be the saddle path reaching $_H$, and the remaining possibility is only the orbit starting from $_L$.

[20]We can equally examine the rate of return to variety R&D for this purpose. But the intuition we obtain is the same, since the rates of the return to quality and variety R&D are identical due to the nancial arbitrage condition ( ).

[21]It is rewritten as

$$\frac{1}{w_H} = \frac{(\gamma-1)+1-a}{(1-\alpha)(1-\kappa-a)}\frac{^{q}}{\gamma}.$$

in the past, since Q re ects the external effects of the past innovations. Bearing these in mind, it is clear in (25) that holding Q constant, ^{q} is strictly decreasing in H^v and increasing in H^q, i.e. there is a one-to-one relationship between the current H^v and H^q for a given Q. On the other hand, if we take Q (i.e. the past innovations) into account, ^{q} is still monotonically increasing in H^q, whereas it has a -shaped relation with H^v. In other words, two different values of H^v are consistent with a given H^q. As (26) shows, if the past H^v has been high, Q is low, which in turn implies $^q > \rho$ for a given H^q in (25). To restore $^q = \rho$, the current H^v must be higher. Similarly if the past H^v has been low, Q is high and the current H^v must be sufficiently low for $^q = \rho$. Thus, the current H^v crucially depends on the past H^v. Compared with the previous case, this case highlights the essential role of the structure of knowledge in generating multiple steady states.

This history-dependent explanation of multiple steady states contrasts with expectational multiple equilibria in Young (1993a) in which the current research intensity is high or low, since it is expected to be so in future. In his model, there are overlaps of two stable arms converging to different steady states, and the economy can be on either trajectory, depending upon agents expectations. In this sense, there are multiple equilibria and an equilibrium is indeterminate. In contrast, our model exhibits only a single trajectory converging to the high growth steady state, and the economy must always be on it in transition. In this sense, there exists a unique equilibrium at each moment of time and equilibrium is determinate in the presence of multiple steady states. This is because the accumulation of the past innovations exerts a dominant force in the selection of an equilibrium, i.e. hysteresis.

4.2.2 Low-Growth Equilibrium and nderdevelopment

Since history plays a dominant role, the economy is trapped in the low growth steady state if it starts from there. On the other hand, as shown in Figure 2, there exists a stable trajectory starting from the low growth steady state to the high growth one. This suggests that a small disturbance to the system such as an R&D subsidy helps the economy to escape from the trap.

We rst consider the case of targetted R&D subsidies. Suppose that the economy is initially located at E_L in Figure 3. Since a variety R&D subsidy makes quality R&D relatively less attractive to entrepreneurs, ξ^q falls for a given Q. As a result, the $\dot{\xi}^q = 0$ schedule shifts downward, moving steady states to E_H^v and E_L^v. On the other hand, a subsidy to quality R&D shifts the $\dot{\xi}^q = 0$ curve shifts upward with steady states moving to E_H^q and E_L^q. New stable paths run from E_L^k to E_H^k, $k = v, q$. Following an industrial policy, the economy must be on a new saddle path at $\overline{Q}$. It should be clear from the gure that the economy can nd itself on a path leading to a high growth steady state if and only if the $\dot{\xi}^q = 0$ curve moves downward, i.e. variety R&D is subsidized or quality R&D is taxed. Note that this result is valid even if a policy shift is temporary.

Another comparative dynamic exercise is to start from the situation where the two schedules do not intersect like a dotted $\dot{\xi}^q = 0$ curve in Figure 3. This arises if variety research technology is relatively inefficient. Although quality innovation occurs, the growth rate asymptotically approaches zero in the absence of the positive externality of variety innovation.[22] To revitalize economic growth, the government should move the $\dot{\xi}^q = 0$ curve downward to make it tangential to or intersect with the $\dot{Q} = 0$ curve. This task is ac-

[22] Since variety R&D is not conducted, $^q =$ and $N = N_0$ for all t. Hence (14) implies $=$ $\gamma a^q \xi^q \int_0^{N_0} q_{ni} di / N_0$. Since q_{ni} rises, ξ^q must converge to zero for this equality to hold, i.e. long-run growth is not sustained.

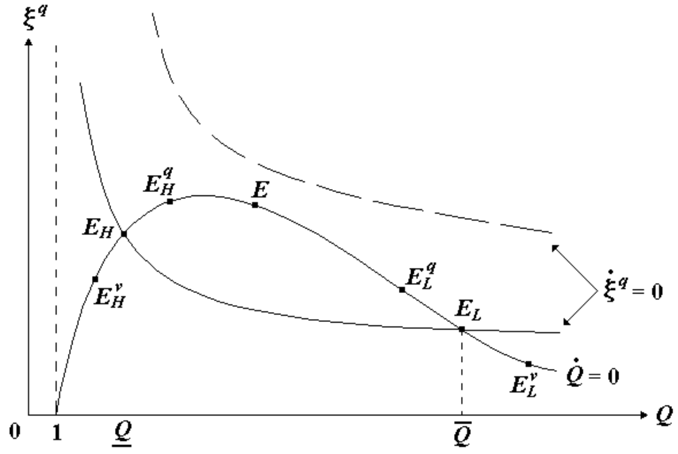

Figure 3: The underdevelopment trap.

complished by a subsidy to variety R&D or a tax on quality R&D. On the other hand, if quality R&D is subsidized or variety R&D is taxed, the $\dot{\xi}^v = 0$ curve shifts upwards with the result that resources are wasted and the situation gets worse.

However, a caveat is in order. If R&D subsidies are untargetted (both R&D are subsidised), the $\dot{\xi}^q = 0$ curve does not move, i.e. industrialisation does not occur. For a successful take-off , subsidies must be targetted. However, a difficulty facing the government is again that it is hard to distinguish between variety and quality R&D. The more difficult this problem, the less unlikely that R&D subsidy is instrumental in bringing about industrialisation. This result stands in contrast with some studies that emphasise R&D subsidy as an important policy (e.g. .Bland and Francois (1996)).

It should be mentioned that an increase in κ induces exactly the same result as a variety R&D subsidy (and tax on quality R&D), since both policies raise an incentive for variety R&D. Thus, the model predicts that how rents are distributed among different innovators is one of crucial determinants of industrialisation of developing economies where rent-seeking activities often pose serious problems.

Noteworthy is also that the revival of growth momentum in the present model is different from the Big Push theory of economic development formalized by Murphy, Shleifer and Vishny (1989) among others. In their model expectations of rms play a decisive role in locking an economy in underdevelopment due to demand externalities, whereas in our model inefficient research technology makes the economy stagnate and there is no role played by expectations.

elfare Analysis

This section examines the efficiency of the market economy by comparing the laissez-faire outcome with the social optimum. The social planner maximizes the intertemporal utility function of a representative consumer. Since ξ^v and ξ^q differ depending on the knowledge structure in the market economy, their socially optimal values also differs. However, we can establish the following proposition.

Proposition 4 *he mar et economy cannot grow e cessively, and in general, the mar et growth rate is lower than the social optimum, irrespective of the nowledge structure*

Without solving the social planner s problems, this proposition should be clear from Proposition 2 which implies that the growth rate is always maximized in the social optimum. This result is despite the fact that the present model captures several types of the business-stealing effects, which tends to make the laissez-faire economy grow excessively in the homogenous R&D model. This is due to the assumption of heterogeneous workers. The optimal allocation of skilled workers is independent of what is going on in the product market where the business-stealing effect is realised.

Another type of a market failure which does not exist is the monopoly distortion effect due to the CES production function (1).[23] A distortion that gives rise to Proposition 4 is the intertemporal spillover effect. As (14) and (15) indicate, the current research successes will raise the future research productivity. Since remunerations for these contributions do not accrue to the original innovators in the market economy, they constitute a positive externality.

An important question, given Proposition 4, is that of which research activity should be subsidized or taxed to achieve the social optimum. Generally the answer depends upon parameter values. We will demonstrate this by considering the case of $K^v = K^q = A$ as an example to avoid taxonomical analysis. Using (21) and (22), the problem facing the social planner is

$$\underset{H \quad H^q}{} \quad \frac{1}{\rho} \quad \ln L + \frac{1-\alpha}{\rho\alpha}\left[\frac{H^v}{a^v} + (1-1/\gamma)\frac{H^q}{a^q}\right] \qquad .t. \;\; H = H^v + H^q. \qquad (27)$$

The solution is

$$H^v = H, \; H^q = 0 \quad \text{for} \;\; a > 1 - 1/\gamma, \qquad (28)$$

$$H^v = 0, \; H^q = H \quad \text{for} \;\; a < 1 - 1/\gamma, \qquad (29)$$

$$H > H^v, \; H^v > 0 \quad \text{for} \; a = 1 - 1/\gamma. \qquad (30)$$

This con rms that the social optimum and the market outcome differ in general, except for the edge-knife case in (30), in which the growth rate is independent of the allocation of skilled workers.[24] Cases of (28) and (29) suggest that the market economy with $H^v > 0$ and $H^q > 0$ can improve welfare by subsidizing variety R&D or taxing quality R&D if $a > 1 - 1/\gamma$ or by subsidizing quality R&D or taxing variety R&D for $a < 1 - 1/\gamma$. The

[23] See Grossman and Helpman (1991, p.70).

[24] A similar knife-edge result would be obtained even if R&D technology exhibits decreasing returns to skilled labour.

intuition is straightforward. If $a > 1 - 1/\gamma$, variety R&D is relatively more productive. It follows that the growth rate and welfare can be increased by shifting skilled workers to variety R&D. The reverse is true for $a < 1 - 1/\gamma$.

Conclusion

Given the successes of the R&D-based growth literature in capturing and revealing many important aspects of modern economic growth, a logical extension is to integrate the two types of the models based on quality and variety innovations. It sheds light on how the technological frontier advances as a result of interactions between different innovations.

We paid close attention to the structure of the knowledge stock, since different types of research activities create different knowledge. It was demonstrated that our synthesized model could exhibit either multiple steady states or a unique steady state, depending upon the knowledge structure. Besides, the knowledge structure affects transitional dynamics, growth rates and comparative statics. The growth process of the economy is crucially in uenced not only by the source of knowledge but also by the composition of the general knowledge.

We examined the effects of industrial policy. The basic message is that such policy may not be as reliable as the literature suggests. Strength of externality, the structure of knowledge and the type of R&D to be subsidised (or taxed) affects the outcome of policy change. The difficulty to predict the policy effect is reinforced by the fact that it is not easy to distinguish between variety and quality innovations ex ante.

A notable feature of our model is that even in the presence of multiple steady states, the equilibrium is unique for a given initial condition. Young (1993a, p.805) notes that if models of endogenous growth are to be built around external effects, it the section of an equilibrium path out of multiple equilibria is an issue that they must surely, sooner or later, confront. Our model offers an answer by focusing on the structure of knowledge. The future is important, since R&D decisions are based upon the expectation of future pro ts and the risk of product obsolescence, but the past is also important, since the externality of knowledge a driving force of the endogenous growth model depends upon the past innovations.

Appendi A Proof of Lemma 2

Due to the de nitions of $V^v(t)$ and $v^v(t)$,

$$V^v(t) - v^v(t) = \int_t^\infty \xi^q(\tau) e^{-\int^\tau \xi^q(\ )d} \int_t e^{-\rho(\ -t)} (1-\kappa)\pi_{0N}(\ )d \quad d\tau \tag{31}$$

$$= V_{ni}^q(t) Z(t) / q_{ni}(t) \tag{32}$$

where (32) uses $(1-\kappa)\pi_{0N}(\ ) = (1-\kappa)(1-\alpha)Z(t)/A(\ ) = \pi_{ni}^q(\ )Z(t)/q_{ni}(t)$. Making use of (8), (9) and (32), equation (16) can be re-expressed as (R). ■

Appendi Growth Rate

This appendix derives equations (17), (18) and (19). First we normalize the initial measure of varieties N_0 to 1, so that $A(t) = \int_0^1 q_{ni}(t)\,di + \int_1^{N(t)} q_{ni}(t)\,di$. Since there are a continuum of intermediate goods industries, this allows us to invoke the Law of Large Numbers and rewrite the quality index as $q_{ni}(t) = \gamma^{n_i(t)} z(\tau)^\varepsilon \quad e^{(\gamma-1)\int_\tau \xi^q(s)ds}$. $e^{\varepsilon(\gamma-1)\int_0^\tau \xi^q(s)ds}$ where the initial quality of the initial variety is assumed to be 1. Besides, we have $i = e^{\int_0^\tau \xi\ (s)ds}$, $\tau \leq t$, where τ is the time when the ith variety is rst introduced. Thus, we can write

$$A(t) = e^{(\gamma-1)\int_0 \xi^q(s)ds} \left(1 + \int_0^t \xi^v(\tau) e^{\int_0^\tau \xi\ (s)-(1-\varepsilon)(\gamma-1)\xi^q(s)\ ds} d\tau\right). \tag{33}$$

Differentiating this gives $\dot{A}(t)/A(t) = (\gamma - 1)\,\xi^q(t) + \xi^v(t)/Q(t)$, which, together with $y(t) = LA(t)^{(1-\alpha)/\alpha}$, leads to (17). This equation and the second equation of (17) give rise to (18). Since $\dot{Q}(t) = 0$ in steady states, (18) leads to the second equation of (19), and substituting this into the rst equation of (17) leads to that of (19).

Appendi C Proof of Proposition 1

Result 1 : For (R_1), we rst derive $-\dot{w}_H/w_H = \dot{H}^q/H^q$ from (14) and (F). Using this, (14), (15) and (S), we rewrite (R) to obtain (R_1). The differential equation (S_1) is derived from (14), (15), (18) and (S).

Result 2 : In steady state, equations (R_1) and (S_1) collapse to

$$\dot{H}^q = 0 \qquad H^q = \frac{aH + \rho a^q}{\Delta}; \qquad \dot{Q} = 0 \qquad H^q = \frac{aH\,(Q-1)}{(1-\varepsilon)\,(1-1/\gamma) + a\,(Q-1)} \tag{34}$$

which are depicted in Figure 1. Uniqueness of an equilibrium is evident from the two equations as long as $a^q\rho < (1 - a/\Delta)H$. Linearization of the system around the steady state generates the following Jacobian matrix

$$= \begin{matrix} \Delta H^q/a^q & 0 \\ Q\left[(1-\varepsilon)\,(1-1/\gamma) + a\,(Q-1)\right]/a^q & -Q\,(H - H^q)\,/a^v \end{matrix}, \tag{35}$$

and its determinant

$$= -\frac{\Delta H^q Q\,(H - H^q)}{a^v a^q} < 0. \tag{36}$$

Hence the equilibrium is a saddle point.

Result 3 : First consider paths leading to the horizontal axis where $H^q = H - H^v = 0$ in Figure 1. The nancial arbitrage condition (F) for $K^v = K^q = A$ is

$$\frac{1}{w_H} = \frac{\varepsilon\,(\gamma-1) + 1 - a}{(1-\alpha)\,(1-\kappa-a)\,\gamma} \cdot H^q, \tag{37}$$

which implies that $w_H \to$, as $H^q \to 0$. Along these paths, we must have $V^v(t)N(t)Q(t)/a^v =$ $w_H(t)$, which implies $V^v(t) \to$. However, after the horizontal axis is hit, we have

$$V^v(t) = \int_t^\infty e^{-\rho(\ \ -t)} \frac{(1-\alpha)\,Z(t)}{A(\tau)} d\tau < \frac{(1-\alpha)\,Z(t)}{\rho A(t)} < \qquad (38)$$

where $A(\tau)$ is rising after t. Clearly perfect foresight is contradicted. Next consider trajectories leading to the horizontal line at $H^q = H$. After this line is reached, the economy moves leftward along the line, i.e. $Q \to$, and free entry in variety R&D implies

$$V^v(t) \frac{N(\ \ )\,Q(t)}{a^v} < w_H(\ \ ), \qquad < t \qquad (39)$$

where denotes the time when the $H^q = H$ line is reached. $N(\ \ )$ remains constant after , and so does $w_H(\ \ )$ due to (37). For (39) to hold, $V^v(t) \to 0$, as $Q(t) \to$. However, $V^v(t) \to 0$ means that a new variety innovator eventually makes zero pro t, which cannot be true. Hence, expectations are contradicted. ■

Appendi D Proof of Proposition 3

Result 1 : We rst rewrite (S) as

$$H = a^v\xi^v + \gamma a^q \xi^q Q \qquad (S')$$

using (14) and (15). Equation (R_2) is derived from (R), (F), (18) and (S'), and equation (S_2) is obtained by substituting (S') into (18).

Result 2 : In steady state, equations (R_2) and (S_2) give

$$\dot{\xi}^q = 0 \qquad \xi^q = \frac{1}{\phi}\left(\frac{H}{a^v Q} + \rho\right), \qquad (40)$$

$$\dot{Q} = 0 \qquad \xi^q = \frac{(Q-1)\,H/a^v}{[(1-\varepsilon)(\gamma-1) + \gamma a\,(Q-1)]\,Q}. \qquad (41)$$

Solving them simultaneously for Q yields $Q^2 + \ Q + d = 0$ where $\ = \rho\gamma a/\phi > 0$, $\ = [(1-\varepsilon)(\gamma-1)-\gamma a]\rho/\phi - H[1/\ -(\gamma-1)]/a^v\phi \gtreqless 0$, $d = H/a^v\phi \ > 0$ where $\ = (1-\kappa-a)/[\varepsilon(\gamma-1)+\kappa]a > 0$. Roots for high and low growth steady states are

$$Q_H = \frac{- \ - \sqrt{\ ^2 - \ d}}{2}, \qquad Q_L = \frac{- \ + \sqrt{\ ^2 - \ d}}{2}. \tag{42}$$

Hence, the necessary and sufficient conditions for two equilibria to exist are

$$\ < 0, \qquad \ ^2 > \ d. \tag{43}$$

Linearizing the system around the steady states, we obtain the Jacobian matrix:

$$\ = \begin{matrix} \phi\xi^q & \frac{H\xi^q}{a} \\ [(1-\varepsilon)(\gamma-1)+\gamma a(Q-1)]Q & \gamma a\xi^q Q - \frac{H}{a} \end{matrix} \tag{44}$$

Using (42), the determinant and trace of this matrix evaluated at the steady states are written as

$$\begin{aligned} {}_H &= \frac{\gamma a\rho\xi^q}{Q}\frac{(\ ^2 - \ d)^{1/2}}{2}\left[\ ^2 - \ d\ ^{1/2} + \ \right] < 0, & (45)\\ {}_L &= \frac{\gamma a\rho\xi^q}{Q}\frac{(\ ^2 - \ d)^{1/2}}{2}\left[\ ^2 - \ d\ ^{1/2} - \ \right] > 0, & (46)\\ t(\ ) &= \rho + \gamma a\xi^q Q > 0. & (47) \end{aligned}$$

where we use (43) in determining the signs of (45) and (46). The high growth steady state is associated with $_H$, and the low growth one with $_L$. Since $_H < 0$, the high growth equilibrium is saddle-path stable, while the low growth equilibrium is unstable because $_L > 0$ and $t(\ ) > 0$.

Result 3 : It is evident from Proposition 2.

Result 4 : First consider paths leading to the horizontal axis where $\xi^q = 0$ in Figure 2. Along this axis, $H = H^v = \xi^v/a^v$. The nancial arbitrage condition (F) for

$K^v = K^q = NZ$ is

$$\frac{1}{w_H} = \frac{\varepsilon(\gamma - 1) + 1 - a}{(1-\alpha)(1-\kappa-a)\gamma} \cdot a^q \xi^q Q, \tag{48}$$

which implies that $w_H \rightarrow$, as $\xi^q \rightarrow 0$. Along these paths, we must have $V^v(t)N(t)/a^v = w_H(t)$, which implies $V^v(t) \rightarrow$. However, after the horizontal axis is reached, (38) applies, so that perfect foresight is contradicted. Next consider trajectories leading upward, which eventually hit the curve

$$H = \frac{\xi^q}{\gamma a^q Q} \tag{49}$$

(not drawn in Figure 2). This is derived by substituting $\xi^v = 0$ into (S'). Once this curve is reached, the economy moves along it rightward with $Q \rightarrow$, and free entry in variety R&D implies

$$V^v(t)\frac{N(\ )}{a^v} < w_H(t), \qquad < t \tag{50}$$

where denotes the time when the curve of (49) is hit. $N(\ )$ remains after . However, (48) and (49) imply that $w_H(t) \rightarrow 0$, as $Q(t) \rightarrow$. Thus, for (50) to hold, $V^v(t) \rightarrow 0$, as $w_H(t) \rightarrow 0$. However, $V^v(t) \rightarrow 0$ means that a new variety innovator eventually makes zero pro t, which cannot be true. Hence, expectations are contradicted. ■

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