Realised and Optimal Monetary Policy Rules in an Estimated Markov-Switching DSGE Model of the United Kingdom

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Abstract

This paper investigates underlying changes in the UK economy over the past thirty-five years using a small open economy DSGE model. Using Bayesian analysis, we find UK monetary policy, nominal price rigidity and exogenous shocks, are all subject to regime shifting. A model incorporating these changes is used to estimate the realised monetary policy and derive the optimal monetary policy for the UK. This allows us to assess the effectiveness of the realised policy in terms of stabilising economic fluctuations, and, in turn, provide an indication of whether there is room for monetary authorities to further improve their policies.

JEL classification: C11; C32; C51; C52.
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1. Introduction

Output and inflation volatility declined in most industrialised countries over the past two decades. Much has been written about the possible causes of this change, which is commonly referred to as the ‘Great Moderation’ (see, Kim and Nelson, 1999, McConnell and Perez-Quiros, 2000 and Stock and Watson, 2003). Amongst various explanations, there has been an ongoing debate about the role of ‘good luck’ versus ‘good monetary policy’. The former is based on the belief that the relative macroeconomic stability observed in recent years is primarily due to good fortune that fewer major economic shocks have occurred. In contrast, the latter suggests that improved monetary policy, such as inflation targeting, has helped moderate swings in inflation and output. Understanding whether monetary policy played a part in stabilising inflation and output has important implications for policy markers. If inflation targeting has been able to stabilise inflation and output, monetary authorities should continue to adopt this policy.

Much of the literature surrounding this debate has focused on the US. However, no consensus has emerged in the literature. Benati and Surico (2007), Davig and Doh (2009) and Cogloy, Primiceri, and Sargent (2010) concluded that US monetary policy targeted inflation more aggressively after the period of Volcker disinflation than during the 1970s. They found that this played a significant role in reducing both the volatility and persistence of inflation. Conversely, Sims and Zha (2006) argue that there is little evidence that changes in monetary policy contributed to the ‘Great Moderation’. Instead they concluded that the economic stabilisation was primarily down to ‘good luck’.

A limited number studies have considered this issue for the UK. Amongst others, Castelnuovo and Surico (2006) investigated possible changes in the transmission of monetary policy, and found that the impact of contractionary monetary policy shocks on inflation was significantly different pre- and post-1992. In addition, Benati (2008) found that UK monetary policy was more responsive to changes in inflation from 1980 onwards. However, their results suggest that a fall in the volatility of demand and supply shocks, rather than inflation targeting, were the main causes of the ‘Great Moderation’ in the UK.

In contrast to the above-VAR based studies for the UK, this paper uses a small-scale open economy Dynamic Stochastic General Equilibrium (DSGE) model proposed by Lubik and Schorfheide (2007). Unlike the VAR models, the DSGE model can explicitly incorporate agents' expectations and provide a clearer interpretation of economic shocks. Furthermore, this paper utilises recent developments in the estimation of DSGE model that allow for Markov-switching structural parameters (see, for example, Davig and Leeper (2007) and Farmer, et al (2008, 2009, 2010b). Such models are generally referred to as Markov-switching rational expectation (MSRE) models and can automatically capture underlying structural changes in an economy over time. The empirical papers based on the MSRE model are limited and have in general been developed for the US (Davig and Doh, 2008; Bianchi, 2010).

This paper extends the 'good luck vs good policy' debate in a number of ways. First, by estimating a number of MSRE versions of the DSGE model developed by Lubik and Schorfheide (2007) for the UK, we can identify underlying changes in the model's structural parameters, such as the standard deviation of exogenous shocks, the monetary policy coefficients and the nominal price rigidity parameter. This allows us to investigate whether there was a change in UK monetary policy towards more aggressive inflation targeting after the Great Inflation period in the 1970s or since the introduction of inflation targeting in 1992. We can also analyse whether the UK monetary authorities implicitly adjusted their policy objectives during the recent financial crises to place a greater emphasis on stabilising output at the expense of achieving their stated inflation target. The model also allows us to identify the role of 'good luck' by capturing changes in the volatility of exogenous shocks.

Second, we use the best fitting MSRE model to derive optimal monetary policy rules. To our knowledge, this is the first paper to evaluate and derive optimal monetary policy rules based on an estimated small open economy MSRE model. By deriving optimal monetary policy rules, we can analyse how effective realised monetary policy has been, in terms of stabilising the macroeconomy, compared to an optimal rule. This allows us to evaluate whether there is room for monetary authorities to further improve their policies.

To preview our results, the MSRE model incorporating shifts in monetary policy, nominal price rigidity and volatility of exogenous shocks is found to best fit the data. This model suggests that UK monetary policy changed during the early

1980s to more aggressively target inflation. It also indicates that the monetary authorities have placed less emphasis on stabilising inflation during the most recent recession, perhaps due to the significant volatility in financial markets and uncertainty about the broader economic outlook. A counterfactual simulation suggests that the monetary policy implemented from the 1980s has helped to stabilise inflation and output. However, the optimal monetary policies we have derived are significantly more effective in stabilising the targeted macroeconomic variables. In particular, this paper presents an optimal monetary policy rule that can stabilise exchange rate movements as effectively as realised monetary policy, whilst more effectively stabilising output and inflation volatility.

The paper proceeds as follows. Section 2 outlines the small open DSGE model proposed by Lubik and Schorfheide (2007), while Section 3 describes the MSRE versions of this model. Section 4 describes the solution method for the MSRE model and in Section 5 we present the data and priors used for the model estimation. Section 6 discusses the Bayesian estimates of the constant parameter model and the MSRE models. The analysis of the optimal monetary policy rules is presented in Section 7. Finally, Section 8 concludes.

2. A small open economy DSGE model

We utilise the small open economy DSGE model proposed by Lubik and Schorfheide (LS, 2007),[1] which, in turn, is a simplified version of the model developed by Gali and Monacelli (2005). LS estimated the model with constant parameters, to investigate whether the UK monetary authorities had responded to exchange rate fluctuations. Our paper focuses on identifying parameter instability in this model and this allows us to capture changes in the structure of the UK economy, and derive optimal monetary policy that can effectively stabilise a number of targeted variables, such as inflation, output and exchange rate movements. The model proposed by LS consists of a forward-looking IS equation, a Phillips curve, an exchange rate equation and a monetary policy rule.

[1] Refer to LS (2007) and Del Negro and Schorfheide (2009) for the derivation of the reduced form equations used. Despite some statistical evidence suggesting that this model may contain some misspecification, the impulse response functions it provides are consistent with those implied by a loosely parameterised DSGE-VAR model (Del Negro and Schorfheide, 2009).

The IS equation is derived from a consumption Euler equation, where consumption is replaced by domestic output using the domestic market clearing condition:

$$y_t = \mathrm{E}_t y_{t+1} - (\tau+\lambda)(R_t - \mathrm{E}_t \pi_{t+1} - \mathrm{E}_t z_{t+1}) + \alpha(\tau+\lambda)\mathrm{E}_t \Delta q_{t+1} + \frac{\lambda}{\tau}\mathrm{E}_t \Delta y^*_{t+1}, \qquad (1)$$

where α is the import share that satisfies $0<\alpha<1$, τ is the intertemporal substitution elasticity and $\lambda = \alpha(2-\alpha)(1-\tau)$. The endogenous variables are aggregate output, y_t, the CPI inflation rate, π_t, q_t is the observable terms of trade, y^*_t is exogenous world output and z_t is the growth rate of global technology process, A_t.

Domestic firms are subject to Calvo-type price setting. A fraction of firms ($1-\theta$) can set prices optimally while the remaining θ firms update their prices by the steady-state inflation rate. Optimal price setting by domestic firms leads to the following Phillips curve

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \alpha\beta \mathrm{E}_t \Delta q_{t+1} - \alpha \Delta q_t + \frac{\kappa}{\tau+\lambda} y_t + \frac{\kappa\lambda}{\tau(\tau+\lambda)} y^*_t, \qquad (2)$$

where β is the discount factor and $\kappa = (1-\theta)(1-\theta\beta)/\theta$ is a "price stickiness" parameter.

Nominal exchange rate depreciation is introduced into the model through the definition of CPI inflation, and the assumption that PPP holds for individual goods at all times. The exchange rate equation is given by

$$\Delta e_t = \pi_t - (1-\alpha)\Delta q_t - \pi^*_t, \qquad (3)$$

where e_t is the nominal exchange rate and π^*_t represents exogenous world inflation.

The model is closed by specifying monetary policy which is conducted according to a generalised Taylor rule. The central bank sets the interest rates in response to movements in CPI inflation, output growth and nominal exchange rate depreciation. This policy rule is specified as follows

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left[\psi_1 \pi_t + \psi_2 (\Delta y_t + z_t) + \psi_3 \Delta e_t\right] + \varepsilon_t^R, \quad \varepsilon_t^R \sim NID(0, \sigma_R^2). \quad (4)$$

We assume that policy coefficients $\psi_1, \psi_2, \psi_3 \geq 0$, and that the smoothing term in the rule is: $0 < \rho_R < 1$.

Exogenous variables $\{z_t, \Delta q_t, y_t^*, \pi_t^*\}$ in the model evolve as AR(1) processes, such that[2]

$$\Delta q_t = \rho_q \Delta q_{t-1} + \varepsilon_t^q, \quad \varepsilon_t^q \sim NID(0, \sigma_q^2), \quad (5)$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \quad \varepsilon_t^z \sim NID(0, \sigma_z^2), \quad (6)$$

$$y_t^* = \rho_{y^*} y_{t-1}^* + \varepsilon_t^{y^*}, \quad \varepsilon_t^{y^*} \sim NID(0, \sigma_{y^*}^2), \quad (7)$$

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \varepsilon_t^{\pi^*}, \quad \varepsilon_t^{\pi^*} \sim NID(0, \sigma_{\pi^*}^2), \quad (8)$$

where z_t, y_t^* and π_t^* are modelled as latent variables. [3]

The small open economy model outlined above can be written as a linear rational expectation system of the form,

$$\Gamma_0(\boldsymbol{\theta}) X_t = \Gamma_1(\boldsymbol{\theta}) X_{t-1} + \Psi(\boldsymbol{\theta}) Z_t + \Pi(\boldsymbol{\theta}) \eta_t, \quad (9)$$

where $X_t = \left[y_t, \pi_t, R_t, \Delta e_t, \Delta q_t, z_t, y_t^*, \pi_t^*, \mathrm{E}_t y_{t+1}, \mathrm{E}_t \pi_{t+1}\right]'$ is a vector of ten state variables, which includes eight predetermined variables and two expectation terms. The vector Z_t stacks exogenous shocks and η_t is composed of rational expectations

[2] IS equation (1) and a Phillips curve (2) are derived under the assumptions of complete asset markets and perfect risk sharing. This implies that $(\tau + \theta)\Delta q_t = \Delta y_t^* - \Delta y_t$. This differs from equation (5). However, as discussed in LS (2007), when the terms of trade process are modelled endogenously, it puts tight cross-equation restrictions on the model. Therefore, consistent with LS (2007), we choose to model this variable as an exogenous AR(1) process.

[3] As noted by LS (2007), when y_t^* and π_t^* are modelled as latent processes, this relaxes the potentially tight cross-equation restrictions embedded in the model. In particular, π_t^* incorporates deviations from PPP.

forecast errors. Γ_0, Γ_1, Ψ and Π are matrices and $\boldsymbol{\theta}$ collects the structural parameters of the model,

$$\boldsymbol{\theta} = \left\{\psi_1, \psi_2, \psi_3, \rho_r, \alpha, \tau, \kappa, \rho_z, \rho_q, \rho_{\pi*}, \rho_{y*}, \sigma_R, \sigma_z, \sigma_q, \sigma_{\pi*}, \sigma_{y*}\right\}.$$

3. Markov-Switching versions of the model

To identify potential structural changes in the UK economy, we estimate different MSRE versions of the above model, all of which allow different structural parameters to evolve according to a two-state Markov-switching process. In particular, three MSRE models are considered. In the first MSRE model, we allow the monetary policy parameters (ρ_R, ψ_1, ψ_2, ψ_3) to be subject to regime shifts. This is to identify potential changes in the UK monetary policy that may due to a number major events, such as joining and leaving the Exchange Rate Mechanism (ERM), introducing inflation targeting in 1992 and the move to an independent Bank of England in 1997.

In the second MSRE model, we only consider the price stickiness parameter, κ, to be a two-state Markov-switching parameter. This is motivated by the idea that firms have more incentive to update prices frequently when the economy faces uncertainty, such as during high inflation periods and recessions.

Third, we identify the 'good luck' factor for the UK economy. The good luck factor is normally presented in the form of small economic disturbances and allowing Markov-switching on the standard deviation of shocks, i.e., $\sigma_R, \sigma_q, \sigma_{y*}, \sigma_{\pi*}, \sigma_z$, can capture the changes in volatility over time.[4]

[4] We also examine changes in the persistence of exogenous shocks. However, the model is not supported by the data.

4. Solving and Estimating Markov-Switching DSGE models

Combining a Markov-switching framework with the assumption of rational expectations in DSGE models is not straightforward. Research into how to identify a full set of solutions to a MSRE model, and what conditions guarantee a unique solution, is ongoing. Recent papers, such as Svensson and Williams (2007a), Davig and Leeper (2007) and Farmer, et al (2008, 2009, 2010b), attempt to incorporate Markov-switching parameters into DSGE models. Most provide theoretical discussion rather than empirical estimation. Davig and Leeper (2007) introduce Markov-switching parameters into a monetary policy rule. They show that some solutions to the MSRE model have a linear representation. They also define the conditions that ensure the solution to the linear representation is unique. However, Farmer, et al (2010a) prove that the conditions outlined in Davig and Leeper (2007) do not apply to the original MSRE model. Instead, Farmer, et al (2008) propose an alternative method that expands the state-space of a MSRE model to an equivalent model with state-invariant parameters. They then define a class of minimal state variable (MSV) solutions (McCallum, 1983) for the latter and prove that any MSV solution is also a solution to the original MSRE model. Farmer, et al (2008) point out that the MSV solution is the most interesting to study as it is often stable under real time learning.

Compared to Svensson and Williams (2007a) and Davig and Leeper (2007), the significant advantage of Farmer, et al (2008) is that it provides a test to indicate the existence and uniqueness of a solution to the extended state-invariant model. In a further paper, Farmer, et al (2010b) move beyond their previous algorithm, that only produces one MSV solution, to identify a full set of MSV equilibria. However, Farmer, et al (2010b) do not offer a clear instruction on how to choose between different solutions. The problem of determinacy/indeterminacy in a MSRE model is another complicated matter that has not yet been solved. Davig and Leeper (2007) and Farmer, et al (2009) made significant contributions to this issue. However, their methods only apply to purely forward-looking models and they do not suit the model used in this study that has lagged interest rates in the monetary policy rule.

Given the above caveats, we adapt the algorithm outlined in Farmer, et al (2008). We find it works well for our model as the iterative procedure converges quickly in all cases. The Markov-Switching models outlined in Section 3 can be recast in the following MSRE system:

$$\Gamma_0\left(\boldsymbol{\theta}_{S_t}\right)X_t = \Gamma_1\left(\boldsymbol{\theta}_{S_t}\right)X_{t-1} + \Psi\left(\boldsymbol{\theta}_{S_t}\right)Z_t + \Pi\left(\boldsymbol{\theta}_{S_t}\right)\eta_t. \tag{10}$$

Compared to its constant variant in equation (9), some of the structural model parameters change depending on the unobserved state variable, S_t, that follows a two-state Markov process with the following transition probabilities:

$$\Pr\left[S_t = 1 \mid S_{t-1} = 1\right] = p_{11}, \;\; \Pr\left[S_t = 2 \mid S_{t-1} = 2\right] = p_{22}.$$

Following Farmer, et al (2008), equation (10) can be written as the following model with regime-invariant parameters:

$$\overline{\Gamma}_0 X_t = \overline{\Gamma}_1 X_{t-1} + \overline{\Psi} Z_t + \overline{\Pi}\eta_t, \tag{11}$$

where $\overline{\Gamma}_0$, $\overline{\Gamma}_1$, $\overline{\Psi}$ and $\overline{\Pi}$ are matrices that are functions of structural parameters and transition probabilities. Farmer, et al (2008) define a MSV solution to equation (11) and prove that it is also a solution to the original MSRE model specified in equation (10). In the case where a unique solution is found, equation (11) can be written as a reduced AR(1) process with Markov-switching parameters: [5]

$$X_t = \Phi_1\left(\boldsymbol{\theta}_{S_t}\right)X_{t-1} + \Phi_2\left(\boldsymbol{\theta}_{S_t}\right)Z_t, \;\; Z_t \sim \mathrm{NID}\left(\mathbf{0}, \boldsymbol{\Sigma}_{S_t}\right). \tag{12}$$

For estimation, equation (12) is related to the observed variables through a measurement equation specified as:

[5] More details of this solution method can be found in Farmer, et al (2008).

$$\begin{bmatrix} \Delta GDP_t \\ INF_t \\ INT_t \\ \Delta EX_t \\ \Delta TOT_t \end{bmatrix} = \begin{bmatrix} \gamma^{(A)} + \Delta y_t + z_t \\ \pi^{(A)} + 4\pi_t \\ r^{(A)} + \pi^{(A)} + 4\gamma^{(A)} + 4R_t \\ \Delta e_t \\ \Delta q_t \end{bmatrix}. \tag{13}$$

The observed variables are output growth (ΔGDP_t), inflation (INF_t), nominal interest rates (INT_t), nominal exchange rate depreciation (ΔEX_t) and changes in the terms of trade (ΔTOT_t). The parameters, $\gamma^{(A)}, \pi^{(A)}$ and $r^{(A)}$ represent the values of output growth, inflation and interest rates when the economy is in its steady state.

We adopt the Bayesian approach to estimate the model. The posterior distribution is obtained through Bayes theorem

$$p\left(\boldsymbol{\theta}, \phi, S^T \mid Y^T\right) = \frac{p\left(Y^T \mid \boldsymbol{\theta}, \phi, S^T\right) p\left(S^T \mid \phi\right) p\left(\phi, \boldsymbol{\theta}\right)}{\int p\left(Y^T \mid \boldsymbol{\theta}, \phi, S^T\right) p\left(S^T \mid \phi\right) p\left(\phi, \boldsymbol{\theta}\right) d\left(\boldsymbol{\theta}, \phi, S^T\right)}, \tag{14}$$

where $p(\phi, \boldsymbol{\theta})$ is the prior for the structural parameters, $\boldsymbol{\theta}$, and the transition probabilities, ϕ, in the MSRE model. $p\left(S^T \mid \phi\right)$ is the prior for the latent state variables and $p\left(Y^T \mid \boldsymbol{\theta}, \phi, S^T\right)$ is the likelihood function. Since it is difficult to characterise the posterior distribution in equation (14), we follow Schorfheide (2005) who factorises the joint posterior as

$$p\left(\boldsymbol{\theta}, \phi, S^T \mid Y^T\right) = p\left(\boldsymbol{\theta}, \phi \mid Y^T\right) p\left(S^T \mid \boldsymbol{\theta}, \phi, Y^T\right). \tag{15}$$

We adopt Schorfheide's (2005) strategy that employs a random walk Metropolis-Hastings algorithm to generate draws from $p\left(\boldsymbol{\theta}, \phi \mid Y^T\right)$. Conditional on the parameter vectors $\boldsymbol{\theta}$ and ϕ, Kim's (1994) smoothing algorithm is then used to generate draws from the history of latent states, S^T. When conducting a Bayesian analysis, the likelihood function is a key element in constructing the posterior distribution. Markov-switching parameters in the state-space model mean that the standard Kalman filter cannot evaluate the likelihood function. Therefore, Kim's (1994) filter is used, which combines the Kalman Filter and the Hamilton filter, along with appropriate approximations. The approximation limits the number of states that can be carried forward in the Kalman filter iteration at each point of time. Therefore, it makes the Kalman filter operable.[6] In particular, in the Bayesian analysis, we combine the prior distribution with the approximated likelihood to obtain the posterior distribution. Sim's optimisation routine CSMINWEL is used to find the posterior mode. The inverse Hessian is then calculated at this posterior mode and is used as the covariance matrix of the proposal distribution. It is scaled to yield a target acceptance rate of 25% to 30%. We use a random walk Metropolis-Hastings algorithm to generate 200,000 draws from the posterior distribution with the first 10,000 draws discarded. Posterior means are obtained by Monte-Carlo averaging. Finally, the log marginal likelihood of each model is approximated with Geweke's (1999) modified harmonic-mean estimator which provides a coherent framework to compare non-nested models.

5. Data and model priors

5.1 Data

Our empirical analysis is based on output growth, inflation, nominal interest rates, the nominal exchange rate and changes in the terms of trade for the UK from 1975Q1 to 2010Q2. All data are seasonally adjusted and at quarterly frequencies. Output growth is the log difference of real GDP, multiplied by 100. Inflation is the log difference of CPI, scaled by 400. The nominal depreciation rate is the log difference of the effective exchange rate index, multiplied by 100. The terms of trade is computed as the relative prices of exports in terms of imports. It is then converted to log differences (scaled by 100). All data are taken from the OECD database. The data used in the estimation are plotted in Figure 1.

{Figure 1 about here}

[6] The details of Kim's (1994) filter and approximate MLE are discussed in Kim and Nelson (1999).

5.2 Choice of priors

The priors are presented in Table 1. These are set to be broadly consistent with the literature on the estimation of New Keynesian models. For example, we use comparatively loose priors for the parameters in the policy rule that are consistent with LS (2007). In addition, the slope coefficient in the Phillips curve, κ, is chosen to be consistent with the range of values typically found in the New Keynesian literature, such as Rotemberg and Woodford (1998) and Gali and Gertler (1999). As for the priors for the exogenous shocks, an AR(1) process is fitted to changes in the terms of trade to obtain priors for the Δq_t process, and priors for the technology process are set according to output growth. Due to a shortage of information about the priors for the foreign output and inflation shocks, they are set to be consistent with LS (2007). The prior means of $\gamma^{(A)}$ and $\pi^{(A)}$ are set to be roughly consistent with the average output growth rate and inflation rate during the sample period. The average real interest rate, $r^{(A)}$, is linked to the discount factor, such that $\beta = \left(1 + r^{(A)}/400\right)^{-1}$. For the switching parameters, the prior distributions are set to be broadly consistent with the estimates in the time-invariant model using two subsamples before and after 1983.[7] In order to examine the consistency of our estimation results, we start the maximisation algorithm from a number of different starting values, before conducting the Bayesian analysis. We find that the optimisation routine always converges to the same values.

{Table 1 about here}

6. Empirical Results

This section presents the Bayesian estimates of the time-invariant parameter model (Model 1) and the four MSRE models (Models 2-5), outlined in Section 3. The

[7] The sample is split in 1983 as this is the approximate point where the inflation volatility observed in the 1970s and early 1980s recedes. This way of setting priors for the switching parameters is motivated by Davig and Doh (2009), to introduce a natural ordering of regime-dependent parameters and to avoid the potential risk of 'label switching' as noted in Hamilton, Waggoner and Zha (2007).

constant parameter model is used as a benchmark, as comparing the log marginal likelihood value of this model with those from the MSRE models gives an indication of whether certain structural parameter changes are supported by the data.

The posterior means and the 90% probability intervals obtained from estimating the constant parameter model are presented in Table 2.[8] The results are generally consistent with the previous literature. In particular, we find that the monetary authorities in the UK pursue a moderately anti-inflationary monetary policy with concerns about output volatility taking precedence over exchange rate movements. A high degree of interest rate smoothing is also found. The posterior mean of κ implies that domestic firms re-optimise prices approximately every two and a half quarters. This degree of nominal price rigidity is comparable with estimates for Canada and New Zealand identified by Justiniano and Preston (2010).

{Table 2 about here}

6.1 Model 2- Markov-switching monetary policy rules

In Model 2 we allow the parameters in the monetary policy rule to shift between two regimes. As shown in Table 3, regime 1 is characterised by strong inflation targeting with the posterior mean of ψ_1 being 1.95 compared to 0.65 in regime 2. In contrast, the differences between other policy parameters (ψ_2, ψ_3, ρ_R) over the two regimes are less significant. It is important to note that, compared to the constant parameter model (Model 1), allowing for the parameter shifts in the monetary policy rule increases the marginal likelihood value as shown in Table 7.

The smoothed and filtered probabilities of regime 2 (where there is less inflation targeting) are plotted in Panel 1 of Figure 2. The result suggests that the switching between monetary policy regimes results in more aggressive inflation targeting after the Great Inflation period in the 1970s. It is also interesting to note that during the most recent recession, the interest rate rule shifts back to regime 2. This implies that during the recession policy makers have placed less emphasis on inflation,

[8] Habit formation in consumption is introduced to the LS model, as a significant autocorrelation pattern is found in the residuals of the IS equation (1) when the LS model is estimated.

perhaps due to significant uncertainty about the broader economic outlook and the introduction of unorthodox policy tools such as quantitative easing.

{Table 3 about here}

{Figure 2 about here}

6.2 Model 3 - Markov-switching price rigidity

Model 3 investigates whether the price stickiness parameter, κ, shifts over time. To identify the timing of shifts in κ, we restrict other structural parameters to be time-invariant. The mean estimates of κ change significantly over the two regimes. κ is 0.07 in regime 1, rising to 0.30 in regime 2. This indicates that prices are optimised approximately every four quarters during regime 1, but every 2 quarters during regime 2.

The filtered and smoothed probabilities of regime 2 are plotted in Panel 2 of Figure 2. It is interesting that during the Great Inflation period in the 1970s regime 2 is the dominant regime. This confirms our presumption that firms have a stronger incentive to change prices more frequently during periods of greater economic uncertainty. Again, the shift in nominal price rigidity is supported by the data.

{Table 4 about here}

6.3 Model 4 - Markov-Switching in volatility of shocks

In Model 4, shifts are only permitted in the standard deviations of exogenous shocks. This model is intended to capture the ‘good luck’ factor. The probabilities presented in Panel 3 of Figure 2 show that fewer domestic and foreign shocks affected the UK economy from the early 1990s until the recent financial crisis. The standard deviations of the nominal interest rate shock, technology growth shock and foreign output shock in the second regime are three times larger than in the first regime. The standard deviations of the foreign inflation shock and terms of trade shock also double in regime 2. There is no overlap in the confidence intervals across the two regimes.

Model 4 is more strongly supported by the marginal likelihood value compared to the time-invariant variances models.

{Table 5 about here}

6.4 Model 5 - Two Markov chains

Comparing the marginal likelihood values of the time-invariant parameter model with the MSRE models suggests that monetary policy parameters, the nominal price rigidity parameter and volatility of shocks are all subject to regime shifts. Whilst it would be preferable to have one model that incorporates three independent Markov chains to govern these changes, this would significantly increase the complexity of the estimation. However, the regime probabilities plotted in Figure 2 show that changes in the monetary policy parameters and the nominal price rigidity parameter occurred at approximately the same time whilst changes in the volatility of shocks occurred separately. Therefore, two independent Markov chains are included in the final MSRE model (Model 5). The changes in the standard deviations of exogenous shocks are dependent on an unobserved state variable, s_t, that has the transition probabilities:

$$\Pr\left[s_t = 1 \mid s_{t-1} = 1\right] = Q_{11}, \quad \Pr\left[s_t = 2 \mid s_{t-1} = 2\right] = Q_{22}.$$

The shifts in the monetary policy parameters and the nominal rigidity parameter are dependent upon the state variable, S_t, with the following transition probabilities

$$\Pr\left[S_t = 1 \mid S_{t-1} = 1\right] = P_{11}, \quad \Pr\left[S_t = 2 \mid S_{t-1} = 2\right] = P_{22}.$$

Since our final MSRE model has two independent Markov-chains, it results in a four state transition matrix given by

$$P^* = Q \otimes P. \tag{16}$$

s_t, S_t and s_{t-1}, S_{t-1} are tracked in the state-space representation. This implies that $4^2 = 16$ states are carried at each iteration.[9]

Model 5 yields the largest marginal likelihood value amongst all the models analysed. It provides the best fit to the UK data, and is therefore used in Section 7 to derive the optimal monetary policy rule. The Markov-switching parameters obtained from Model 5 are generally in line with the corresponding parameters estimated from Models 2-4. More aggressive inflation targeting is again identified in regime 1, with the posterior mean of ψ_1 being 2.10 compared to 0.78 in regime 2. In addition, as implied by the posterior means of κ across the two regimes, the domestic firms re-optimise prices approximately every four quarters during regime 1, but every two and a half quarters during regime 2.

The filtered and smoothed probabilities plotted in Panel 1 of Figure 3, again indicate that regime 2 prevails from the mid 1970s to early 1980s, a period characterised by a less aggressive inflation targeting and lower price rigidity. In addition, regime 2 reappears during the recent financial crisis. The filtered and smoothed probabilities of the high volatility regime, plotted in Panel 2 of Figure 3, again suggest that less domestic and foreign shocks have hit the economy since the early 1990s until the recent financial crisis.

{Table 6 about here}

{Figure 3 about here}

7. Realised and optimal monetary policy

In this section, we move away from the empirical estimation, to optimal monetary policy design based on our best fitting model, Model 5. The posterior means of all the structural parameters of this model, other than the monetary policy parameters, are used to derive optimal monetary policies within a generalised Taylor rule framework given by

[9] The choice of sixteen states is due to the consideration that the marginal increasing in efficiency from carrying more states is small and likely does not exceed the marginal cost of increasing computation time (Kim and Nelson, 1998).

$$R_t = \rho_R R_{t-1} + \psi_\pi \pi_t + \psi_y \left(\Delta y_t + z_t\right) + \psi_e \Delta e_t . \tag{17}$$

The optimal monetary policy specified in equation (17) responds to the same set of variables as the estimated rule in equation (4). The optimal monetary policy parameters are chosen to minimise the following intertemporal loss function at period t:

$$W = \mathrm{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} L_{t+\tau} , \tag{18}$$

where $0 < \beta \leq 1$ coincides with the household's discount factor and the period loss function is given by

$$L_t = Y_t' \Lambda Y_t , \tag{19}$$

where $Y_t \equiv \left\{\pi_t, y_t, \Delta R_t, \Delta e_t\right\}'$ is a vector of targeted variables. Here, we consider an unconditional welfare loss function where β goes to unity. The weighting matrix Λ is a diagonal matrix with the diagonal elements of $\left(1, \Lambda_y, \Lambda_{\Delta R}, \Lambda_{\Delta e}\right)$. These weights determine the relative priority given to each of the targeted variables. We consider a range of different weights on the variances of inflation, output, interest rates and exchange rates. The structural parameters of Model 5, used to derive optimal monetary policy rules, contain Markov-switching parameters for nominal price rigidity and standard deviations of exogenous shocks. As such, we use the Markov-Jump-Linear-Quadratic (MJLQ) system to derive the optimal monetary policy rules. This has been popularised in a number of papers, including Blake and Zampolli (2006) and Svensson and Williams (2007a, 2007b, 2008) and can accommodate the Markov-switching parameters in the MSRE model to derive optimal monetary policy rules. In particular, we adopt the MJLQ model proposed by Svensson and Williams (2007a) to derive the optimal monetary policy.[10] Using this algorithm we derive optimal monetary policy rules specified as the generalised Taylor rule using a number of weights, as illustrated in the following subsections.

7.1. Unconditional standard deviations

Initially, we restrict the weights on exchange rate volatility to zero, i.e. $\Lambda_{\Delta e} = 0$, and allow the other weights $\left(\Lambda_y, \Lambda_{\Delta R}\right)$ to vary over a fine grid on the unit square. We calculate the optimal monetary policy rule and the unconditional variances of the targeted variables for each set of weights used. The grey lines at the bottom of Figure 4 are the output volatility/inflation volatility frontiers for the optimal rules derived under a subset of weights. These are obtained under the assumption that $\Lambda_{\Delta R} = 0.2$, 0.5 or 0.9, while Λ_y ranges from 10^{-5} to unity. The frontiers are convex, meaning that the central bank lowers inflation at the expense of raising output volatility. It is interesting to note that when $\Lambda_{\Delta R}$ increases, the output volatility/inflation volatility frontier moves towards the north-east corner of Figure 4. We also note that the realised policy rule estimated from Model 5 lies in the far north-east corner of Figure 4. This suggests it is extremely suboptimal in stabilising output and inflation compared to the optimal rules derived under the assumption that zero weight is put on exchange rate variation.

As a counterfactual we also considered the impact of the realised monetary policy rule in regime 1 (characterised by more aggressive inflation targeting) that was adopted from the early 1980s onwards being used for the entire sample period. The results show that applying the regime 1 policy monetary policy rule to the whole sample more effectively stabilises inflation and output volatility compared to the realised policy rule. This implies that at least some of the observed business cycle moderation reflects the adoption of a more effective monetary policy. However, as illustrated in Figure 4, although this counterfactual significantly reduces inflation

[10] The detailed algorithm used to derive optimal monetary policy rules are described in Svensson and Williams (2007a). We focus on the scenario where no learning occurs and the central bank and private agents can observe the different regimes of the economy.

volatility, compared to the realised rule, it is still considerably less effective than the optimal rules.

{Figure 4 about here}

It should be noted, however, that the realised monetary policy rule estimated from Model 5 produces an exchange rate variance of 3.79. This is smaller than the average exchange rate variance of 4.15 produced by the optimal rules derived under the assumption of zero weight being placed on exchange rate movements. This suggests that policy makers in the UK were concerned with exchange rate volatility.

A key question is therefore whether we can have an optimal monetary policy rule that achieves the same level of exchange rate volatility as the realised rule, whilst keeping inflation and output stable. To do so, we set the weight on the exchange rate variance ($\Lambda_{\Delta e}$) to 0.1, whilst again varying the weights on the output and interest rate variances $\left(\Lambda_y, \Lambda_{\Delta R}\right)$ over a fine grid on the unit square. The black lines in Figure 4 are the output volatility/inflation volatility frontiers for the optimal rules derived under these assumptions. We find that putting a smaller weight on controlling exchange rate volatility sharply increases output and inflation variance. Nevertheless, it is still possible to obtain a number of optimised rules that can achieve better policy outcomes than the realised rule. For instance, we present an optimised simple rule derived under the assumption that $\Lambda_y = 0.2$, $\Lambda_{\Delta R} = 0.6$ and $\Lambda_{\Delta e} = 0.1$ in Table 8. It generates the same level of exchange rate volatility as the realised rule, but successfully reduces output and inflation volatility, as shown in Table 9. Compared to the realised monetary policy rule, this optimal monetary rule has a higher degree of interest rate smoothing, and is more responsive to fluctuations in output, inflation and exchange rates across all regimes.

{Table 8 about here}

{Table 9 about here}

7.2. Impulse response functions

To further understand the dynamics of the model under different monetary policies, we simulate the unconditional impulse responses of the four targeted variables: output, inflation, the interest rate and exchange rate to exogenous shocks in technology, the terms of trade, foreign inflation and foreign output. We conduct 10,000 simulations of 12 quarters each, and plot the median responses of these targeted variables. The solid lines in Figure 5 plot the impulse responses of these targeted variables under the realised monetary policy rule. The dashed lines show the impulse responses obtained under the optimal rule used in Table 8.

{Figure 5 about here}

The model presents similar dynamics under both the optimal and realised monetary policy rules: a positive technology shock is observed to raise output growth permanently, lower inflation, increase the interest rate, and as a result, appreciate the currency. An improvement in the terms of trade raises output and lowers inflation via a nominal exchange rate appreciation. The world inflation shock has a direct effect on currency appreciation, and indirectly increases inflation if monetary policy reacts to changes in the exchange rate. Finally, the foreign output shock appears to lower domestic output; it also appears to reduce domestic inflation and therefore serves to loosen monetary policy.

It should be noted, that whilst the targeted variables also exhibit similar responses to exogenous shocks under the realised and optimal monetary policy rules, the magnitude of their response is generally smaller under the latter. The exception is interest rates for which the reaction to shocks is more aggressive under the optimal rule than the realised monetary policy rule.

8. Conclusions

This paper uses a small-scale open economy DSGE model to identify underlying structural changes in the UK economy. Using Bayesian analysis, we conclude that a number of MSRE models can provide a better fit for the UK data than the constant parameter model. We find the MSRE model incorporating shifts in monetary policy parameters, the nominal price rigidity parameter and volatility of shocks (Model 5) best fits the data. It identifies the changing dynamics of the UK economy and monetary policy over the past three decades. In particular, it highlights that UK monetary policy started to more aggressively target inflation after the Great Inflation period in the 1970s. It also suggests that less emphasis has been placed on targeting inflation during the recent financial crisis.

The best fitting MSRE model, Model 5, is then used to derive optimal monetary policy in the form of a generalised Taylor rule. To our knowledge, this is the first paper that evaluates and designs UK monetary policy based on an estimated open economy MSRE model. The results can be summarised as follows. First, the counterfactual simulation whereby the regime 1 monetary policy rule was used for the whole sample period, suggests that the monetary policy used from 1980s onwards has been more effective in stabilising economic activity than the policy used in the 1970s. This indicates that at least some of the ‘Great Moderation’ is due to the adoption of a more effective monetary policy.

However, the results also suggests that both the realised monetary policy and the counterfactual appear to be suboptimal in terms of stabilising output and inflation, compared to a number of optimal monetary policy rules derived using Model 5. This implies that there is room for monetary authorities to further improve their policies. In particular, this paper presents an optimal rule that can stabilise exchange rate movements as effectively as the realised monetary policy rule, whilst keeping output and inflation more stable than both the realised monetary policy rule and the counterfactual. This optimal policy rule requires a higher degree of interest rate smoothing, and is more responsive to fluctuations in output, inflation and exchange rates across all regimes.

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Table 1: Prior Distribution

Parameters	Domain	Density	Para(1)	Para(2)	Model Spec
τ	$[0,1)$	Beta	0.50	0.20	
κ	$\mathbb{R}^+$	Gamma	0.30	0.20	Models 3 & 5
ψ_1	$\mathbb{R}^+$	Gamma	1.50	0.50	Models 2 & 5
ψ_2	$\mathbb{R}^+$	Gamma	0.25	0.15	Models 2 & 5
ψ_3	$\mathbb{R}^+$	Gamma	0.25	0.15	Models 2 & 5
ρ_R	$[0,1)$	Beta	0.50	0.25	Models 2 & 5
ρ_z	$[0,1)$	Beta	0.40	0.10	
ρ_q	$[0,1)$	Beta	0.20	0.10	
$\rho_{\pi*}$	$[0,1)$	Beta	0.80	0.10	
ρ_{y*}	$[0,1)$	Beta	0.90	0.10	
α	$[0,1)$	Beta	0.20	0.05	
h	$[0,1)$	Beta	0.50	0.25	
$r^{(A)}$	$\mathbb{R}^+$	Gamma	2.50	1.00	
$\pi^{(A)}$	$\mathbb{R}^+$	Normal	5.00	2.00	
$\gamma^{(A)}$	$\mathbb{R}^+$	Gamma	0.52	0.20	
σ_R	$\mathbb{R}^+$	Inverse Gamma	0.50	5.00	Models 4 & 5
σ_z	$\mathbb{R}^+$	Inverse Gamma	0.85	5.00	Models 4 & 5
σ_{y*}	$\mathbb{R}^+$	Inverse Gamma	1.50	5.00	Models 4 & 5
$\sigma_{\pi*}$	$\mathbb{R}^+$	Inverse Gamma	0.55	5.00	Models 4 & 5
σ_q	$\mathbb{R}^+$	Inverse Gamma	1.20	5.00	Models 4 & 5
P_{11}	$[0,1)$	Beta	0.90	0.05	
P_{22}	$[0,1)$	Beta	0.90	0.05	
Q_{11}	$[0,1)$	Beta	0.90	0.05	
Q_{22}	$[0,1)$	Beta	0.90	0.05	

Notes: (a) Para (1) and Para (2) indicate the means and the standard deviations of Beta, Gamma, and Normal distributions;

(b) s and v for the Inverse Gamma distribution, where $pIG(\sigma \mid v,s) \propto \sigma^{-v-1} e^{-vs2/2\sigma 2}$.

(c) Model Spec indicates the parameters that are allowed to switch in the MSRE model specified.

Table 2: Model 1 with the time-variant parameters

Parameter	Mean	90% interval
τ	0.381	[0.174, 0.597]
κ	0.294	[0.038, 0.621]
ψ_1	1.344	[1.061, 1.605]
ψ_2	0.606	[0.262, 0.926]
ψ_3	0.176	[0.088, 0.250]
ρ_R	0.855	[0.803, 0.905]
ρ_z	0.196	[0.060, 0.350]
ρ_q	0.082	[0.012, 0.154]
$\rho_{\pi*}$	0.272	[0.169, 0.374]
ρ_{y*}	0.838	[0.749, 0.933]
α	0.114	[0.071, 0.157]
h	0.599	[0.290, 0.933]
$r^{(A)}$	1.247	[0.576, 1.842]
$\pi^{(A)}$	5.132	[3.989, 6.240]
$\gamma^{(A)}$	0.498	[0.366, 0.629]
σ_R	0.275	[0.237, 0.311]
σ_z	0.842	[0.735, 0.941]
σ_{y*}	1.557	[0.854, 2.250]
$\sigma_{\pi*}$	3.240	[2.940, 3.544]
σ_q	1.226	[1.099, 1.345]

Note: The table reports posterior means and 90% probability interval.

Table 3: Model 2 with Markov-Switching parameters in policy rules

	Regime 1: aggressive inflation targeting		Regime 2: weak inflation targeting	
Parameter	Mean	90% interval	Mean	90% interval
τ	0.424	[0.285, 0.556]		
κ	0.342	[0.039, 0.651]		
ψ_1	1.953	[1.545, 2.331]	0.646	[0.324, 0.885]
ψ_2	0.348	[0.096, 0.605]	0.336	[0.061, 0.594]
ψ_3	0.121	[0.064, 0.180]	0.196	[0.056, 0.333]
ρ_R	0.779	[0.720, 0.842]	0.811	[0.643, 0.914]
ρ_z	0.134	[0.036, 0.226]		
ρ_q	0.090	[0.009, 0.160]		
$\rho_{\pi*}$	0.259	[0.157, 0.357]		
ρ_{y*}	0.917	[0.877, 0.957]		
α	0.093	[0.053, 0.132]		
h	0.553	[0.314, 0.799]		
$r^{(A)}$	1.330	[0.577, 2.040]		
$\pi^{(A)}$	3.762	[2.851, 4.630]		
$\gamma^{(A)}$	0.484	[0.362, 0.625]		
σ_R	0.264	[0.227, 0.303]		
σ_z	0.904	[0.813, 0.995]		
σ_{y*}	0.618	[0.371, 0.871]		
$\sigma_{\pi*}$	3.237	[2.910, 3.535]		
σ_q	1.214	[1.095, 1.332]		
P_{11}	0.953	[0.923, 0.986]		
P_{22}	0.810	[0.736, 0.887]		

Notes: (a) The table reports posterior means and 90% probability interval.
(b) Regime 1 is characterised as more aggressive targeting inflation rate than regime 2.

Table 4: Model 3 with Markov-Switching parameters in nominal rigidity

	Regime 1: High price rigidity		Regime 2: Low price rigidity	
Parameter	Mean	90% interval	Mean	90% interval
τ	0.261	[0.151,0.373]		
κ	0.068	[0.026,0.113]	0.304	[0.129,0.472]
ψ_1	1.227	[0.962, 1.468]		
ψ_2	0.863	[0.460, 1.299]		
ψ_3	0.176	[0.081, 0.263]		
ρ_R	0.889	[0.858, 0.920]		
ρ_z	0.217	[0.082, 0.352]		
ρ_q	0.086	[0.010, 0.157]		
ρ_{π^*}	0.250	[0.151, 0.348]		
ρ_{y^*}	0.857	[0.800,0.913]		
α	0.091	[0.060, 0.122]		
h	0.617	[0.364, 0.880]		
$r^{(A)}$	1.220	[0.559, 1.866]		
$\pi^{(A)}$	4.397	[3.476, 5.429]		
$\gamma^{(A)}$	0.506	[0.366, 0.643]		
σ_R	0.260	[0.232, 0.287]		
σ_z	0.856	[0.769, 0.942]		
σ_{y^*}	0.840	[0.427, 1.251]		
σ_{π^*}	3.238	[2.935, 3.553]		
σ_q	1.214	[1.093, 1.339]		
P_{11}	0.925	[0.881, 0.971]		
P_{22}	0.756	[0.668, 0.842]		

Notes: (a) The table reports posterior means and 90% probability interval.
(b) Regime 1 is characterised as a high level of nominal price rigidity compared to Regime 2.

Table5: Model 4 with Markov-switching in standard deviations of shocks

	Regime 1: Low volatility		Regime 2: High volatility	
Parameter	Mean	90% interval	Mean	90% interval
τ	0.482	[0.287,0.646]		
κ	0.284	[0.037,0.629]		
ψ_1	1.750	[1.274, 2.191]		
ψ_2	0.882	[0.470, 1.304]		
ψ_3	0.138	[0.067, 0.211]		
ρ_R	0.847	[0.804,0.892]		
ρ_z	0.312	[0.157, 0.466]		
ρ_q	0.095	[0.015, 0.179]		
ρ_{π^*}	0.217	[0.120, 0.312]		
ρ_{y^*}	0.905	[0.828, 0.992]		
α	0.100	[0.054, 0.144]		
h	0.426	[0.167, 0.668]		
$r^{(A)}$	1.227	[0.441, 1.898]		
$\pi^{(A)}$	4.423	[3.419, 5.415]		
$\gamma^{(A)}$	0.576	[0.460, 0.688]		
σ_R	0.187	[0.159, 0.215]	0.558	[0.404, 0.703]
σ_z	0.474	[0.399, 0.548]	1.331	[1.100, 1.566]
σ_{y^*}	0.859	[0.567, 1.172]	2.615	[1.599, 3.590]
σ_{π^*}	2.344	[1.999, 2.692]	4.395	[3.935, 4.993]
σ_q	0.838	[0.728, 0.955]	1.769	[1.462, 2.090]
P_{11}	0.917	[0.878, 0.961]		
P_{22}	0.817	[0.750, 0.886]		

Notes: (a) The table reports posterior means and 90% probability interval.
(b) Regime 1 is characterised as low volatility compared to Regime 2.

Table 6: Model 5 with Markov-switching in standard deviation of shocks, nominal rigidity and monetary polices parameters

	Regime 1		Regime 2	
Parameter	Mean	90% interval	Mean	90% interval
τ	0.287	[0.171, 0.387]		
κ	0.096	[0.020,0.173]	0.247	[0.085, 0.441]
ψ_1	2.102	[1.674,2.577]	0.783	[0.377, 1.215]
ψ_2	0.447	[0.105, 0.757]	0.354	[0.052, 0.646]
ψ_3	0.116	[0.036,0.188]	0.217	[0.055, 0.379]
ρ_R	0.858	[0.801, 0.921]	0.890	[0.810, 0.966]
ρ_z	0.355	[0.220,0.497]		
$\rho_{\pi*}$	0.176	[0.071, 0.278]		
ρ_{y*}	0.814	[0.754, 0.877]		
ρ_q	0.089	[0.012,0.162]		
$r^{(A)}$	1.271	[0.596, 1.896]		
$\pi^{(A)}$	3.759	[3.113, 4.402]		
$\gamma^{(A)}$	0.574	[0.456, 0.687]		
α	0.078	[0.044,0.108]		
h	0.659	[0.413, 0.895]		
σ_R	0.172	[0.149, 0.197]	0.411	[0.323, 0.489]
σ_z	0.451	[0.378, 0.523]	1.327	[1.075, 1.576]
σ_{y*}	0.963	[0.628, 1.275]	2.136	[1.386, 2.877]
$\sigma_{\pi*}$	2.325	[1.988, 2.660]	4.461	[4.011, 5.000]
σ_q	0.825	[0.719, 0.931]	1.788	[1.456, 2.106]
P_{11}	0.900	[0.853, 0.948]		
P_{22}	0.781	[0.712, 0.861]		
Q_{11}	0.923	[0.876, 0.970]		
Q_{22}	0.815	[0.741, 0.888]		

Notes: (a) The table reports posterior means and 90% probability interval.
(b) Two Markov chains are included in Model 5, with one governing the shifts of monetary policy parameters and nominal price rigidity and the other governs standard deviations of shocks.
(c) P_{ii} is the transition probability for the first Markov-chain, whilst Q_{ii} is the transition probability for the second Markov-chain.

Table 7: Log marginal likelihood values

Model 1:	Constant parameter model	-1333.450
Model 2:	Markov-switching monetary policy rule parameters	-1330.321
Model 3:	Markov-switching in the price stickiness parameter	-1332.370
Model 4:	Markov-switching in volatility of shocks	-1263.371
Model 5:	The model with two Markov chains	-1253.290

Note: the log marginal likelihood for each model is presented in the table, which provides a coherent framework to compare non-nested models.

Table 8: Realised and optimal monetary policy rules

Realised policy rule				
	R_{t-1}	$\Delta y_t + z_t$	π_t	Δe_t
regime 1	0.858	0.063	0.298	0.016
regime 2	0.890	0.039	0.086	0.024
Optimal monetary policy rule				
	R_{t-1}	$\Delta y_t + z_t$	π_t	Δe_t
regime 1	0.999	0.114	0.839	0.064
regime 2	0.917	0.097	0.638	0.069
regime 3	0.999	0.080	0.700	0.055
regime 4	0.954	0.069	0.534	0.059

Notes: (a) The realised rule in the upper panel of this table is obtained from estimating Model 5.
(b) The monetary policy parameters on output, inflation and exchange rates are calculated as $(1-\rho_R)\psi_i, i=1,2,3$.
(c) The optimal monetary policy rule presented in the lower panel is derived under the assumption that $\Lambda_{\Delta e}=0.1$, $\Lambda_y=0.2$ and $\Lambda_{\Delta R}=0.6$. Regime 1 corresponds to low volatility of shocks and high degree of price rigidity ($\kappa=0.096$); Regime 2 corresponds to low volatility of shocks and a low degree of price rigidity ($\kappa=0.247$); Regime 3 is the opposite scenario to regime 2 and regime 4 is the opposite scenario to regime 1.

Table 9: Unconditional variances

	output	inflation	interest-rate	exchange rate
Realised policy	0.563	0.048	0.027	3.795
Optimal rule	0.421	0.013	0.028	3.795
Regime1 rule	0.504	0.026	0.032	3.976

Note: (a) Under different monetary policies, unconditional variances of the four targeting variables are presented in this table.
(b) Realised policy rule is obtained from estimating Model 5. Regime 1 rule indicates the scenario that the realised regime 1 monetary policy rule (aggressive inflation targeting) is used for the entire sample period.
(c)The optimal rule is derived under the assumption that $\Lambda_{\Delta e}=0.1$, $\Lambda_y=0.2$ and $\Lambda_{\Delta R}=0.6$. It is presented in the lower panel of Table 8.

Figure 1: data

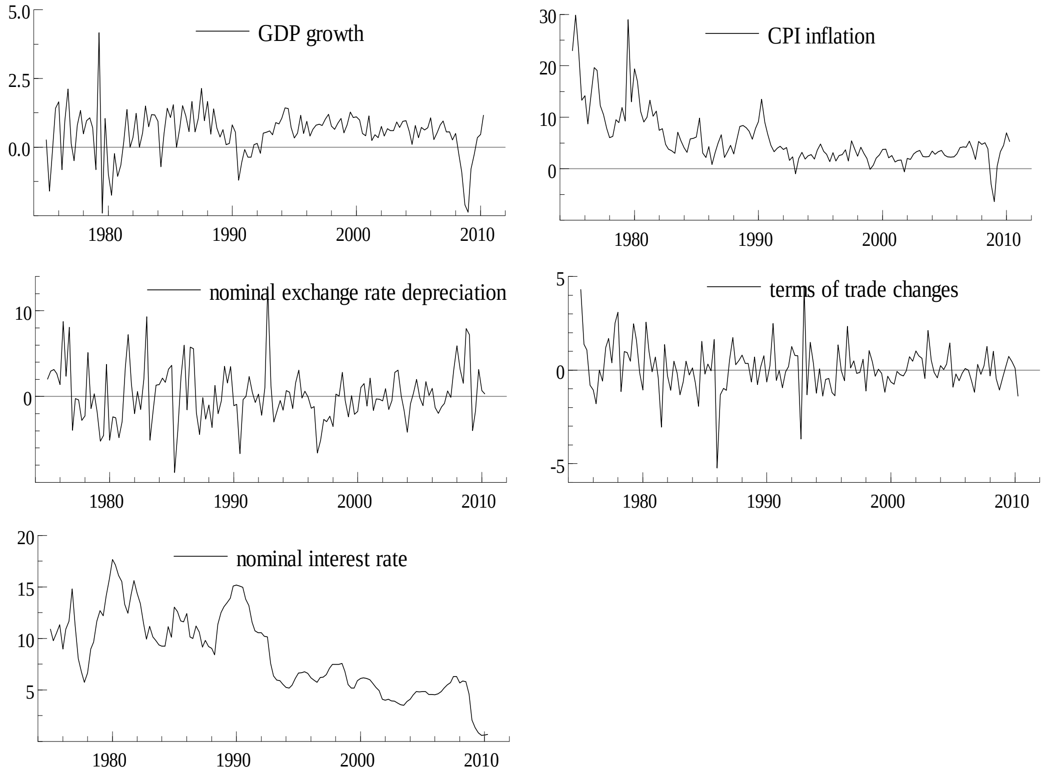


Figure 2: filtered and smoothed probabilities of regime 2

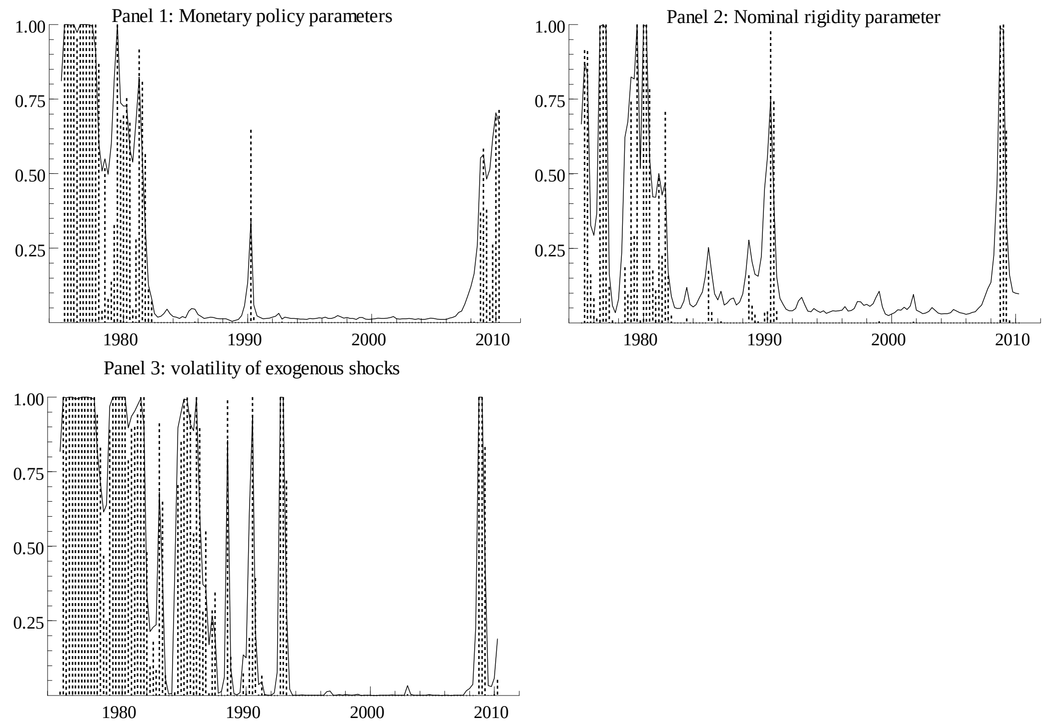


Figure 3: filtered and smoothed probabilities of regime 2 for Model 5

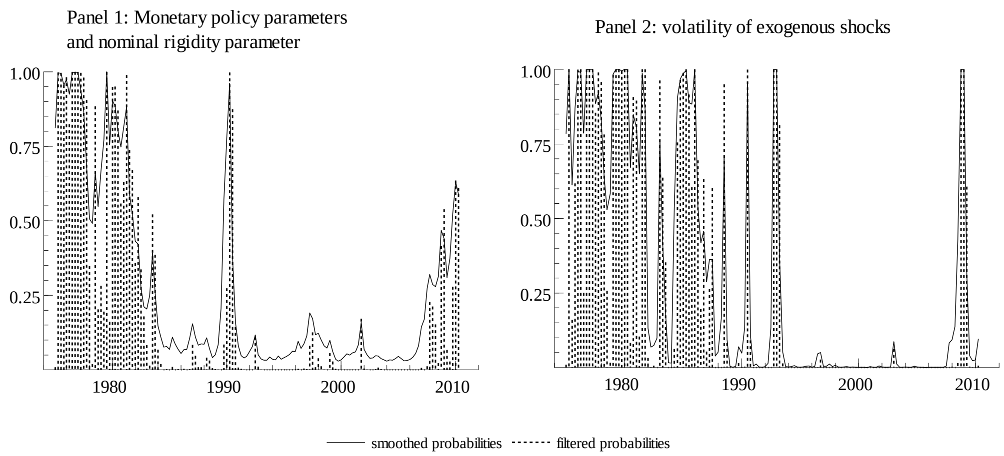
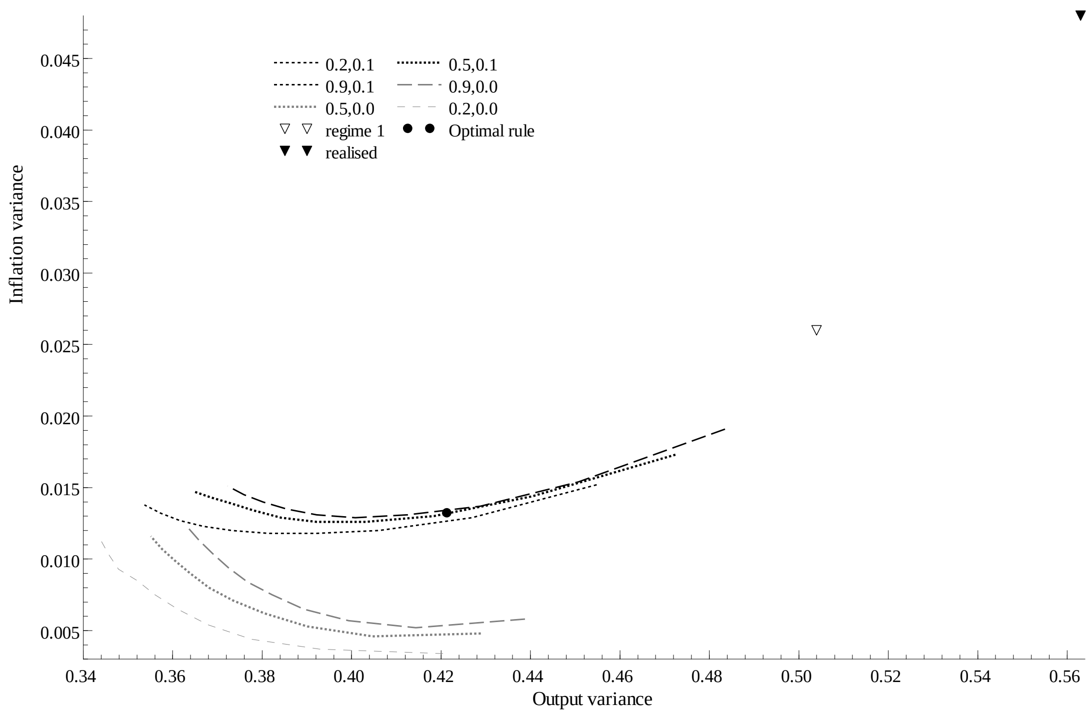
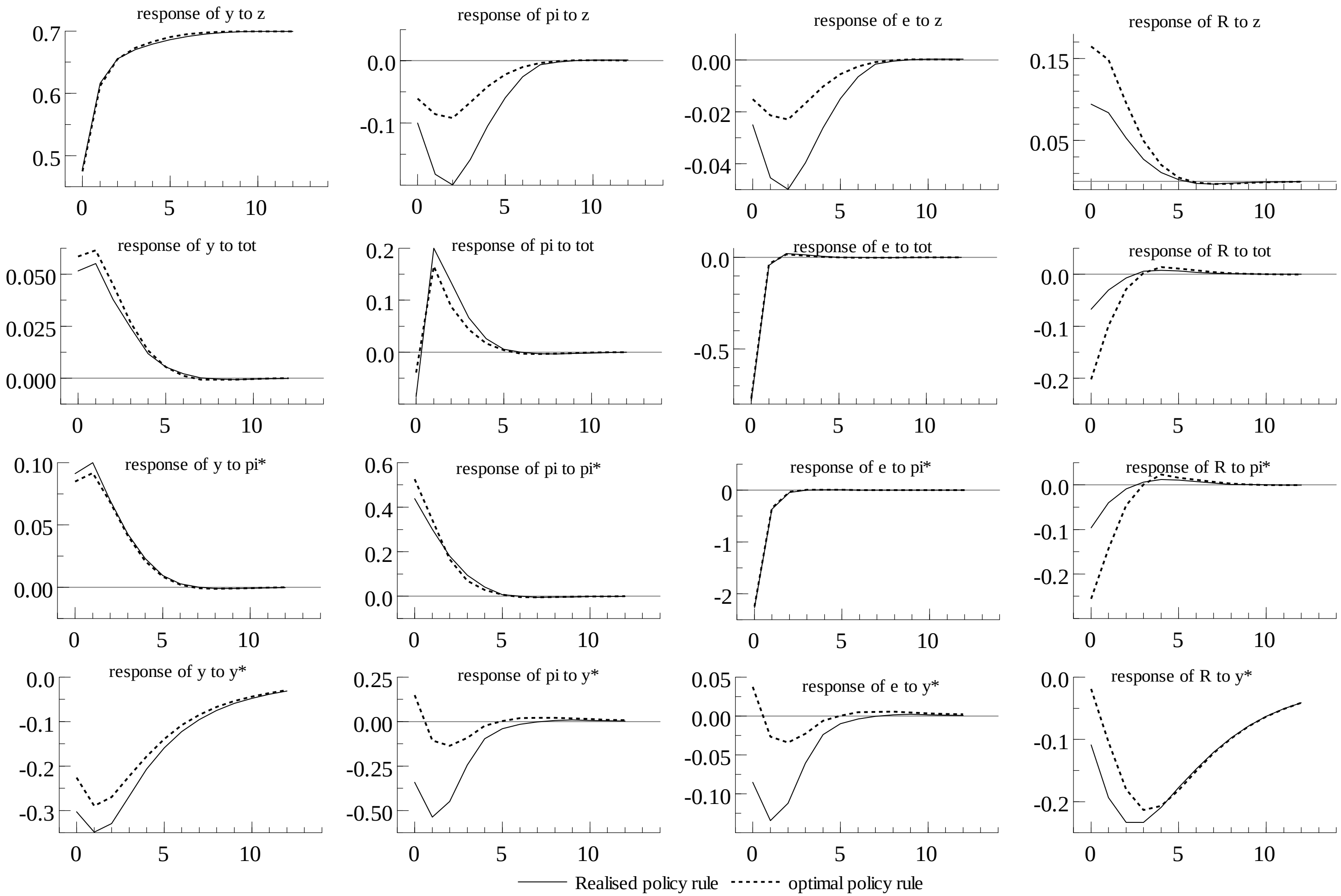


Figure 4: Output/inflation volatility frontiers for optimised simple rules



Notes: (a) The frontier is derived under the assumption of weights on interest rate and exchange rate at the values indicated in the legend, whilst allowing the weight on output to range from 10^{-5} to 1.
(b) Number in legend: the first number is the weight on the interest rate variance, while the second number is the weight put on exchange rate variance.
(c) Inverted solid triangle denotes the realised monetary policy rule from estimation Model 5, whilst the inverted hollow triangle is the realised regime 1 policy being used for the whole sample period. The dot marks the optimal rule derived under the assumption that weights on interest rate, exchange rate and output volatilities are 0.6, 0.1 and 0.2 respectively.

Figure 5: Impulse responses of targeting variables under the realised and optimal monetary policy rules



Notes**:** unconditional impulse responses to shocks under realised and optimal monetary policy rules are plotted in this figure. Solid line in each panel indicates median responses under realised monetary policy rule, while the dashed line indicates the median responses under optimal monetary policy rule.