# Monetary and Fiscal Policy under Deep Habits<sup>\*</sup>

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#### Abstract

Recent work on optimal policy in sticky price models suggests that demand management through fiscal policy adds little to optimal monetary policy. We explore this consensus assignment in an economy subject to 'deep' habits at the level of individual goods where the counter-cyclicality of mark-ups this implies can result in government spending crowding-in private consumption in the short run. We explore the robustness of this mechanism to the existence of price discrimination in the supply of goods to the public and private sectors. We then describe optimal monetary and fiscal policy in our New Keynesian economy subject to the additional externality of deep habits and explore the ability of simple (but potentially nonlinear) policy rules to mimic fully optimal policy.

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### 1 Introduction

Recent work looking at optimal monetary and fiscal policy in sticky-price New Keynesian models (see, for example, Schmitt-Grohe and Uribe (2004a) and Benigno and Woodford (2003)) typically finds that fiscal policy should be largely devoted to ensuring fiscal solvency, while monetary policy plays a demand management role.<sup>1</sup> In the context of a benchmark sticky wage and price New Keynesian economy, Eser, Leith, and Wren-Lewis (2009) demonstrate analytically that the government spending gap should always be zero even if nominal frictions imply that it is optimal for monetary policy to support

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<sup>&</sup>lt;sup>1</sup>For a discussion of this consensus assignment in the light of these and other papers, see Kirsanova, Leith, and Wren-Lewis (2009).

a non-zero output gap.<sup>2</sup> In other words, provided there are no constraints on monetary policy, demand management through changes in government spending relative to its efficient level contributes little or nothing to the stabilisation problem in a benchmark New Keynesian economy. The current paper explores the robustness of this result in the context of a New Keynesian model of monetary and fiscal policy where households possess 'deep habits' as in Ravn, Schmitt-Grohe, and Uribe (2006), where the habits are formed at the level of individual goods rather than the households' overall level of consumption ('superficial' habits).

The introduction of deep habits to the benchmark New Keynesian model is important in two key respects. Firstly, when habits exist at the level of individual goods it implies that price setters face dynamic demand functions such that pricing decisions are dynamic and mark-ups become countercyclical in line with the empirical evidence. The counter-cyclicality of mark-up behaviour in turn implies that consumption and wages can respond positively to positive government spending shocks, in contrast to models without deep habits where government spending typically crowds out private consumption. In other words, the fiscal policy transmission mechanism can be quite different from that when habits are either absent or superficial, and there are a number of empirical studies which argue that such correlations are more consistent with the data -Ravn, Schmitt-Grohe, Uribe, and Uuskula (2008) review this empirical evidence and argue that deep habits are an effective mechanism for capturing this feature of the data.<sup>3</sup> Secondly, the habits externality, whereby households do not take account of the impact of their consumption decisions on the welfare of others, implies that there is a significant additional distortion in the economy beyond those associated with monopolistic competition and nominal inertia. As a result, there may be a potential role for fiscal stabilisation policy alongside monetary policy in such an economy.

In order to explore the relative contribution of monetary and fiscal policy to macroeconomic and fiscal stabilisation, we construct a sticky-price New Keynesian economy along the lines of Benigno and Woodford (2003), where households provide labour to imperfectly competitive firms who are only able to change prices after a random interval of time. As in Benigno and Woodford (2003), taxes are distortionary. We begin exploring the fiscal policy transmission mechanism by varying the relative extent of habits in private and public goods consumption and by allowing firms to discriminate between pricing for private and public goods. In light of these results, we then assess the ability of fiscal policy to stabilise an economy with price-stickiness, monopolistic competition and deep habits in private and public consumption. In doing so, unlike Benigno and Woodford (2003), we also allow government spending to be used as a policy instrument,

 $<sup>^{2}</sup>$ Where, in the context of describing optimal policy, we define a gap as being the deviation of a variable from its efficient level, i.e. the level which would be chosen by a social planner unconstrained by the decentralised equilibrium.

<sup>&</sup>lt;sup>3</sup>Although Ramey (2008) argues such results are driven by the failure of VARs to correctly capture the timing of fiscal shocks.

rather than treating it as an exogenous stream which needs to be financed.

In the next section, we describe our model. Section 3 then examines the fiscal policy transmission mechanism, before we explore optimal stabilisation policy. We find that although government spending shocks can crowd in private sector consumption in the short-run, such effects really only emerge at high levels of deep habits formation and common pricing across goods sold to the private and public sectors. When we turn to augmenting optimal monetary policy with the government spending instrument, we find that this instrument adds very little to stabilisation policy even in our sticky price economy with a significant consumption externality.

Further enriching the policy problem to include government debt and consider distortionary taxation to be a policy instrument, we find that it remains optimal to allow steady-state government debt to follow a random walk.<sup>4</sup> At the same time monetary policy essentially acts to stabilise the consumption gap in the face of technology shocks and tax policy deals with the mark-up shocks and the consumption externality, without generating inflation in either case.

In the final section, we examine the determinacy properties of simple monetary and fiscal rules, before assessing the ability of such rules to mimic the fully-optimal Ramsey policy. We find that the usual classification of the determinate active and passive policy rules due to Leeper (1991) depends upon the extent of deep habits formation present. Our analysis also shows that while optimised simple monetary policy rules imply little role for linear terms in output, allowing the fiscal rules to respond to output AND allowing either the monetary or fiscal rules to be non-linear, such that they respond differently to booms and recessions, enables them to better mimic the Ramsey policy.

## 2 The Model

The economy is comprised of households, two monopolistically competitive production sectors, and the government. The households derive utility from the consumption of both private and public goods. There is a continuum of final goods of each type, with each final good being produced as an aggregate of a continuum of intermediate goods. Households form external consumption habits at the level of the individual private/public final goods in their baskets - Ravn, Schmitt-Grohe, and Uribe (2006) call this type of habits 'deep'. Furthermore, we assume price inertia at the level of intermediate goods producers.

### 2.1 Households

The economy is populated by a continuum of households, indexed by k and of measure 1. Households derive utility from consumption of composite private and public goods and disutility from hours spent working.

 $<sup>^4\</sup>mathrm{As}$  in, for example, Benigno and Woodford (2003), Schmitt-Grohe and Uribe (2004a), and Leith and Wren-Lewis (2007).

**Deep Habits** When habits are of the deep kind, each household's private consumption basket,  $X_t^k$ , is an aggregate of a continuum of habit-adjusted final goods, indexed by *i* and of measure 1,

$$X_{t}^{k} = \left(\int_{0}^{1} \left(C_{it}^{k} - \theta C_{it-1}\right)^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}},$$

where  $C_{it}^k$  is household k's consumption of good i and  $C_{it} \equiv \int_0^1 C_{it}^k dk$  denotes the crosssectional average consumption of this good.  $\eta$  is the elasticity of substitution between habit-adjusted final goods ( $\eta > 1$ ), while the parameter  $\theta$  measures the degree of external habit formation in the consumption of each individual private good i. Setting  $\theta$  to 0 returns us to the usual case of no habits in private consumption.

The composition of the consumption basket is chosen in order to minimize expenditures, and the resultant demand for an individual final goods is

$$C_{it}^{k} = \left(\frac{P_{it}^{C}}{P_{t}^{C}}\right)^{-\eta} X_{t}^{k} + \theta C_{it-1}, \qquad \forall i$$

where  $P_t^C$  represents the overall price index (or CPI), defined as an average of prices of private final goods,  $P_t^C \equiv \left(\int_0^1 \left(P_{it}^C\right)^{1-\eta} di\right)^{1/(1-\eta)}$ . Aggregating across households yields the total demand for good  $i, i \in [0, 1]$ ,

$$C_{it} = \left(\frac{P_{it}^C}{P_t^C}\right)^{-\eta} X_t + \theta C_{it-1}.$$
(1)

Due to the presence of habits, this demand is dynamic in nature, as it depends not only on current period elements but also on the lagged value of consumption. This, in turn, will make the pricing/output decisions of the firms producing these final goods, intertemporal.

**Remainder of the Household's Problem** For the remainder of the households' problem, households choose the habit-adjusted private consumption aggregate,  $X_t^k$ , hours worked,  $N_t^k$ , and the portfolio allocation,  $D_{t+1}^k$ , to maximize expected lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(X_t^k)^{1-\sigma}}{1-\sigma} - \chi \frac{(N_t^k)^{1+\upsilon}}{1+\upsilon} + \chi^G \frac{(X_t^G)^{1-\sigma}}{1-\sigma} \right]$$

subject to the budget constraint,

$$\int_0^1 P_{it}^C C_{it}^k di + E_t Q_{t,t+1} D_{t+1}^k = (1 - \tau_t) W_t N_t^k + D_t^k + \Phi_t$$
(2)

and the usual transversality condition.  $E_t$  is the mathematical expectation conditional on information available at time t,  $\beta$  is the discount factor ( $0 < \beta < 1$ ),  $\chi$  the relative weight on disutility from time spent working,  $\chi^G$  the relative weight on utility from consumption of public goods, and  $\sigma$  and v are the inverses of the intertemporal elasticities of habit-adjusted consumption and work ( $\sigma, v > 0$ ;  $\sigma \neq 1$ ). The household's period-tincome includes: after-tax wage income from providing labour services to intermediate goods producing firms  $(1 - \tau_t) W_t N_t^k$ , dividends from the monopolistically competitive firms  $\Phi_t$ , and payments on the portfolio of assets  $D_t^k$ . Financial markets are complete and  $Q_{t,t+1}$  is the one-period stochastic discount factor for nominal payoffs.  $\tau_t$  is the labor income tax rate. In the maximization problem, households take as given the processes for  $C_{t-1}$ ,  $W_t$ ,  $\Phi_t$ , and  $\tau_t$ , as well as the initial asset position  $D_{-1}^k$ .

The first order conditions for labour and habit-adjusted consumption are:

$$\frac{\chi\left(N_t^k\right)^{\upsilon}}{\left(X_t^k\right)^{-\sigma}} = (1 - \tau_t)w_t$$

and

$$Q_{t,t+1} = \beta \left(\frac{X_{t+1}^k}{X_t^k}\right)^{-\sigma} \frac{P_t^C}{P_{t+1}^C}$$

where  $w_t \equiv \frac{W_t}{P_t^C}$  is the real wage (see Appendix A for further details). The Euler equation for consumption can be written as

$$1 = \beta E_t \left[ \left( \frac{X_{t+1}^k}{X_t^k} \right)^{-\sigma} \frac{P_t^C}{P_{t+1}^C} \right] R_t$$

where  $R_t^{-1} = E_t [Q_{t,t+1}]$  denotes the inverse of the risk-free gross nominal interest rate between periods t and t + 1.

### 2.2 The Government

**Deep Habits** While it is natural to think of households failing to internalise the impact of their consumption decisions on the utility of others, it is less obvious that government spending decisions are subject to a similar externality if spending is on global public goods. However, if public goods are local then the externality in government consumption can occur at a local level. Controversies over 'post-code lotteries' in health care and other local services (Cummins, Francis, and Coffey (2007)) and comparisons of regional per capita government spending levels (MacKay (2001)) suggest that households care about their local government spending levels relative to those in other constituencies. We therefore allow for these effects in public consumption, but will assess how optimal policy varies as we alter the extent of such externalities. It is important to note that, although the national government is aware of the externality in the households' perception of public goods provision, in allocating public spending across goods, they are bound by the experience of that spending within each house-

hold. In other words, this is not a model of pork-barrel politics where local politicians over-provide local services which are financed from universal taxation<sup>5</sup>, but simply one in which public goods are local in nature and households care about the provision of individual public goods in their constituency relative to other constituencies.

Assuming, for simplicity, that each household defines an area associated with a local public good, the government decides for each household on the provision of individual public goods so as to maximize the aggregate  $X_t^{G,k}$  that enters household k's utility function, given the allocated level of aggregate spending,  $G_{it-1}$ , from the previous period,

$$\max_{\left\{G_{it}^{k}\right\}_{i}} X_{t}^{G,k} = \left(\int_{0}^{1} \left(G_{it}^{k} - \theta^{G}G_{it-1}\right)^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}} \\ s.t. \int_{0}^{1} P_{it}^{G}G_{it}^{k} di \le P_{t}^{G}G_{t}^{k}.$$

Since final goods producers could potentially discriminate between sales to the private and the public sectors, we allow for a distinct set of public final goods prices,  $\{P_{it}^G\}_i$ , and a corresponding price index,  $P_t^G$ .  $\theta^G$  gives a measure of the level of habits formation in the consumption of public goods. In the maximization problem, the government takes as given the past consumption of individual public goods, as it respects the habits formation behaviour of households. The demand for public goods  $i, i \in [0, 1]$ , by household/constituency k is

$$G_{it}^k = \left(\frac{P_{it}^G}{P_t^G}\right)^{-\eta} X_t^{G,k} + \theta^G G_{it-1},$$

where  $P_t^G$  is the average of public goods prices,  $P_t^G = \left(\int_0^1 \left(P_{it}^G\right)^{1-\eta} di\right)^{\frac{1}{1-\eta}}$ . Aggregating across all households/constituencies obtains the overall demand for public goods i,

$$G_{it} = \int_0^1 G_{it}^k dk = \left(\frac{P_{it}^G}{P_t^G}\right)^{-\eta} X_t^G + \theta^G G_{it-1}$$

which is also dynamic in nature.

**Government Budget Constraint** Combining the series of the representative consumer's flow budget constraints, (2), with borrowing constraints that rule out Ponzischemes, gives the intertemporal budget constraint (see Woodford (2003), chapter 2, page 69),

$$\sum_{T=t}^{\infty} E_t [P_T C_T] \le D_t + \sum_{T=t}^{\infty} E_t [Q_{t,T} (\Phi_T + W_T N_T (1 - \tau_T))].$$

 $<sup>{}^{5}</sup>$ For a model of pork-barrel politics with vote-trading and alternative voting mechanisms, see Chari and Cole (1995).

Noting the equivalence between factor incomes and national output,

$$P_t^C Y_t^C + P_t^G Y_t^G = W_t N_t + \Phi_T,$$

and the definition of aggregate demand, we can rewrite the private sector's budget constraint as,

$$D_t = -\sum_{T=t}^{\infty} E_t [Q_{t,T} (P_T^G G_T - W_T N_T \tau_T)]$$

which implies that some combination of monetary accommodation, distortionary taxation and spending adjustments is required to service government debt as well as stabilise the economy.<sup>6</sup> Noting that, in aggregate, the households' net portfolio consists of government bonds  $D_t = B_t$  allows us to write the flow budget constraint as,

$$B_{t+1} = R_t (B_t + P_t^G G_t - W_t N_t \tau_t)$$
(3)

or in real terms,

$$b_{t+1} = R_t \left[ b_t (\pi_t^C)^{-1} + \frac{P_t^G}{P_t^C} G_t - w_t N_t \tau_t \right],$$

where  $b_t = \frac{B_t}{P_{t-1}^C}$ .

#### 2.3 Firms

In this subsection we consider the behaviour of firms. These are split into two kinds: final and intermediate goods producing firms, respectively. Final goods producers may also differentiate their output/pricing decisions according to the which sector they are supplying, private or public. Intermediate goods firms produce a differentiated intermediate good and are subject to nominal inertia in the form of Calvo (1983) contracts. This structure is adopted for reasons of tractability, allowing us to easily explore different degrees of price discrimination and habits across private and public consumption goods in a sticky price environment. Additionally, combining optimal price setting under both Calvo contracts and dynamic demand curves would undermine the desirable aggregation properties of the Calvo model as each firm given the signal to re-set prices would set a different price dependent on the level of consumption habits their product enjoyed relative to other firms'. By separating the two pricing decisions we avoid reintroducing the history-dependence in price setting the Calvo set-up is designed to avoid. In contrast, Ravn, Schmitt-Grohe, Uribe, and Uuskula (2008) use the quadratic adjustment cost model of Rotemberg (1982) to introduce nominal inertia to a model with deep habits - while this allows them to introduce price stickiness at the level of

<sup>&</sup>lt;sup>6</sup>In sections 3.1 and 3.2 below, we temporarily abstract from the fiscal financing needs of the government by allowing access to lump-sum taxation. We do so in order to explore the implications of removing government debt from the policy problem, before excluding lump-sum taxes and returning to the more realistic case where all taxes are distortionary.

final goods firms, it removes the price dispersion that is a major cost in our model with Calvo (1983) contracts.<sup>7</sup> This is not a concern in their analysis as they are interested in the empirical properties of the deep habits model, rather than describing optimal policy as we do.

#### 2.3.1 Final Goods Producers

We assume that final goods (of either type) are produced by monopolistically competitive firms as an aggregate of a set of intermediate goods (indexed by j), according to the function

$$Y_{it} = \left(\int_0^1 \left(Y_{jit}\right)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}},\tag{4}$$

where  $\varepsilon$  is the constant elasticity of substitution between inputs in production ( $\varepsilon > 1$ ) and  $Y_{it}$  represents the total amount of final goods produced (i.e. the sum of private and public final goods,  $Y_{it} = Y_{it}^C + Y_{it}^G$ ).

Taking as given intermediate goods prices  $\{P_{jit}\}_j$  and subject to the available technology (4), firms first choose the amount of intermediate inputs  $\{Y_{jit}\}_j$  that minimize production costs  $\int_0^1 P_{jit}Y_{jit}dj$ . The first order conditions yield the demand functions

$$Y_{jit} = \left(\frac{P_{jit}}{P_{it}^m}\right)^{-\varepsilon} Y_{it}, \quad \forall j, \ \forall i,$$
(5)

where  $P_{it}^m \equiv \left(\int_0^1 P_{jit}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$  is the aggregate of intermediate goods prices in sector *i* and represents the nominal marginal cost of producing an additional unit of the final good *i*.

Deep Habits The nominal profits from producing private goods are given as,

$$\Phi_{it}^C \equiv \left( P_{it}^C - P_{it}^m \right) Y_{it}^C,$$

and those from producing public goods as,

$$\Phi_{it}^G \equiv \left( P_{it}^G - P_{it}^m \right) Y_{it}^G,$$

where we distinguish between the prices charged for public goods and private goods to allow final goods producing firms to price discriminate between the two sectors. We also consider what happens when such price discrimination is not possible.

When habits in both types of goods are of the deep kind, the respective demand functions are dynamic in nature, which renders the firms' profit maximization problem intertemporal – in setting prices, firms take account of the subsequent effects on market

<sup>&</sup>lt;sup>7</sup>Lombardo and Vestin (2008) show that the welfare costs of Calvo (1983) contracts are typically higher than under Rotemberg (1982) when the steady-state is inefficient, as is the case in our model.

share driven by habit formation. Firms choose processes for  $P_{it}^Z$  and  $Y_{it}^Z$  (for Z = C, G) to maximize the present discounted value of profits,  $E_t \sum_{s=0}^{\infty} Q_{t,t+s} \Phi_{it+s}^Z$ , under the restriction that all demand be satisfied at the chosen price,  $Z_{it} = Y_{it}^Z$ . Formally, we have:

$$\max_{\{P_{it}^{Z}, Y_{it}^{Z}\}_{i}} E_{t} \sum_{s=0}^{\infty} Q_{t,t+s} \Phi_{it+s}^{Z} = E_{t} \sum_{s=0}^{\infty} Q_{t,t+s} \left(P_{it+s}^{Z} - P_{it+s}^{m}\right) Y_{it+s}^{Z}$$

$$s.t.Y_{it+s}^{Z} = \left(\frac{P_{it+s}^{Z}}{P_{t+s}^{Z}}\right)^{-\eta} X_{t+s}^{Z} + \theta^{Z} Y_{it+s-1}^{Z}$$

$$Q_{t,t+s} = \beta^{s} \left(\frac{X_{t+s}}{X_{t}}\right)^{-\sigma} \frac{P_{t}^{C}}{P_{t+s}^{C}}, \quad \text{with } Z = C, G$$

and the first order conditions are:

$$v_{it} = \left(P_{it}^C - P_{it}^m\right) + \theta E_t \left[Q_{t,t+1}v_{it+1}\right]$$
$$v_{it}^G = \left(P_{it}^G - P_{it}^m\right) + \theta^G E_t \left[Q_{t,t+1}v_{it+1}^G\right]$$
$$Y_{it}^C = v_{it} \left[\eta \left(\frac{P_{it}^C}{P_t^C}\right)^{-\eta-1} \left(P_t^C\right)^{-1} X_t\right]$$
$$Y_{it}^G = v_{it}^G \left[\eta \left(\frac{P_{it}^G}{P_t^G}\right)^{-\eta-1} \left(P_t^G\right)^{-1} X_t^G\right]$$

and

where 
$$v_{it}$$
 and  $v_{it}^G$  are the Lagrange multipliers on the dynamic demand constraints and  
represent the shadow prices of producing private and public good *i*, respectively. These  
shadow values equal the marginal benefit of additional profits from each type of good,  
 $P_{it}^C - P_{it}^m$  and  $P_{it}^G - P_{it}^m$ , respectively, plus the discounted expected payoffs from higher  
future sales,  $\theta E_t [Q_{t,t+1}v_{it+1}]$  and  $\theta^G E_t [Q_{t,t+1}v_{it+1}^G]$ . In the presence of deep habits in  
consumption increasing sales to the private (public) sector leads to an increase in sales  
of  $\theta$  ( $\theta^G$ ) in the next period. The other first order conditions indicate than an increase  
in price  $P_{it}^C$  ( $P_{it}^G$ ) brings additional revenues of  $Y_{it}^C$  ( $Y_{it}^G$ ) while simultaneously causing  
a decline in demand given by the terms in square brackets and valued at the respective  
shadow prices.

In contrast, if we do not allow final goods producers to discriminate between private and public purchases of their products then the first order conditions reduce to,

$$v_{it} = \left(P_{it}^C - P_{it}^m\right) + \theta E_t \left[Q_{t,t+1}v_{it+1}\right]$$
$$v_{it}^G = \left(P_{it}^G - P_{it}^m\right) + \theta^G E_t \left[Q_{t,t+1}v_{it+1}^G\right]$$

and

$$Y_{it}^C + Y_{it}^G = \eta \left(\frac{P_{it}^C}{P_t^C}\right)^{-\eta-1} \left(P_t^C\right)^{-1} \left[v_{it}X_t + v_{it}^G X_t^G\right]$$

with the additional constraint that  $P_{it}^G = P_{it}^C$ . The combined first order condition indicates that the common price should be increased until the extra revenue generated by selling to both sectors,  $Y_{it}^C + Y_{it}^G$ , matches the value of the decline in demand.

#### 2.3.2 Intermediate Goods Producers

The intermediate goods sectors consist of a continuum of monopolistically competitive firms indexed by j and of measure 1. Each firm j produces a unique good using only labour as input in the production process

$$Y_{jit} = A_t N_{jit}.$$
 (6)

Total factor productivity,  $A_t$ , affects all firms symmetrically and follows an exogenous stationary process,  $\ln A_t = \rho_A \ln A_{t-1} + \varrho_t^A$ , with persistence parameter  $\rho_A \in (0, 1)$  and random shocks  $\varrho_t^A \sim iidN(0, \sigma_A^2)$ .

Firms choose the amount of labour that minimizes production costs,  $W_t N_{jit}$ . The minimization problem gives a demand for labour  $N_{jit} = \frac{Y_{jit}}{A_{jit}}$  and a nominal marginal cost  $MC_t = \frac{W_t}{A_t}$ , which is the same across firms. Nominal profits are expressed as  $\Phi_{jit} \equiv (P_{jit} - MC_t) Y_{jit}$ .

We further assume that intermediate goods producers are subject to the constraints of Calvo (1983)-contracts such that, with fixed probability  $(1 - \alpha)$  in each period, a firm can reset its price and with probability  $\alpha$  the firm retains the price of the previous period. When a firm can set the price, it does so in order to maximize the present discounted value of profits,  $E_t \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s} \Phi_{jit+s}$ , and subject to the demand for its own good (5) and the constraint that all demand be satisfied at the chosen price. Profits are discounted by the *s*-step ahead stochastic discount factor  $Q_{t,t+s}$  and by the probability of not being able to set prices in future periods.

Optimally, the relative price satisfies the following relationship:

$$\frac{P_{jit}^*}{P_t^C} = \left(\frac{\xi_t}{\xi_t - 1}\right) \frac{E_t \sum_{s=0}^{\infty} \left(\alpha\beta\right)^s \left(X_{t+s}\right)^{-\sigma} mc_{t+s} \left(P_{it+s}^m\right)^{\varepsilon} Y_{it+s}}{E_t \sum_{s=0}^{\infty} \left(\alpha\beta\right)^s \left(X_{t+s}\right)^{-\sigma} \left(\frac{P_{t+s}}{P_t}\right)^{-1} \left(P_{it+s}^m\right)^{\varepsilon} Y_{it+s}}$$

where  $mc_t = \frac{MC_t}{P_t^C}$  is the real marginal cost. Following Ireland (2004), we allow the desired mark-up,  $\frac{\xi_t}{\xi_t - 1}$ , to be time varying by allowing for shocks to the mark-up implied by the elasticity of substitution between intermediate goods, where  $\ln(\xi_t) = (1 - \rho_m) \ln(\varepsilon) + \rho_m \ln(\xi_{t-1}) + \varrho_t^m$  and  $\varrho_t^m$  is an i.i.d. shock with zero mean and a vari-

ance of  $\sigma_m^2$ . Ireland (2004) finds, in the context of a benchmark New Keynesian model, that mark-up shocks are more important than technology shocks in driving movements in inflation and, for anything beyond relatively short time horizons, the output gap. Furthermore, the mark-up shock is also used as a device to generate policy trade-offs between output gap and inflation stabilisation in the benchmark New Keynesian model, where the nominal inertia costs of technology shocks can easily be avoided through an appropriate monetary policy response. While our model with deep habits and fiscal policy contains additional distortions, such that technology shocks also present policy makers with trade-offs, it is interesting to consider the impact of mark-up shocks on the description of optimal policy.

 $P_{it}^m$  represents the price at the level of sector *i* and is an average of intermediate goods prices within that sector. With  $\alpha$  of firms keeping last period's price and  $(1 - \alpha)$  of firms setting a new price, the law of motion of this price index is:

$$(P_{it}^m)^{1-\varepsilon} = \alpha \left(P_{it-1}^m\right)^{1-\varepsilon} + (1-\alpha) \left(P_{jit}^*\right)^{1-\varepsilon}$$

This description of intermediate goods firms is the same irrespective of nature of final goods pricing to consumers or the government.

#### 2.4 Equilibrium

In the absence of sector-specific shocks or other forms of heterogeneity, final goods producers are symmetric and so are households. However, symmetry does not apply to intermediate goods producers who face randomness in price setting. There is a distribution of intermediate goods prices and aggregate output is therefore determined as (see Appendix B for details on aggregation)

$$Y_t \equiv Y_t^C + Y_t^G = A_t \frac{N_t}{\Delta_t}.$$
(7)

 $\Delta_t \equiv \int_0^1 \left(\frac{P_{jt}}{P_t^m}\right)^{-\varepsilon} dj \text{ is the measure of price dispersion, which can be shown (see Wood-ford (2003), Chapter 6) to follow an AR(1) process given by$ 

$$\Delta_t = (1 - \alpha) \left(\frac{P_t^*}{P_t^m}\right)^{-\varepsilon} + \alpha \left(\pi_t^m\right)^{\varepsilon} \Delta_{t-1}.$$
(8)

Note that we have three measures of aggregate prices, a producer price index  $P_t^m$ , the usual consumer price index  $P_t^C$ , and the index of the prices of goods supplied to the government  $P_t^G$ , and consequently, there will be three measures of inflation. We define:  $\pi_t^m \equiv \frac{P_t^m}{P_{t-1}^m}$ ,  $\pi_t^G \equiv \frac{P_t^G}{P_{t-1}^G}$  and  $\pi_t^C \equiv \frac{P_t^C}{P_{t-1}^C}$ . Furthermore, the three inflation variables are

related according to the following relationship

$$\pi_t^m = \pi_t^C \frac{\mu_{t-1}^C}{\mu_t^C} = \pi_t^G \frac{\mu_{t-1}^G}{\mu_t^G},\tag{9}$$

where  $\mu_t^C \equiv \frac{P_t^C}{P_t^m}$  and  $\mu_t^G \equiv \frac{P_t^G}{P_t^m}$  are the markups of final private and public goods producers, respectively. In the presence of deep habits, these markups are time-varying. The overall markup in the economy is given by the product of markups in the intermediate goods and final goods sectors.

Finally, the aggregate version of the household's budget constraint (2) combines with the government budget constraint (3) and the definition of aggregate profits ( $\Phi_t = P_t^C Y_t^C + P_t^G Y_t^G - W_t N_t$ ) to obtain the usual aggregate resource constraint,

$$P_t^C Y_t^C + P_t^G Y_t^G = P_t^C C_t + P_t^G G_t.$$
 (10)

The equilibrium is then characterized by equations (7) - (10), together with the government budget constraint and the equilibrium conditions defining the households' and the firms' behaviour (Appendix B lists the entire set of equilibrium conditions), to which we add the monetary and fiscal policy specification (as detailed in Sections 3 and 4 below).

### 2.5 Solution Method and Model Calibration

Since we are ultimately interested in assessing the welfare benefits of allowing fiscal policy to contribute to the stabilisation of our New Keynesian economy featuring deep habits, we cannot rely on linear approximations to our model's equilibrium conditions when evaluating optimal policy. Kim and Kim (2003) have shown that such approximations can give rise to spurious welfare rankings amongst alternative policies. Instead, we employ the perturbation methods of Schmitt-Grohe and Uribe (2004b) to obtain a second order accurate solution to the model which can be used to validly rank the welfare consequences of alternative policies. Levine, Pearlman, and Pierse (2008) show, in the context of a New Keynesian model subject to superficial habits of a magnitude similar to the deep habits we consider here, that attempting to compute optimal monetary policy using a linearised model can introduce significant errors relative to the case where the optimal policy is based on a second-order accurate solution to the model.

In order to solve the model, we must select numerical values for some key structural parameters. Table 1 reports our choices, which are similar to those of other studies using a New Keynesian economy with habits in consumption. The model is calibrated to a quarterly frequency and we assume an annual real rate of interest of 4%, which implies a discount factor  $\beta$  of 0.9902. The risk aversion parameter  $\sigma$  is set at 2.0, while v equals 0.25<sup>8</sup>, and the relative weight on labour in the utility function  $\chi$  is assumed

 $v^{8}$  is the inverse of the Frisch labour supply elasticity. While estimates of this elasticity vary quite

to be 3.0, while  $\chi^G$  is 0.111. Consistent with the empirical evidence, the level of price inertia  $\alpha$  is 0.75 and we set  $\eta = \varepsilon = 11$  which implies a degree of market power of 1.21, split approximately equally between the two monopolistically competitive sectors of our economy. For the habits formation parameters  $\theta$  and  $\theta^G$ , we use a benchmark value of 0.65, which falls within the range of estimates identified in the literature<sup>9</sup>, but we allow for different possible values in the [0, 1) interval as we conduct sensitivity analyses of our results. We further assume a steady state government debt to GDP ratio that corresponds to an annual average of 60%. Under the Ramsey optimal policy, the implicit steady state tax rate takes an empirically plausible value of 0.35 under no habits and 0.31 under the benchmark calibration of habits, reflecting primarily the fiscal financing role of taxes.<sup>10</sup> Technology shocks are assumed persistent with persistence parameter  $\rho_A = 0.9$ and standard deviation  $\sigma_A = 0.009$ , while the mark-up shocks in the intermediate goods sector follow the estimated process in Ireland (2004),  $\rho_m = 0.9625$  and  $\sigma_m = 0.0012$ . In the case of an exogenous government spending process, its characteristics are given by  $\rho_G = 0.9$  and  $\sigma_G = 0.014$ .

# 3 Optimal Ramsey Policy

In this section, we consider the nature of optimal policy in response to exogenous shocks. The optimal policy problem can be set up in terms of a Lagrangian as,

$$L_0 = \max_{\mathbf{y}_t} E_0 \sum_{t=0}^{\infty} \beta^t [U(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t) - \boldsymbol{\lambda}_t f(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t)]$$

where  $\mathbf{y}_t$  and  $\mathbf{u}_t$  are vectors of the model's endogenous and exogenous variables, respectively,  $U(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t) = \frac{(X_t)^{1-\sigma}}{1-\sigma} - \chi \frac{(N_t)^{1+\upsilon}}{1+\upsilon} + \chi^G \frac{(X_t^G)^{1-\sigma}}{1-\sigma}$ ,  $f(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t) = 0$ are the model's equilibrium conditions (equations (19) - (39) in Appendix B), and  $\boldsymbol{\lambda}_t$  is a vector of lagrange multipliers associated with these constraints.

The optimisation implies the following first order conditions,

$$E_t \left[ \frac{\partial U(.)}{\partial \mathbf{y}_t} + \beta F \frac{\partial U(.)}{\partial \mathbf{y}_{t-1}} + \beta^{-1} \boldsymbol{\lambda}_{t-1} F^{-1} \frac{\partial f(.)}{\partial \mathbf{y}_{t+1}} + \boldsymbol{\lambda}_t F^{-1} \frac{\partial f(.)}{\partial \mathbf{y}_t} + \beta \boldsymbol{\lambda}_{t-1} F \frac{\partial f(.)}{\partial \mathbf{y}_{t-1}} \right] = 0$$
(11)

where F is the lead operator, such that  $F^{-1}$  is a one-period lag. A second-order accurate widely, we follow the macroeconomic literature in using a larger value, similarly to Gali, Lopez-Salido,

widely, we follow the macroeconomic interature in using a larger value, similarly to Gali, Lopez-Salido, and Valles (2007).

<sup>&</sup>lt;sup>9</sup>Macro-based estimates of habits formation of the superficial type range from 0.59 as in Smets and Wouters (2003) to very high values of 0.98 as reported by Bouakez, Cardia, and Ruge-Murcia (2005). For the deep type of habits, Ravn, Schmitt-Grohe, and Uribe (2006) give a value of 0.86. Micro-based estimates (see, for example, Ravina (2007)) are substantially lower, with a range of 0.29-0.5.

 $<sup>^{10}</sup>$ In the case where the government has access to lump-sum taxes to balance the budget, the optimal steady state tax rate would be -0.21 with no habits, reflecting the long-run inefficiency due to monopolistic competition, and a very large 0.57 under the benchmark value of habits, reflecting the consumption externality.

solution to optimal policy then involves solving these first order conditions in combination with the non-linear equilibrium conditions of the model,  $f(\mathbf{y}_{s+1}, \mathbf{y}_s, \mathbf{y}_{s-1}, \mathbf{u}_s) = 0$ , using the perturbation methods of Schmitt-Grohe and Uribe (2004b).

We measure the welfare cost of a particular policy as the fraction of permanent consumption that must be given up in order to equal welfare in the stochastic economy to that of the steady state,  $E_0 \sum_{t=0}^{\infty} \beta^t u\left(X_t, N_t, X_t^G\right) = (1-\beta)^{-1} u\left((1-\theta)(1-\zeta)\overline{C}, \overline{N}, \overline{X^G}\right)$ . Given the utility function adopted, the expression for  $\zeta$  in percentage terms is

$$\zeta = \left[1 - \frac{\left[(1-\sigma)\Theta\right]^{\frac{1}{1-\sigma}}}{(1-\theta)\overline{C}}\right] \times 100,$$

where  $\Theta \equiv (1-\beta)W + \chi \frac{\overline{N}^{1+\nu}}{1+\nu} - \chi^G \frac{\left(\overline{X^G}\right)^{1-\sigma}}{1-\sigma}$  and  $W \equiv E_0 \sum_{t=0}^{\infty} \beta^t u\left(X_t, N_t, X_t^G\right)$  represents the expectation of lifetime utility in the stochastic equilibrium, conditional on the economy being in the Ramsey non-stochastic steady state in the first period.<sup>11</sup>

In order to explore the contribution of fiscal policy instruments to optimal stabilisation in a sticky price economy featuring deep habits, we gradually introduce fiscal considerations to the policy problem. To begin with, we consider the nature of the fiscal policy transmission mechanism by introducing exogenous government spending shocks to a model variant where monetary policy is optimal. This allows us to explore the crowding-in results of Ravn, Schmitt-Grohe, and Uribe (2006) in an economy where monetary policy is conducted optimally and where we can make different assumptions about the pricing of private and public goods. We then allow government spending to be varied as part of optimal policy, to assess whether or not government spending (as a proxy for the manipulation of aggregate demand through fiscal policy) contributes to stabilisation policy. In both cases, we temporarily abstract from fiscal solvency issues by assuming the policy maker has access to a lump-sum tax through which to balance the budget. We then relax this assumption and consider the optimal policy response to technology and cost-push shocks, when taxes are distortionary and Ricardian equivalence no longer holds. In all cases, we consider optimal policies with commitment. Finally, in Section 4 we explore the ability of potentially non-linear, but simple policy rules to replicate the Ramsey policy.

### 3.1 Exogenous Government Spending and Optimal Monetary Policy

We first consider the case when fiscal policy is exogenous, while monetary policy is set optimally under commitment, and the government has access to lump-sum taxes to balance its budget. We assume that government spending follows an exogenous stationary process,  $\ln G_t = (1 - \rho_G) \ln G + \rho_G \ln G_{t-1} + \varrho_t^G$ , with persistence parameter

<sup>&</sup>lt;sup>11</sup>We opt for a conditional measure of welfare because the random walk properties of Ramsey policy, that emerge in the presence of government debt (see Section 3.3), do not allow for the computation of an unconditional welfare measure.

 $\rho_G \in (0, 1)$  and random shocks  $\varrho_t^G \sim iidN(0, \sigma_G^2)$ . Even though government spending is exogenous, households still derive utility from the consumption of public goods and form habits accordingly. The monetary authority sets the nominal interest rate to maximize households' welfare subject to the private sector's response and given the exogenous processes. We analyze the implications of this policy in terms of impulse responses to a government spending shock.

A Positive Government Spending Shock Figure 1 details the impulse responses to a positive government spending shock in three cases - no habits, habits of  $\theta = \theta^G = 0.65$  and common pricing across private and public goods and, finally, the same degree of habits, but with price discrimination across public and private goods. Consider the case without habits: the increase in government spending results in an increase in aggregate demand which the monetary authority offsets by raising real interest rates and discouraging household consumption. The policy maker does this until consumption falls so much that labour supply increases more than labour demand and the marginal costs of production actually fall.

We then consider the case where household preferences include deep habits over both private and public goods ( $\theta = \theta^G = 0.65$ ) and where the suppliers of these goods are constrained to supply to the private and public sectors at the same price. Here, the increased demand for goods tempts final goods suppliers to reduce their mark-ups in order to capture a larger share of the increased overall product demand. Ceteris paribus, this will tend to stimulate consumption. The policy maker encourages such behaviour in the initial period by cutting the real interest rate and further encouraging final goods suppliers to cut mark-ups and increase household consumption. This is sufficient to actually result in the increase in government spending crowding in private consumption. The reason why the policy maker behaves in this way is that the interest rate cut boosts initial consumption which then supports subsequent consumption due to the habits effect. This is desirable as it then allows the policy maker to toughen subsequent policy without inducing significant deflation through falling marginal costs.

Finally, we consider a variant with the same degree of habits but where the final goods producers can charge different prices to the private and public sectors. Here the markup charged to the public sector is substantially reduced, but this does not affect other variables as the government's activities are financed by lump-sum taxation in these impulse responses. Accordingly, the government spending shock does not have a direct impact on the markup charged to the private sector's consumption. This means that there is no attempt by final goods suppliers to cut their mark-up and encourage private sector consumption. As a result, despite the initial cut in the real interest rate, there is no crowding in of private sector consumption.

The crowding-in effects are effectively the result of the common pricing behaviour by final goods producers combined with sufficient degrees of habits formation in either private, or private and public goods consumption. The crowding-in effects disappear if, for example, private goods consumption habits are at their benchmark level,  $\theta = 0.65$ , but habits in public goods consumption are smaller, for example  $\theta^G = 0.2$ . However, as the markup effects on private consumption are important in generating these results, higher degrees of habits formation in private consumption can restore the crowdingin effects (the dash-dot impulse responses in Figure 2 show a crowding in effect when  $\theta = 0.85$ , even in the absence of habits in public goods consumption,  $\theta^G = 0$ ). It should be noted, however, that these effects are quantitatively small.

#### 3.2 Endogenous Government Spending and Optimal Monetary Policy

In this subsection, we analyse the optimal policy response to technology and markup shocks, where the nominal interest rate and government spending serve as policy instruments. We continue to ignore the budgetary consequences of policy by assuming fiscal authorities have access to a lump-sum tax with which to balance the budget.

A Technology Shock Figure 3 analyses the response to a positive technology shock and includes three cases - no habits effects and the case of deep habits with either common or discriminatory pricing across private and public goods. In the absence of habits effects, policy seeks to eliminate the inflationary consequences of the shock, leaving consumption, government spending and output suboptimally low due to the distortionary effects of monopolistic competition. If the policy maker were forced to behave in a time consistent manner, then this permanent distortion would result in an inflationary bias, but under commitment the policy maker is able to resist the temptation to introduce policy surprises in order to offset this distortion.

When we introduce significant deep habits effects, the nature of the distortion changes as households now over-consume, due to the habits externality, thus implying a significant consumption and output gap (the difference between actual output and the efficient level of output, as a percentage of the efficient level<sup>12</sup>) of almost 30%. In the face of this enormous externality, monetary policy no longer seeks to solely stabilise inflation. Real interest rates are slowly relaxed, and consumption and output rise. The rates are not cut as aggressively as in the absence of deep habits, as the markup would fall further encouraging more undesirable consumption, and the policy maker is prepared to suffer an initial fall in inflation. In fact, monetary policy may even be tightened initially, when habits formation effects are substantial, as is the case under the benchmark calibration. The government spending gap is very small relative to the massive consumption/output gaps, mirroring the findings in Eser, Leith, and Wren-Lewis (2009) who demonstrate, in the benchmark New Keynesian model without habits, that government spending contributed little to macroeconomic stabilisation. Given the

<sup>&</sup>lt;sup>12</sup>See Appendix C for the details of the social planner's problem.

size of the consumption externality in this model with habits, it is difficult to think of circumstances where aggregate demand management through fiscal policy is likely to contribute significantly to macroeconomic stabilisation unless use of monetary policy is constrained, for example due to the zero lower bound on nominal interest rates or due to participation in a monetary union.

A Mark-Up Shock We then consider the response of policy to a markup shock, taken as a 1% increase in  $\xi_t$ , which represents a decrease in the intermediate goods producers' desired markup. In this case, the policy maker faces a trade-off between inflation and output stabilization even in the absence of habits as inflation falls while output rises. With little change in government spending, interest rates are initially raised in order to reduce aggregate demand and the size of the output gap, while allowing for additional deflation. This is illustrated by the dotted lines in Figure 4.

In the presence of deep habits, the initial tightening of monetary policy is even stronger, as the policy maker attempts to curb the large output gap that can ensue due to over-consumption effects. The increase in real interest rates discourages consumption but also makes final goods producers raise markups, as they discount the lost future profits that such price increases entail more strongly. This increase in markups further subdues consumption, while at the same time being sufficiently strong to make inflation increase in equilibrium. The subsequent relaxation of policy reduces markups and hence inflation but at the cost of increased consumption and output gaps. At the same time, the government spending gap remains small relative to the consumption and output gaps. While the ability to price discriminate across private and public goods does not have much bearing on the results<sup>13</sup>, the time-varying markups that arise under deep habits are shown to play an important role in the optimal policy response to cost-push shocks.

### 3.3 Optimal Monetary and Fiscal Policy

We now turn to the analysis of optimal Ramsey policy when policy makers have control over monetary policy and both fiscal policy instruments - government spending and income taxes, but where they no longer have access to lump-sum taxes to satisfy the government's budget constraint. It is important to note that, if we did continue to remove the need to adjust either government spending or distortionary taxes to satisfy the intertemporal budget constraint, then the policy maker can achieve the first best allocation using the income tax instrument to offset the consumption externality and the mark-up shocks, while using the interest rate to offset the nominal inertia costs of technology shocks.

Before considering the response to technology and mark-up shocks, it is interesting to consider the initial steady-state of the Ramsey policy. This is computed by solving

<sup>&</sup>lt;sup>13</sup>Impulse responses across the two types of pricing behaviour are virtually the same.

the steady-state of the Ramsey first order conditions and the equilibrium conditions describing our New Keynesian economy, conditional on an initial government debt to GDP ratio. In the case of our model without habits, the combination of the monopolistic competition and tax distortions suppresses output below its socially efficient level. Interestingly, the optimal policy implies that the absolute size of the government spending gap is significantly smaller than the consumption gap. The intuition for this pattern lies in the desire to support the debt stock with the optimal combination of gap variables without generating any steady-state inflation. In the case of habits, the consumption externality renders the level of output too high despite the presence of monopolistic competition and distortionary taxation. As a result the consumption and government spending gaps.

We now turn to consider the case where the policy maker utilises monetary and fiscal instruments to stabilise both the economy and the government's finances in the face of technology and cost-push shocks, in an environment where the policy maker faces multiple trade-offs. Figure 5 details the response to a 1% positive technology shock. A key element of the policy response is that the steady-state of government debt follows a random walk as in Benigno and Woodford (2003), Schmitt-Grohe and Uribe (2004a), and Leith and Wren-Lewis (2007). The basic intuition for this is that in a sticky-price environment adjusting fiscal instruments to offset fiscal shocks is costly, such that policy makers ensure that policy instruments are adjusted to service the new steady-state debt that emerges following shocks, but the policy maker commits to not attempting to do more. In the absence of habits, gap variables are adjusted to their new steady-state values from the second period onwards, and debt slowly evolves to its new steady-state consistent with those variables. Real interest rates are adjusted in the face of the technology shock to maintain consumption at its new constant gap value. With a positive technology shock, tax rates fall and government spending, consumption and output rise to support the lower steady-state debt stock without affecting inflation. As shown in Leith and Wren-Lewis (2007), behaviour in the initial period is slightly different as the policy maker exploits the fact that expectations are given to reduce the impact of the shock on debt. Accordingly, in the initial period real interest rates rise (to raise debt service costs and offset the increase in the tax base) and encourage a surprise deflation in the initial period (although taxes rise to partially offset this deflation) - the combined impact of this is to reduce the eventual decline in debt that would otherwise emerge.<sup>14</sup>

When there are deep habits in consumption, the policy maker needs to minimise both the consumption externality and the costs of nominal inertia. Despite this additional

<sup>&</sup>lt;sup>14</sup>Leith and Wren-Lewis (2007) show that the combination of instruments used in the initial period depends crucially upon the degree of price stickiness and the steady state debt-GDP ratio. In our benchmark calibration, debt service costs and inflationary suprises are particularly effective in influencing the level of government debt.

trade-off, the assignment of instruments remains similar, although the stabilisation of gap variables at their new long-run levels is no longer immediately after the initial period (it should be noted that the transition to the new steady-state still retains the property that producer price inflation is effectively zero). Monetary policy adjusts interest rates to help stabilise the consumption gap in the face of the technology shock, and tax rates are adjusted to largely offset the extra consumption generated by the technology shock in the presence of habits, while together ensuring that producer price inflation is near zero from the second period onwards. Once again, in the initial period instruments are used to reduce the long-run impact of the technology shock on government debt, most noticeably through the increase in real interest rates, which has the added advantage of reducing the initial boost to consumption. There are negligible differences when final goods producers can price discriminate between private and public goods.

We now consider the mark-up shock detailed in Figure 6. In the absence of habits, the tax rate is employed to mitigate the impact of the mark-up shock while maintaining the consumption, government spending and output gaps close to their new steady-state values. In the initial period, there is an attempt to offset the long-run reduction in government debt following the negative mark-up shock, primarily through tightening monetary policy (which increases debt service costs, reduces the size of the tax base and supports a surprise deflation). When we introduce deep habits, the policy maker has to consider both the consumption externality and the mark-up shock. As a result, the tax rate is raised more aggressively than in the absence of habits and producer price inflation rises rather than falls. Nevertheless, gap variables are very quickly driven to their new steady-state values which support the new steady-state value of debt without generating inflation.

We note that an economy where final goods producers can price discriminate between private and public consumption is very similar to the one in which they are constrained to a common price policy. The responses to technology and mark-up shocks under optimal Ramsey policies are almost identical across the two types of pricing behaviour.

## 4 Optimal Simple Rules

In this section we consider the ability of simple monetary and fiscal rules to achieve the welfare outcomes commensurate with the fully optimal Ramsey policy. In addition to analysing simple log-linear rules, our second order approximation of a heavily distorted economy may also support the use of non-linear rules.

Our general interest rate rule is given as,

$$\ln(R_t/\overline{R}) = \left(\phi_R - (2\phi_{R^2})^{1/2}\right) \ln(R_{t-1}/\overline{R}) + (R_{t-1}/\overline{R})^{(2\phi_{R^2})^{1/2}} \\ + \left(\phi_{\pi} - (2\phi_{\pi^2})^{1/2}\right) \ln(\pi_t^m/\overline{\pi}^m) + (\pi_t^m/\overline{\pi}^m)^{(2\phi_{\pi^2})^{1/2}} \\ + \left(\phi_y - (2\phi_{y^2})^{1/2}\right) \ln(Y_t/\overline{Y}) + (Y_t/\overline{Y})^{(2\phi_{y^2})^{1/2}} - 3,$$

with the following second order approximation,

$$\widehat{R}_{t} = \phi_{R}\widehat{R}_{t-1} + \phi_{R^{2}}\widehat{R}_{t-1}^{2} + \phi_{\pi}\widehat{\pi}_{t}^{m} + \phi_{\pi^{2}}\left(\widehat{\pi}_{t}^{m}\right)^{2} + \phi_{y}\widehat{Y}_{t} + \phi_{y^{2}}\widehat{Y}_{t}^{2}.$$
(12)

The second order terms allow us to capture non-linearities in the otherwise log-linear rules. For example, a positive coefficient on the second order term implies that the response to positive deviations of that variable from steady-state is higher than negative deviations. Conversely, when the coefficients are negative there are bigger responses to negative deviations from the steady-state of the particular variable. Note that we consider the adjustment of interest rates to changes in producer price inflation  $\pi_t^m$  (rather than a more general inflation measure), as this is the inflation measure that captures the costs of nominal inertia.<sup>15</sup> The log-linear special case of this rule is similar to the monetary policy rules considered in, for example, Rotemberg and Woodford (1999) and Schmitt-Grohe and Uribe (2007), while the second order terms allow us to capture the asymmetric interest rate smoothing considered by Florio (2006) and the general asymmetric behaviour either driven by non-linearities in the model (Dolado, Maria-Dolores, and Naveira (2005)) or policy maker preferences (Srinivasan, Mahambare, and Ramachandran (2006)).

We also consider similar rules for fiscal policy. Firstly describing the tax rate,

$$\begin{aligned} \ln(\tau_t/\overline{\tau}) &= \left(\gamma_b - (2\gamma_{b^2})^{1/2}\right) \ln(b_t/\overline{b}) + (b_t/\overline{b})^{(2\gamma_{b^2})^{1/2}} \\ &+ \left(\gamma_y - (2\gamma_{y^2})^{1/2}\right) \ln(Y_t/\overline{Y}) + (Y_t/\overline{Y})^{(2\gamma_{y^2})^{1/2}} - 2 \end{aligned}$$

for which the second order approximation is given by,

$$\widehat{\tau}_t = \gamma_b \widehat{b}_t + \gamma_{b^2} \widehat{b}_t^2 + \gamma_y \widehat{Y}_t + \gamma_{y^2} \widehat{Y}_t^2$$
(13)

and a similarly constructed rule for government spending, which in approximated form is given by,

$$\widehat{G}_t = \kappa_b \widehat{b}_t + \kappa_{b^2} \widehat{b}_t^2 + \kappa_y \widehat{Y}_t + \kappa_{y^2} \widehat{Y}_t^2.$$
(14)

The log-linear special cases of these fiscal rules are similar to those considered in Schmitt-Grohe and Uribe (2004a), Linnemann (2006), and Leith and von Thadden (2008) for

<sup>&</sup>lt;sup>15</sup>See Kirsanova, Leith, and Wren-Lewis (2006) for a discussion of the importance of targeting the rate of inflation which captures the costs of the price dispersion associated with nominal inertia.

the tax rule, and Leith and Wren-Lewis (2000) for the government spending rule.

### 4.1 Determinacy Analysis

While we are considering potentially non-linear policy rules in our second order accurate model solution, the determinacy properties of those rules in the neighbourhood of the steady-state can be assessed by considering a log-linearised description of our economy. Therefore we embed our policy rules in a log-linearised version of our equilibrium conditions described in Appendix D.

Here, the benchmark results in the literature stem from Leeper (1991) who provides the original characterisation of policy rules as being 'active' or 'passive'. An active monetary policy rule is one in which the monetary authority satisfies the Taylor principle in that they adjust nominal interest rates such that real interest rates rise in response to excess inflation. Conversely, a passive monetary rule is one which fails to satisfy this principle. In Leeper (1991)'s terminology a passive fiscal policy is one in which the fiscal instrument is adjusted to stabilise the government's debt stock, while an active fiscal policy fails to do this. Leeper (1991) demonstrated, in the context of a lump sum tax instrument, that it is only active/passive policy combinations that ensure determinacy of the rational expectations equilibrium. A similar characterisation<sup>16</sup> emerges in the context of economies where Ricardian equivalence does not hold and the policy instrument is government spending (Leith and Wren-Lewis (2000)) or distortionary taxation (Linnemann (2006)). We now revisit these results in our New Keynesian economy with deep habits. Since earlier results in this literature either do not consider feedback from output to the policy instrument or find that such feedback does not significantly improve welfare in an optimal simple rule (see Schmitt-Grohe and Uribe (2004a)), we set  $\phi_y = \gamma_y = \kappa_y = 0$  to exclude output from the policy rules when considering determinacy. Accordingly, our simple rules reduce to,

$$\widehat{R}_t = \phi_R \widehat{R}_{t-1} + \phi_\pi \widehat{\pi}_t^m \tag{15}$$

$$\widehat{\tau}_t = \gamma_b \widehat{b}_t \tag{16}$$

$$\widehat{G}_t = \kappa_b \widehat{b}_t. \tag{17}$$

In Figure 7 we plot the combinations of the fiscal feedback to government spending,  $\kappa_b$ , and the monetary response to inflation,  $\alpha_{\pi}$ , for various degrees of interest rate inertia,  $\alpha_R$ , and deep habits,  $\theta = \theta^G$ , assuming that final goods producing firms charge the same price to both the private and public sectors. Moving across each row increases the extent of deep habits, while moving down each column increases the extent of interest

<sup>&</sup>lt;sup>16</sup>However, the presence of non-Ricardian elements can affect the critical value of fiscal response required to render the fiscal policy rule 'passive' (see Leith and Wren-Lewis (2000)) and, in models with a richer supply side, can lead to bifurcations in the policy combinations required for determinacy, as in Leith and von Thadden (2008).

rate inertia. The picture in the top left corner therefore mimics the analysis of Leith and Wren-Lewis (2000). If the monetary policy is active,  $\alpha_{\pi} > 1$ , then fiscal policy must cut government spending in response to increased government debt. If fiscal policy fails to respond to deviations of debt from steady-state, then the active monetary policy will give rise to a debt interest spiral which implies instability. Meanwhile if monetary policy is passive then this can help stabilise debt when fiscal policy fails to do so, as the saddlepath solution delivers a path for real interest rates which offsets the instability in debt which would otherwise emerge. Finally, if fiscal policy is acting to stabilise debt, then a passive monetary policy will lead to indeterminacy in the usual manner as inflationary expectations become self-fulfilling (see Woodford (2003)). Moving down the first column where we increase the degree of nominal interest rate inertia, then the critical value for the interest rate response to inflation necessary for monetary policy to be described as active falls below one. This is because it is the long-run response to inflation,  $\frac{\alpha_{\pi}}{1-\alpha_R} > 1$ , which is key to defining the Taylor principle in an inertial rule.

As we move across the columns, the determinacy region in the South-East quadrant associated with a combination of an active monetary policy and a passive fiscal policy is reduced for large fiscal responses to debt, rendering that part of the policy space unstable. The intuition for this change is as follows: imagine a shock which raises the debt stock. Government spending is reduced which implies that the final goods firms will seek to increase their mark-ups. If the degree of habits is sufficiently large, the increasing mark-ups will raise consumer prices, labour costs, and inflation, resulting in higher real interest rates which will destabilise the debt stock. Similarly, the previously indeterminate combination of a passive monetary policy combined with a passive fiscal policy which adjusts spending in response to deviations of debt from steady-state becomes increasingly determinate as the extent of deep habits is increased. Intuitively, a shock which raises debt implies that government spending falls, mark-ups increase, raising prices and, with a passive monetary policy, resulting in falls in real interest rates which stabilise the debt.

In Figure 8 we perform the same analysis in a model variant where final goods firms can price discriminate between the private and public sector. As a result the fiscal rule does not directly affect the mark-up charged on final goods for the private sector. Accordingly, the presence of deep habits does not matter until the degree of deep habits becomes very high.

In Figure 9 we consider the case of fiscal feedback to the tax rate rather than government spending in an economy where final goods firms charge the same price to the private and public sectors. As before, for an economy without habits we find that an active/passive policy combination is necessary to ensure determinacy, although for a strong fiscal response to debt disequilibrium the response of interest rates to inflation needs to be higher - this is because raising tax rates fuels inflation through their impact on marginal costs. Interestingly, an active monetary policy combined with a fiscal policy which fails to raise taxes in response to higher debt is not always unstable, but can be indeterminate if the inappropriate fiscal response is sufficiently aggressive, due to the supply side effects of variations in tax rates. As the degree of habits is increased this does not affect this analysis until the extent of habits passes a critical value (see Figure 10). At this point almost all regions become indeterminate - for example, for the usual active monetary policy and passive fiscal policy combination, the higher interest rates in response to inflation, imply that the profits from investing in habits are lower so that mark-ups increase, validating the increase in inflation.

In Figure 11 the determinacy properties of the tax rule is considered in an economy where final goods firms can price discriminate between the private and public sectors - the analysis is largely unchanged relative to Figure 9, although the bifurcation in determinacy regions occurs earlier than under common pricing - see Figure 12.

### 4.2 Optimal Simple Rules

We search across the rule parameter space using the Simplex method employed by the Fminsearch algorithm in Matlab (see, Lagarias, Reeds, Wright, and Wright (1998)) in order to minimise the conditional welfare losses associated with the rule. In searching, we explored both the active monetary/passive fiscal and passive monetary/active fiscal policy determinate regions of the policy space. Table 2 details the optimal parameters for the simple linear rules in (15) - (17). Interestingly, the combination of a passive monetary rule and an active government spending rule is preferable to the active monetary rule/passive fiscal rule combination, although the costs of employing such simple rules is significant with welfare costs that are at least 39% higher than fully optimal policy. However, this simply reflects the fact that government spending does not contribute to stabilisation. When we consider the combination of interest rate and tax rate rules there is a significant improvement in welfare - the costs of following this pair of simple rules amounts to 15.8% of welfare costs under Ramsey. Combining government spending and tax rules marginally improves welfare further.

We then consider a richer set of rules which introduce non-linearities to monetary policy and terms in output to the interest rate, government spending and tax rules. As the results are very similar across the two forms of pricing, we concentrate on the assumption of common pricing when exploring the non-linear refinement to the rules. The optimal parameterisation of these rules is given by,

$$\begin{split} \widehat{R}_t &= 1.2309 \widehat{R}_{t-1} - 0.47092 \widehat{R}_{t-1}^2 + 2.849 \widehat{\pi}_t^m + 3.7092 \, (\widehat{\pi}_t^m)^2 + 0.035696 \widehat{Y}_t - 2.9665 \widehat{Y}_t^2 \\ \widehat{\tau}_t &= 0.18386 \widehat{b}_t + 1.2049 \widehat{Y}_t \\ \widehat{G}_t &= 0.020784 \widehat{b}_t + 1.0953 \widehat{Y}_t. \end{split}$$

This combination of rules enables us to mimic the welfare levels attained by the Ramsey

optimal policy, with the welfare costs under the optimal rule,  $\zeta$ , only 0.23% higher than under the Ramsey policy. The optimised rule coefficients imply that government spending is not being used to stabilise debt, but that government spending is moving with output in order to ensure a stable government spending gap. The fiscal adjustment is taking place through the tax rule, which also raises the tax rate in response to higher output as a means of off-setting the consumption habits externality.<sup>17</sup> The latter result reflects the findings of Ljungqvist and Uhlig (2000) who show, in the context of a real economy with superficial habits, that contractionary tax rates can help align output with the efficient level in response to technology shocks. The monetary policy rule in its linear terms is fairly standard, implying that the rule is super inertial, with a strong response to inflation and a relatively muted response to output. The non-linear terms imply that interest rate inertia is more muted when interest rates are cut and interest rates respond more to higher, than lower, inflation, but react more aggressively to falls, than rises, in output. The latter may be thought to be surprising since the consumption externality implies that output is suboptimally high such that booms are more likely to concern the policy maker than recessions. However, the large coefficient on output in the tax rule implies that the tax rule is more than compensating for the consumption externality.

We further assess the relative importance of adding output to the rules rather than allowing policy to behave asymmetrically over the business cycle. We set the coefficients on quadratic terms to zero and obtain the following optimal parameterisation,

$$\begin{aligned} \widehat{R}_t &= 1.0984 \widehat{R}_{t-1} + 3 \widehat{\pi}_t^m + 0.012589 \widehat{Y}_t \\ \widehat{\tau}_t &= 0.12961 \widehat{b}_t + 0.07563 \widehat{Y}_t \\ \widehat{G}_t &= 0.017091 \widehat{b}_t + 0.957 \widehat{Y}_t. \end{aligned}$$

Here we find that, although the rules come close to the Ramsey policy, they are not fully able to mimic it, with a welfare cost of 0.0289% of permanent consumption which is 2.04% higher than under the Ramsey policy. The key difference is that the absence of the asymmetric monetary policy response does not enable the tax rule to fully deal with the consumption habits externality and the optimised coefficient on output in the tax rule is significantly lower, falling from 1.2 to 0.076.

Finally we consider introducing the non-linearity to the fiscal rules rather than the monetary rule. We find the optimised rules are given by,

$$\widehat{R}_t = 1.0959 \widehat{R}_{t-1} + 2.9011 \widehat{\pi}_t^m + 0.010749 \widehat{Y}_t$$
$$\widehat{\tau}_t = 0.14379 \widehat{b}_t - 0.015854 \widehat{b}_t^2 + 1.1088 \widehat{Y}_t - 0.056154 \widehat{Y}_t^2$$

<sup>&</sup>lt;sup>17</sup>Since this rule is defined in terms of the tax rate, the response of the tax rate to output implies an implicit stabilisation benefit to progressive taxation.

$$\widehat{G}_t = 0.010151 \widehat{b}_t + 0.033293 \widehat{b}_t^2 + 1.0543 \widehat{Y}_t + 0.0014314 \widehat{Y}_t^2$$

where again the asymmetries in the fiscal rules enable the tax rule to respond to the consumption externality more effectively. Here the non-linearity in the tax rule implies that tax rates decrease more in response to falls in debt and output, than they rise in response to increases. While the government spending rule does nothing to stabilise debt (government spending continues to rise in response to an increase in debt), it moves in line with output, with a slight tendency to rise more in a boom, in order to stabilise the underlying government spending gap. The non-linearity enables this combination of optimal rules to come closer to the welfare outcomes under the Ramsey policy. However, with a welfare cost in terms of permanent consumption 0.39% higher than under Ramsey, this set of optimal rules is slightly inferior to the optimal combination that features a non-linear monetary policy rule.

For the two combinations of non-linear rules, we also assess their ability to mimic the Ramsey policy in terms of impulse responses. Figures (13) and (14) plot the impulse responses to a technology shock and a markup shock under the Ramsey optimal policy (solid lines) and under the optimal rules: quadratic monetary rule with linear fiscal rules (dash lines) and linear monetary rule with quadratic fiscal rules (dash-dot lines). We notice that the optimal rules are better able to mimic the Ramsey policy when considering a technology shock, whereas slightly larger differences relative to Ramsey policy emerge following a markup shock. For both shocks and sets of rules, the rules, while being fully determinate, are able to achieve a very slow adjustment of debt which is close to the random walk in steady-state debt observed under the Ramsey policy. At the same time, the strong response of the fiscal rules to output ensures that the government spending gap is held fairly constant, while the tax rate moves to offset the consumption externality associated with deep habits.

### 5 Conclusion

In this paper we explored optimal monetary and fiscal policy in a New Keynesian economy subject to deep habits in consumption, where the habits externality exists at the level of individual goods. Employing second order approximation techniques we consider various forms of optimal policy of increasing richness in the context of a significantly distorted economy. We begin by considering the consumption response to government spending shocks, when monetary policy is optimal. We show that earlier findings, that deep habits can account for empirical results whereby public spending crowds in private consumption, are not robust to relatively modest declines in the extent of habits formation and allowing firms to price discriminate when supplying goods to the public and private sectors. Furthermore, we find that government spending, despite being the fiscal instrument that directly feeds into aggregate demand, contributes very little to the stabilisation of the economy following technology and mark-up shocks, when monetary policy is conducted in an optimal manner.

When we consider the trade-offs between business cycle stabilisation and fiscal solvency, we find that it remains optimal to allow steady-state debt to follow a random walk following shocks, although the transition to that steady-state is more gradual than that observed in simpler models, due to the additional consumption externality faced by the policy maker when consumers possess deep habits. In terms of the operation of individual instruments, monetary policy largely ensures that the consumption gap is stabilised in the face of technology shocks, while the income tax instrument serves to offset the consumption externality associated with habits and any shocks to the imperfectly competitive firm's desired markups.

Finally, we assessed the ability of simple linear and non-linear monetary and fiscal policy rules to achieve the levels of welfare associated with the Ramsey policy. Relatively simple interest rate and tax rate rules perform reasonably well, but are not able to fully mimic the Ramsey policy. However, a combination of non-linearities in either the monetary policy rule or the fiscal rules AND linear terms in output in the fiscal rules is able to capture the welfare levels observed under the Ramsey policy.

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# A Analytical Details (not for publication)

### A.1 Households

**Cost Minimization** Households decide the composition of the consumption basket to minimize expenditures

$$\begin{split} \min_{\left\{C_{it}^{k}\right\}_{i}} \int_{0}^{1} P_{it}^{C} C_{it}^{k} di \\ s.t. \left(\int_{0}^{1} \left(C_{it}^{k} - \theta C_{it-1}\right)^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}} \geq \overline{X}_{t}^{k} \end{split}$$

The demand for individual goods i is

$$C_{it}^{k} = \left(\frac{P_{it}^{C}}{P_{t}^{C}}\right)^{-\eta} X_{t}^{k} + \theta C_{it-1}.$$

where  $P_t^C$  can be expressed as an aggregate of the private goods *i* prices,  $P_t^C = \left(\int_0^1 \left(P_{it}^C\right)^{1-\eta} di\right)^{\frac{1}{1-\eta}}$ . Averaging across all households gives the overall demand for private final goods as,

$$C_{it} = \int_0^1 C_{it}^k dk = \left(\frac{P_{it}^C}{P_t^C}\right)^{-\eta} X_t + \theta C_{it-1}.$$

Utility Maximization The solution to the utility maximization problem is obtained by solving the Lagrangian function:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[ u \left( X_t^k, N_t^k, X_t^{G,k} \right) - \lambda_t^k \left( P_t^C X_t^k + P_t \vartheta_t + E_t Q_{t,t+1} D_{t+1}^k - (1 - \tau_t) W_t N_t^k - D_t^k - \Phi_t \right) \right]$$
(18)

In the budget constraint, we have re-expressed the total spending on the private consumption basket,  $\int_0^1 P_{it}^C C_{it}^k di$ , in terms of quantities that affect the household's utility,  $\int_0^1 P_{it}^C C_{it}^k di = P_t^C X_t^k + P_t^C \vartheta_t$ , where under deep habits  $\vartheta_t$  is given as  $\vartheta_t \equiv \theta \int_0^1 \left(\frac{P_{it}^C}{P_t^C}\right) C_{it-1} di$ . Households take  $\vartheta_t$  as given when maximising utility.

The first order conditions are then,

- $(X_t^k): \qquad u_X(t) = \lambda_t^k P_t^C$
- $\left(N_{t}^{k}\right): \qquad -u_{N}\left(t\right) = u_{X}(t)\left(1 \tau_{t}\right)\frac{W_{t}}{P_{t}^{C}}$
- $(D_t^k): \qquad 1 = \beta E_t \left[ \frac{u_X(t+1)}{u_X(t)} \frac{P_t^C}{P_{t+1}^C} \right] R_t$

where  $R_t = \frac{1}{E_t[Q_{t,t+1}]}$  is the one-period gross return on nominal riskless bonds.

With utility given by  $u(X, N, X^G) = \frac{X^{1-\sigma}}{1-\sigma} - \chi \frac{N^{1+\nu}}{1+\nu} + \chi^G \frac{(X^G)^{1-\sigma}}{1-\sigma}$ , the first derivatives are  $u_X(\cdot) = X^{-\sigma}$  and  $u_N(\cdot) = -\chi N^{\nu}$ 

### A.2 Intermediate Goods Producers

The cost minimization of intermediate goods producers involves the choice of labour input  $N_{jit}$  subject to the available production technology

$$\begin{split} \min_{N_{jit}} & W_t N_{jit} \\ s.t. \ A_t N_{jit} = Y_{jit} \end{split}$$

and yields a labour demand,  $N_{jit} = \frac{Y_{jit}}{A_t}$ , and a nominal marginal cost which is the same across all intermediate goods producing firms,  $MC_t = \frac{W_t}{A_t}$ . Profits are defined as:  $\Phi_{jit} \equiv P_{jit}Y_{jit} - W_t N_{jit} = (P_{jit} - MC_t)Y_{jit}$ .

The profit maximization is subject to the Calvo-style of price setting behaviour where, with fixed probability  $(1 - \alpha)$  each period, a firm can set its price and with probability  $\alpha$  the firm keeps the price from the previous period. When a firm can set the price it does so in order to maximize the present discounted value of profits, subject to the demand for its own goods. Profits are discounted by the stochastic discount factor, adjusted for the probability of not being able to set prices in future periods:

$$\begin{aligned} \max_{P_{jit}^*} E_t \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s} \Phi_{jit+s} &= E_t \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s} \left[ \left( P_{jit}^* - MC_{t+s} \right) Y_{jit+s} \right] \\ s.t. Y_{jit+s} &= \left( \frac{P_{jit}^*}{P_{it+s}^m} \right)^{-\varepsilon} Y_{it+s} \\ Q_{t,t+s} &= \beta^s \left( \frac{X_{t+s}}{X_t} \right)^{-\sigma} \frac{P_t^C}{P_{t+s}^C} \end{aligned}$$

Optimally, the relative price is set at

$$\frac{P_{jit}^*}{P_t^C} = \left(\frac{\xi_t}{\xi_t - 1}\right) \frac{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s (X_{t+s})^{-\sigma} mc_{t+s} \left(P_{it+s}^m\right)^{\varepsilon} Y_{it+s}}{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s (X_{t+s})^{-\sigma} \left(\frac{P_{t+s}^C}{P_t^C}\right)^{-1} \left(P_{it+s}^m\right)^{\varepsilon} Y_{it+s}}$$

where  $mc_t = \frac{MC_t}{P_t^C}$  is the real marginal cost, expressed in terms of private consumption goods, and  $Y_{it} = Y_{it}^C + Y_{it}^G$  represents the total production of good *i*, including both private and public goods. We allow the desired mark-up,  $\frac{\xi_t}{\xi_t-1}$ , to be time varying allowing for shocks to the mark-up implied by the elasticity of substitution between intermediate goods, where  $\ln(\xi_t) = (1-\rho_m)\ln(\varepsilon) + \rho_m \ln(\xi_{t-1}) + \varrho_t^m$  and  $\varrho_t^{m^{\sim}}iidN(0, \sigma_m)$ . The relative price can also be expressed as

$$\frac{P_{jit}^*}{P_t^C} = \left(\frac{\xi_t}{\xi_t - 1}\right) \frac{K_{1t}}{K_{2t}}$$

where  $K_{1t}$  and  $K_{2t}$  have the following recursive representation:

$$K_{1t} \equiv E_t \sum_{s=0}^{\infty} (\alpha \beta)^s (X_{t+s})^{-\sigma} mc_{t+s} \left(\frac{P_{it+s}^m}{P_t^C}\right)^{\varepsilon} Y_{it+s}$$
$$= X_t^{-\sigma} mc_t (\mu_t^C)^{-\varepsilon} Y_{it} + \alpha \beta E_t \left[K_{1t+1} (\pi_{t+1}^C)^{\varepsilon}\right]$$

and

$$K_{2t} \equiv E_t \sum_{s=0}^{\infty} (\alpha \beta)^s (X_{t+s})^{-\sigma} \left(\frac{P_{t+s}}{P_t}\right)^{-1} \left(\frac{P_{it+s}}{P_t^C}\right)^{\varepsilon} Y_{it+s}$$
$$= X_t^{-\sigma} (\mu_t^C)^{-\varepsilon} Y_{it} + \alpha \beta E_t \left[K_{2t+1} \left(\pi_{t+1}^C\right)^{\varepsilon-1}\right]$$

and  $\mu_t^C = \frac{P_t^C}{P_t^m}$  represents the markup of private goods prices over the nominal marginal cost of production.

## **B** Equilibrium Conditions

### **B.1** Aggregation and Symmetry

**Aggregate Output:** The market clearing condition at the level of intermediate goods is

$$\left(\frac{P_{jit}}{P_{it}^m}\right)^{-\varepsilon} Y_{it} = A_t N_{jit}, \qquad \forall j, \ \forall i$$

which upon aggregation across the j firms becomes

$$Y_{it}\Delta_{it} = A_t N_{it}, \qquad \forall i$$

where  $Y_{it} \equiv Y_{it}^C + Y_{it}^G$  is the total production of good *i*, including sales to both the private and the public sectors, and  $\Delta_{it} \equiv \int_0^1 \left(\frac{P_{jit}}{P_{it}^m}\right)^{-\varepsilon} dj$  represents intermediate goods price dispersion in sector *i*. With final goods producing sectors being symmetric, we can drop the *i* subscript and write the aggregate production as,

$$Y_t \equiv Y_t^C + Y_t^G = \frac{A_t}{\Delta_t} N_t.$$

**Aggregate Profits:** Aggregate profits from intermediate goods producers are given by

$$\begin{split} \int_{0}^{1} \int_{0}^{1} \Phi_{jit} dj di &= \int_{0}^{1} \int_{0}^{1} \left( P_{jit} - MC_{t} \right) Y_{jit} dj di \\ &= \int_{0}^{1} \int_{0}^{1} \left( P_{jit} - \frac{W_{t}}{A_{t}} \right) \left( \frac{P_{jit}}{P_{it}^{m}} \right)^{-\varepsilon} \left( Y_{it}^{C} + Y_{it}^{G} \right) dj di \\ &= \int_{0}^{1} \int_{0}^{1} \left[ \frac{P_{jit}^{1-\varepsilon}}{(P_{it}^{m})^{-\varepsilon}} \left( Y_{it}^{C} + Y_{it}^{G} \right) - \frac{W_{t}}{A_{t}} \left( A_{t} N_{jit} \right) \right] dj di \\ &= \int_{0}^{1} \left( P_{it}^{m} \right)^{\varepsilon} \left( Y_{it}^{C} + Y_{it}^{G} \right) \left( \int_{0}^{1} P_{jit}^{1-\varepsilon} dj \right) di - W_{t} \int_{0}^{1} \int_{0}^{1} N_{jit} dj di \\ &= \int_{0}^{1} P_{it}^{m} \left( Y_{it}^{C} + Y_{it}^{G} \right) di - W_{t} N_{t} \end{split}$$

while aggregate profits from final goods producers can be written as

$$\int_{0}^{1} \Phi_{it}^{C} di + \int_{0}^{1} \Phi_{it}^{G} di = \int_{0}^{1} \left( P_{it} - P_{it}^{m} \right) Y_{it}^{C} di + \int_{0}^{1} \left( P_{it}^{G} - P_{it}^{m} \right) Y_{it}^{G} di$$
$$= \int_{0}^{1} \left( P_{it}^{C} Y_{it}^{C} + P_{it}^{G} Y_{it}^{G} \right) di - \int_{0}^{1} P_{it}^{m} \left( Y_{it}^{C} + Y_{it}^{G} \right) di$$

Then, the economy wide profits are

$$\begin{split} \Phi_t &= \int_0^1 \Phi_{it}^C di + \int_0^1 \Phi_{it}^G di + \int_0^1 \int_0^1 \Phi_{jit} dj di \\ &= \int_0^1 \left( P_{it}^C Y_{it}^C + P_{it}^G Y_{it}^G \right) di - \int_0^1 P_{it}^m \left( Y_{it}^C + Y_{it}^G \right) di \\ &+ \int_0^1 P_{it}^m (Y_{it}^C + Y_{it}^G) di - W_t N_t \\ &= P_t^C Y_t^C + P_t^G Y_t^G - W_t N_t \end{split}$$

where we have used the assumption of symmetric final goods sectors to obtain the final result.

**Aggregate resource constraint:** Combining the households' budget constraint with the government budget constraint and the above definition of profits gives the aggregate resource constraint

$$P_t^C C_t + P_t^G G_t = P_t^C Y_t^C + P_t^G Y_t^G.$$

# B.2 System of Equilibrium Conditions

Consumers:

$$X_t = C_t - \theta C_{t-1} \tag{19}$$

$$X_t^G = G_t - \theta^G G_{t-1} \tag{20}$$

$$\frac{\chi N_t^{\upsilon}}{X_t^{-\sigma}} = (1 - \tau_t) w_t \tag{21}$$

$$X_t^{-\sigma} = \beta E_t \left[ X_{t+1}^{-\sigma} \frac{P_t^C}{P_{t+1}^C} \right] R_t$$
(22)

Final goods firms:

When final goods firms can price discriminate between private and public purchasers of their product their behaviour is described by,

$$C_t = \eta \omega_t X_t \tag{23}$$

$$\omega_t = \left(1 - \frac{1}{\mu_t^C}\right) + \theta \beta E_t \left[ \left(\frac{X_{t+1}}{X_t}\right)^{-\sigma} \omega_{t+1} \right]$$
(24)

$$G_t = \eta \omega_t^G X_t^G \tag{25}$$

$$\omega_t^G = \left(1 - \frac{1}{\mu_t^G}\right) + \theta^G \beta E_t \left[ \left(\frac{X_{t+1}}{X_t}\right)^{-\sigma} \frac{\pi_{t+1}^G}{\pi_{t+1}^C} \,\omega_{t+1}^G \right] \tag{26}$$

While, when no such price discrimination is possible, then the mark-ups are the same across sectors,

$$\mu_t^G = \mu_t^C \tag{27}$$

and equations (23) and (25) are combined as,

$$C_t + G_t = \eta(\omega_t X_t + \omega_t^G X_t^G).$$
(28)

Intermediate goods producers:

$$\left(P_t^m\right)^{1-\varepsilon} = \alpha \left(P_{t-1}^m\right)^{1-\varepsilon} + (1-\alpha) \left(P_{jt}^*\right)^{1-\varepsilon}$$
(29)

$$\frac{P_{jt}^*}{P_t^C} = \left(\frac{\xi_t}{\xi_t - 1}\right) \frac{K_{1t}}{K_{2t}} \tag{30}$$

where : 
$$K_{1t} = X_t^{-\sigma} mc_t \left(\mu_t^C\right)^{-\varepsilon} \left(Y_t^C + Y_t^G\right) + \alpha \beta E_t \left[K_{1t+1} \left(\pi_{t+1}^C\right)^{\varepsilon}\right]$$
 (31)

: 
$$K_{2t} = X_t^{-\sigma} \left(\mu_t^C\right)^{-\varepsilon} \left(Y_t^C + Y_t^G\right) + \alpha \beta E_t \left[K_{2t+1} \left(\pi_{t+1}^C\right)^{\varepsilon-1}\right]$$
 (32)

$$mc_t = \frac{w_t}{A_t} \tag{33}$$

$$\ln A_t = \rho_A \ln A_{t-1} + \varrho_t^A \tag{34}$$

$$\ln \xi_t = (1 - \rho_m) \ln \left(\varepsilon\right) + \rho_m \ln \xi_{t-1} + \varrho_t^m \tag{35}$$

Aggregate production:

$$Y_t = A_t \frac{N_t}{\Delta_t} \tag{36}$$

$$\Delta_t = (1 - \alpha) \left(\frac{P_t^*}{P_t^m}\right)^{-\varepsilon} + \alpha \left(\pi_t^m\right)^{\varepsilon} \Delta_{t-1}$$
(37)

Government budget constraint:

$$\frac{B_{t+1}}{R_t} = B_t + P_t^G G_t - W_t N_t \tau_t \tag{38}$$

And the following relationships linking the inflation measures,

$$\pi_t^m = \pi_t^C \frac{\mu_{t-1}^C}{\mu_t^C} = \pi_t^G \frac{\mu_{t-1}^G}{\mu_t^G}.$$
(39)

### B.3 The Deterministic Steady State

The non-stochastic long-run equilibrium is characterized by constant real variables and nominal variables growing at a constant rate. The equilibrium conditions (19) - (28) reduce to:

$$X = (1 - \theta) C \tag{40}$$

$$X^G = \left(1 - \theta^G\right)G\tag{41}$$

$$\chi N^{\upsilon} X^{\sigma} = (1 - \tau) w \tag{42}$$

$$1 = \beta R \left( \pi^C \right)^{-1} \tag{43}$$

$$C = \eta \omega X \tag{44}$$

$$\mu^{C} = [1 - (1 - \theta\beta)\omega]^{-1}$$
(45)

$$G = \eta \omega^G X^G \tag{46}$$

$$\mu^{G} = \left[1 - \left(1 - \theta^{G}\beta\right)\omega^{G}\right]^{-1} \tag{47}$$

Without price discrimination, there is a common markup,

$$\mu^G = \mu^C \tag{48}$$

and the steady-state equations (44) and (46) combine in,

$$C + G = \eta(\omega X + \omega^G X^G) \tag{49}$$

$$1 = \alpha \left(\pi^m\right)^{\varepsilon - 1} + \left(1 - \alpha\right) \left(\frac{P^*}{P^C} \mu^C\right)^{1 - \varepsilon}$$
(50)

$$\frac{P^*}{P^C} = \frac{\xi}{\xi - 1} \frac{K_1}{K_2} = \left[ \frac{\varepsilon}{\varepsilon - 1} \frac{1 - \alpha\beta \left(\pi^C\right)^{\varepsilon - 1}}{1 - \alpha\beta \left(\pi^C\right)^{\varepsilon}} \right] mc$$
(51)

$$K_1 = \frac{u_X \ mc \ \left(\mu^C\right)^{-\varepsilon} Y}{1 - \alpha\beta \ (\pi^C)^{\varepsilon}}$$
(52)

$$K_2 = \frac{u_X \left(\mu^C\right)^{-\varepsilon} Y}{1 - \alpha\beta \left(\pi^C\right)^{\varepsilon - 1}}$$
(53)

$$mc = \frac{w}{A} \tag{54}$$

$$\xi = \varepsilon \tag{55}$$

$$A = 1 \tag{56}$$

$$Y = \frac{A}{\Delta}N\tag{57a}$$

$$\Delta = \frac{1 - \alpha}{1 - \alpha \left(\pi^m\right)^{\varepsilon}} \left(\frac{P^*}{P^C} \mu^C\right)^{-\varepsilon}$$
(58)

$$\frac{B}{P^{C}} \equiv b = \frac{\left(P^{G}/P^{C}\right)G - wN\tau}{R^{-1} - (\pi^{C})^{-1}}$$
(59)

$$\pi^m = \pi^C = \pi^G \tag{60}$$

Table 1 contains the imposed calibration restrictions. We assume values for the Frisch labour supply elasticity (1/v), and the following parameters,  $\beta$ ,  $\sigma$ ,  $\eta$ ,  $\varepsilon$ ,  $\alpha$ ,  $\theta$ ,  $\theta^G$ ,  $\chi$  and  $\chi^G$ . In describing optimal policy, we take the second order approximation around the Ramsey steady-state, which is obtained by the solving the steady-state of the model (as given by equations (40) - (60)), conditional on the optimal rate of inflation and levels of taxation and government spending (for a given government debt to GDP ratio) which are obtained by simultaneously solving the Ramsey first order conditions in (11).

## C The Social Planner's Problem

In order to assess the trade-offs facing the policy maker in a sticky-price economy subject to tax, monopolistic competition and consumption externality distortions, it is helpful to compute the efficient allocation that would be chosen by a social planner. The social planner ignores the nominal inertia and all other inefficiencies and chooses real allocations that maximize the representative consumer's utility subject to the aggregate production function and the law of motion for habit-adjusted private and public consumption:

$$\max_{\{X_t, C_t, N_t, X_t^G, G_t\}} E_0 \sum_{t=0}^{\infty} \beta^t u \left( X_t^*, N_t^*, X_t^{G*} \right)$$
  
s.t.  $C_t^* + G_t^* = A_t N_t^*$   
 $X_t^* = C_t^* - \theta C_{t-1}^*$   
 $X_t^{G*} = G_t^* - \theta^G G_{t-1}^*$ 

The optimal choice implies the following relationship between the marginal rate of substitution between labour and habit-adjusted private consumption and the intertemporal marginal rate of substitution in habit-adjusted private consumption

$$\frac{\chi\left(N_{t}^{*}\right)^{\nu}}{\left(X_{t}^{*}\right)^{-\sigma}} = A_{t} \left[1 - \theta\beta E_{t} \left(\frac{X_{t+1}^{*}}{X_{t}^{*}}\right)^{-\sigma}\right].$$

In addition, the balance between private and public consumption is given by,

$$(X_t^*)^{-\sigma} - \theta\beta E_t \left(X_{t+1}^*\right)^{-\sigma} = \chi^G [\left(X_t^{G*}\right)^{-\sigma} - \theta^G \beta E_t \left(X_{t+1}^{G*}\right)^{-\sigma}]$$

The deterministic steady state equivalent of these expressions are  $\chi (N^*)^{\upsilon} (X^*)^{\sigma} = A (1 - \theta \beta)$  and  $\left(\frac{C^*}{G^*}\right)^{-\sigma} = \chi^G \left(\frac{1 - \theta^G \beta}{1 - \theta \beta}\right) \left(\frac{1 - \theta^G}{1 - \theta}\right)^{-\sigma}$ , which upon further substitutions can be written as,

$$\chi (N^*)^{\upsilon + \sigma} \left[ (1 - \theta) \Psi^* A \right]^{\sigma} = A \left( 1 - \theta \beta \right)$$

and

$$\left(\frac{\Psi^*}{1-\Psi^*}\right)^{-\sigma} = \chi^G \left(\frac{1-\theta^G \beta}{1-\theta\beta}\right) \left(\frac{1-\theta^G}{1-\theta}\right)^{-\sigma},$$

where  $\Psi^*$  is the optimal steady state share of private consumption,  $\Psi^* \equiv \frac{C^*}{C^*+G^*}$ . In the case of equal habits in the two types of consumption goods, the last expression simplifies to  $\left(\frac{\Psi^*}{1-\Psi^*}\right)^{-\sigma} = \chi^G$ .

## **D** Log-linear Representation

Our log-linearised economy can be described as follows. Firstly, we have the IS curve in terms of habit-adjusted consumption,

$$\widehat{X}_t = E_t \widehat{X}_{t+1} - \frac{1}{\sigma} \widehat{R}_t + \frac{1}{\sigma} E_t \widehat{\pi}_{t+1}^C, \tag{61}$$

and the New Keynesian Phillips Curve (NKPC) written in terms of producer price inflation

$$\widehat{\pi}_t^m = \beta E_t \widehat{\pi}_{t+1}^m + \kappa \left( \widehat{mc}_t + \widehat{\mu}_t^C \right)$$
(62)

where  $\kappa \equiv \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}$ . The dynamic equations describing changes in the final goods markups can be written as,

$$\widehat{\omega}_t = \frac{1}{\mu^C \omega} \widehat{\mu}_t^C + \theta \beta E_t \widehat{\omega}_{t+1} + \theta \beta \sigma \left( \widehat{X}_t - E_t \widehat{X}_{t+1} \right)$$
(63)

$$\widehat{\omega}_{t}^{G} = \frac{1}{\mu^{G}\omega^{G}}\widehat{\mu}_{t}^{G} + \theta^{G}\beta E_{t}\left(\widehat{\omega}_{t+1}^{G} + \widehat{\pi}_{t+1}^{G} - \widehat{\pi}_{t+1}^{C}\right) + \theta^{G}\beta\sigma\left(\widehat{X}_{t} - E_{t}\widehat{X}_{t+1}\right)$$
(64)

where the shadow values of producing another unit of a final good for the private and public sectors are given by,

$$\widehat{\omega}_t = \widehat{C}_t - \widehat{X}_t \tag{65}$$

and

$$\widehat{\omega}_t^G = \widehat{G}_t - \widehat{X}_t^G. \tag{66}$$

We also have the following expressions defining habit-adjusted private and public goods consumption  $\hat{X}_t$  and  $\hat{X}_t^G$ , CPI inflation  $\hat{\pi}_t^C$ , public goods price inflation  $\hat{\pi}_t^G$ , and the real marginal cost  $\widehat{mc}_t$ :

$$\widehat{X}_{t} = \frac{1}{1-\theta} \left( \widehat{C}_{t} - \theta \widehat{C}_{t-1} \right)$$
(67)

$$\widehat{X}_{t}^{G} = \frac{1}{1 - \theta^{G}} \left( \widehat{G}_{t} - \theta^{G} \widehat{G}_{t-1} \right)$$
(68)

$$\widehat{\pi}_t^C = \widehat{\pi}_t^m + \widehat{\mu}_t^C - \widehat{\mu}_{t-1}^C \tag{69}$$

$$\widehat{\pi}_t^G = \widehat{\pi}_t^m + \widehat{\mu}_t^G - \widehat{\mu}_{t-1}^G \tag{70}$$

$$\widehat{mc}_t = \sigma \widehat{X}_t + \upsilon \left[ \Upsilon \widehat{C}_t + (1 - \Upsilon) \,\widehat{G}_t \right] + \frac{\tau}{1 - \tau} \widehat{\tau}_t - (1 + \upsilon) \,\widehat{A}_t, \tag{71}$$

where  $\Upsilon$  is the steady state share of private consumption out of the total production,

 $\Upsilon \equiv \frac{C}{C+G}.$  And finally, there is the government budget constraint,

$$\widehat{b}_{t+1} = \widehat{R}_t + R\left(\pi^C\right)^{-1} \left(\widehat{b}_t - \widehat{\pi}_t^C\right) + \frac{R}{b} \left[\frac{P^G}{P^C} G\left(\widehat{G}_t + \widehat{\mu}_t^G - \widehat{\mu}_t^C\right) - wN\tau\left(\widehat{w}_t + \widehat{N}_t + \widehat{\tau}_t\right)\right].$$
(72)

Parameter	Value	Description				
r	$(1.04)^{1/4}$	Real interest rate				
$\sigma$	2	Inverse of intertemporal elasticity of substitution				
$\epsilon_{Nw} = (1/\upsilon)$	4.0	Frisch labour supply elasticity				
$\eta$	11.0	Elasticity of substitution across final goods				
ε	11.0	Elasticity of substitution across intermediate goods				
$\alpha$	0.75	Degree of price stickiness				
heta	0.65	Degree of habits formation in private goods consumption				
$ heta^G$	0.65	Degree of habits formation in public goods consumption				
$\chi$	3.0	Relative weight on labour in the utility function				
$\chi _{\chi ^{G}}$	0.111	Relative weight on utility from public goods consumption				
B/GDP	$0.6 \times 4$	Debt to GDP ratio				
$ ho_A$	0.9	Persistence of technology shock				
$ ho_m$	0.9625	Persistence of markup shock				
$ ho_G$	0.9	Persistence of government spending shock				
$\sigma_A$	0.009	Standard deviation of technology shocks				
$\sigma_m$	0.0012	Standard deviation of markup shock				
$\sigma_G$	0.014	Standard deviation of government spending shocks				

Table 1: Parameter values used in simulations

	$\alpha_R$	$\alpha_{\pi}$	$\gamma_b$	$\kappa_b$	$\left(\frac{\zeta^{rules}}{\zeta^{Ramsey}} - 1\right) \times 100$
Tax, active, common	1.0335	3.2685	0.1119		15.78
Tax, active, discriminating	1.0347	3.1713	0.11181		17.24
Tax, passive, common	0.15868	-11.396	-1.1438		37.62
Tax, passive, discriminating	0.17937	-10.827	-1.1528		39.07
G, active, common	1.1021	1.1936		-0.16841	40.53
G, active, discriminating	1.0125	1.1833		-0.16749	43.44
G, passive, common	0.17985	-9.1775		0.50357	39.07
G, passive, discriminating	0.20985	-9.0365		0.52252	39.07
Tax, G, active, common	1.0244	3.7333	0.133354	-0.049243	14.33
Tax, G, active, discriminating	1.0262	3.6057	0.13351	-0.04859	14.33
Tax, G, passive, common	0.13472	-8.3309	-1.2867	0.68082	21.61
Tax, G, passive, discriminating	0.16432	-8.3398	-1.196	0.65231	21.61

Table 2: The optimal parameterisation of the simple log-linear rules with no response to output. Each row details the fiscal rule (by instrument type), the active/passive nature of the monetary policy rule, and the type of pricing of final private and public goods. The last column gives the percent increase in welfare costs relative to the Ramsey optimal policy.

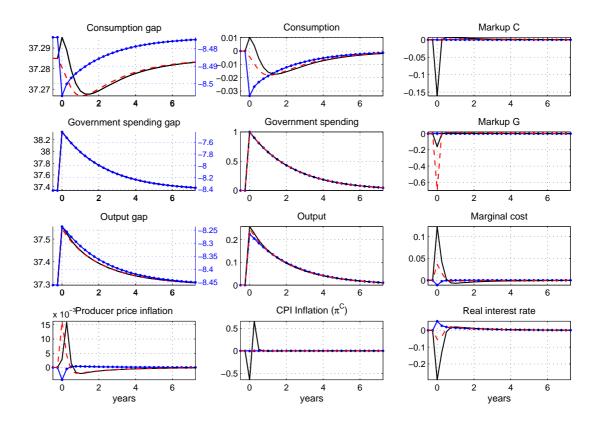


Figure 1: Impulse responses to a +1% government spending shock with optimal monetary policy: no habits (dots) and deep habits ( $\theta = \theta^G = 0.65$ ) with common pricing (solid line) and with discriminating pricing (dash lines). The inflation and interest rate variables are expressed in annualized terms. Gap variables under no habits read off the right y-axis.

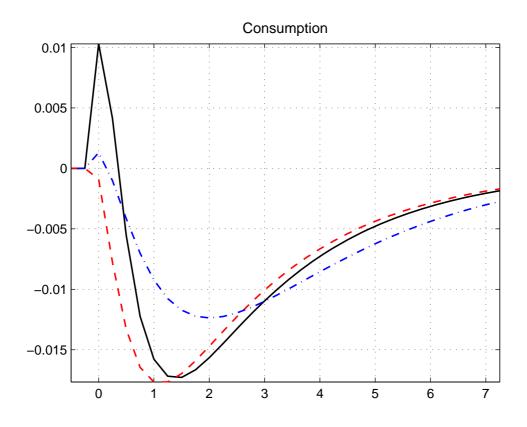


Figure 2: Consumption responses to a +1% government spending shock under optimal monetary policy, with common pricing of private and public goods:  $\theta = \theta^G = 0.65$  (solid line),  $\theta = 0.65$  and  $\theta^G = 0.2$  (dashed line),  $\theta = 0.85$  and  $\theta^G = 0$  (dash-dot line).

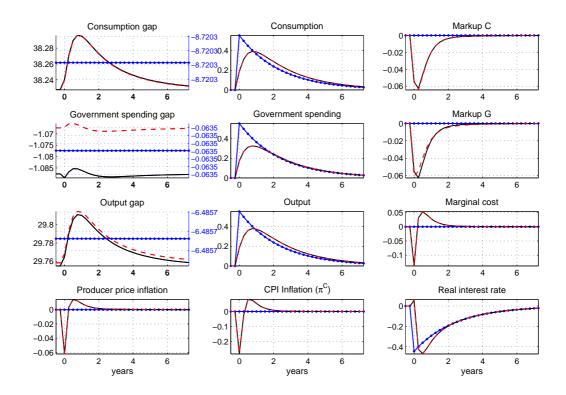


Figure 3: Impulse responses to a +1% technology shock with optimal monetary and fiscal policy, the case of *lump-sum taxes*: no habits (dots) and deep habits ( $\theta = \theta^G = 0.65$ ) with common pricing (solid line) and with discriminating pricing (dash lines). The inflation and interest rate variables are expressed in annualized terms. Gap variables under no habits read off the right y-axis.

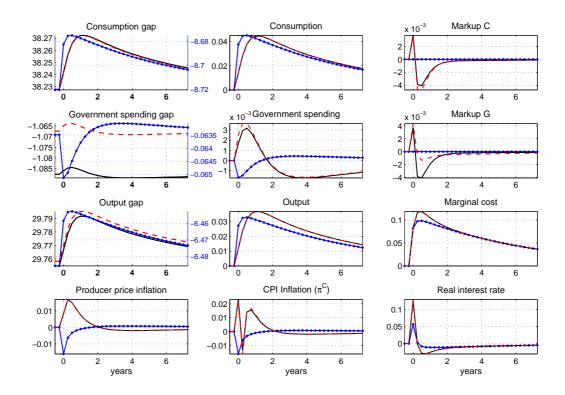


Figure 4: Impulse responses to a negative markup shock  $(+ 1\% \text{ change in } \xi_t)$  under optimal monetary and fiscal policy, the case of *lump-sum taxes*: no habits (dots) and deep habits ( $\theta = \theta^G = 0.65$ ) with common pricing (solid line) and with discriminating pricing (dash lines). The inflation and interest rate variables are expressed in annualized terms. Gap variables under no habits read off the right y-axis.

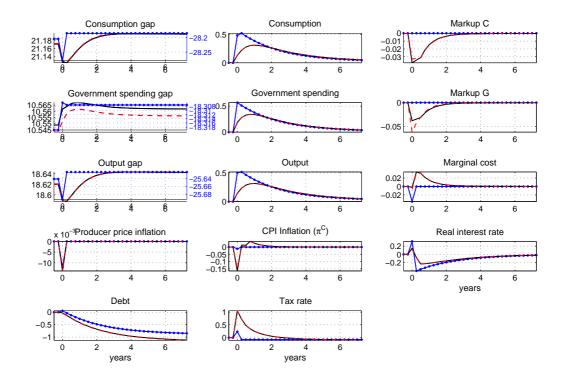


Figure 5: Impulse responses to a +1% technology shock under optimal monetary and fiscal policy: no habits (dots) and deep habits ( $\theta = \theta^G = 0.65$ ) with common pricing (solid lines) and with discriminating pricing (dash lines). The inflation and interest rate variables are expressed in annualized terms. Gap variables under no habits read off the right y-axis.

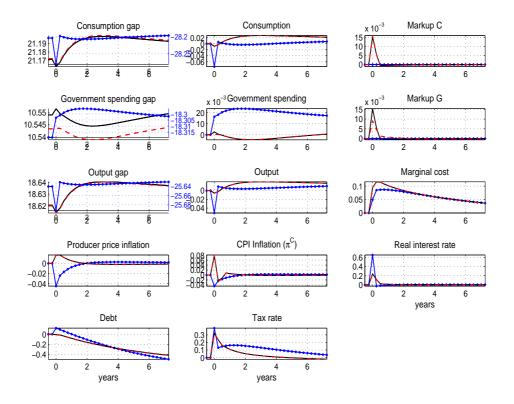


Figure 6: Impulse responses to a negative markup shock  $(+ 1\% \text{ change in } \xi_t)$  under optimal monetary and fiscal policy: no habits (dots) and deep habits ( $\theta = \theta^G = 0.65$ ) with common pricing (solid lines) and with discriminating pricing (dash lines). The inflation and interest rate variables are expressed in annualized terms. Gap variables under no habits read off the right y-axis.

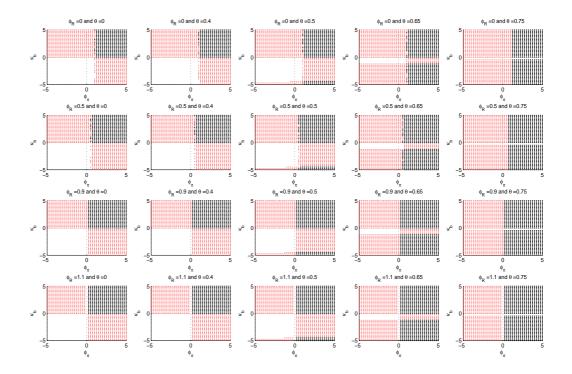


Figure 7: Determinacy Properties of the Government Spending Rule with Common Pricing: determinacy (light grey), indeterminacy (blanks), and instability (dark grey).

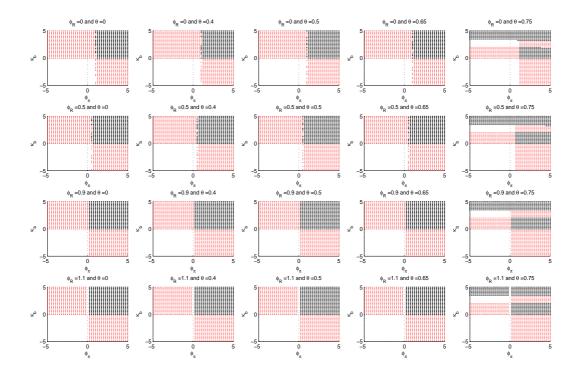


Figure 8: Determinacy Properties of the Government Spending Rule with Price Discrimination: determinacy (light grey), indeterminacy (blanks), and instability (dark grey).

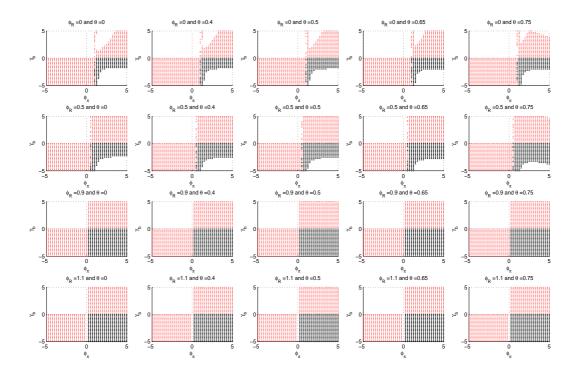


Figure 9: Determinacy Properties of the Tax Rule with Common Pricing: determinacy (light grey), indeterminacy (blanks), and instability (dark grey).

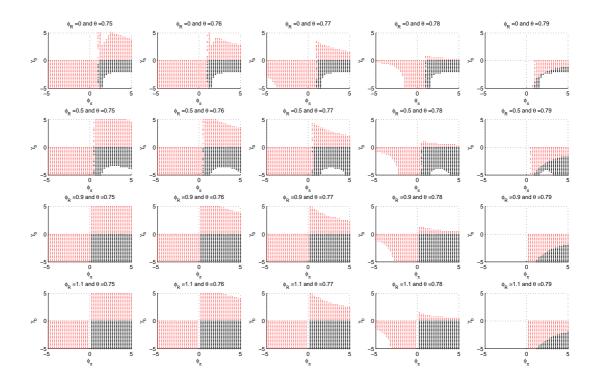


Figure 10: Determinacy Properties of the Tax Rule with Common Pricing under high levels of habits formation: determinacy (light grey), indeterminacy (blanks), and instability (dark grey).

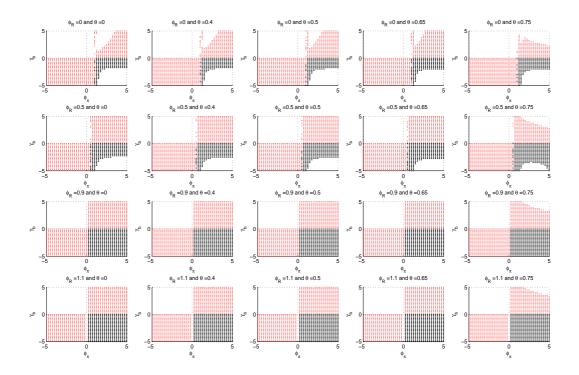


Figure 11: Determinacy Properties of the Tax Rule with Price Discrimination: determinacy (light grey), indeterminacy (blanks), and instability (dark grey).

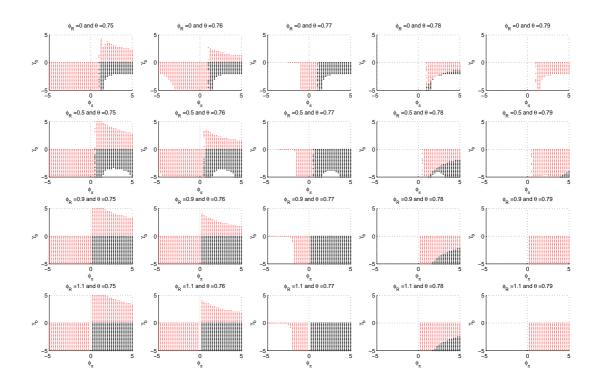


Figure 12: Determinacy Properties of the Tax Rule with Price Discrimination under high levels of habits formation: determinacy (light grey), indeterminacy (blanks), and instability (dark grey).

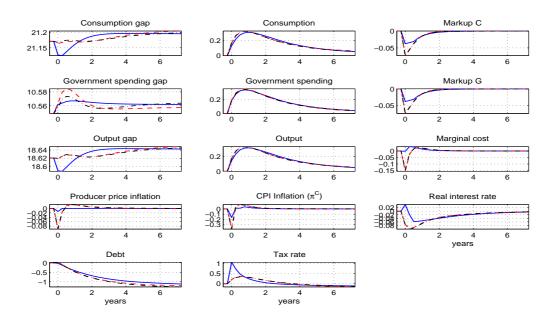


Figure 13: Impulse responses to a +1% technology shock under the Ramsey optimal policy (solid lines) and the optimal simple rules: quadratic monetary policy with linear fiscal policy (dash lines) and linear monetary policy with quadratic fiscal policy (dash dot lines), the benchmark calibration for deep habits with common pricing. The inflation and interest rate variables are expressed in annualized terms.

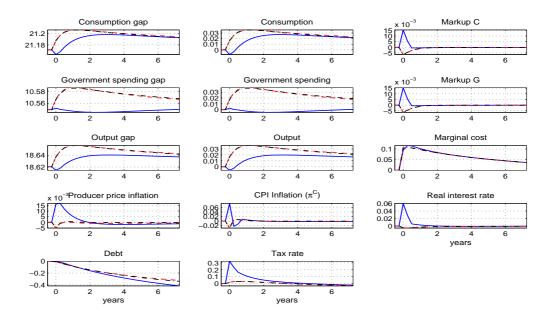


Figure 14: Impulse responses to negative markup shock  $(+1\% \text{ change in } \xi_t)$  under the Ramsey optimal policy (solid lines) and the optimal simple rules: quadratic monetary policy with linear fiscal policy (dash lines) and linear monetary policy with quadratic fiscal policy (dash-dot lines), the benchmark calibration for deep habits with common pricing. The inflation and interest rate variables are expressed in annualized terms.