# Why the Rich Should Like R&D Less (revised version)

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**Abstract**

It is well known that research and development (R&D) is an important engine for economic growth. Also, initial wealth inequality and subsequent economic growth are well known to be related. This paper links inequality and R&D-driven growth. It shows that in a class of economies where R&D is the main engine for growth, different wealth groups differ in their desire for aggregate innovative efforts: the higher the profit share of the individual's incomes the lower their ideal aggregate R&D and innovation. If rich shareholders were able to pursue their common interest and to discourage too much R&D compared, then a pro-labour government able to impose distortionary progressive taxation, by minimizing the difference between the rich and the poor can maximize growth. Such predicted negative relationship between desired R&D and dynastic wealth is robust to any subsidy rate lower than 100%.

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## 1. Introduction

In the political economy literature, the relationship between inequality and growth has long been studied (Acemoglu 2008), without reference to research and development (R&D), despite the well known importance of R&D and innovation for economic growth itself[1]. This paper first computes analytically the ideal steady state

[1] Notable exceptions are Chou and Talmain (1996) and Foellmi and Zweimueller (2006), exploiting the elastic labour supply channel and, respectively, the demand size effects with non-homotetic preferences. Garcia-Penalosa and Wen (2008) more recently obtained a positive relationship between taxation and R&D based on the insurance effect of progressivity on risk-averse researchers. Potentially positive effect of progressivity on R&D cooperation are stressed by Cozzi (2005).

aggregate growth path for different wealth groups, in a simple R&D-driven growth model *a la* Jones (1995), and then tests the robustness of the results under alternative assumptions. I will show that the higher the family profit share of income (relative to its labour income share) the lower their preferred aggregate R&D employment. The reason is that more R&D employment leads to faster innovation and growth, which is beneficial to both wage and profit earners, but it also implies less manufacturing employment and less profits in the aggregate, which is detrimental to the profit component of income. The profit-richer the individual the more important her/his dividend incomes, and hence the stronger the perceived negative externality of aggregate R&D employment.

In so far as richer shareholders are better able to solve their free riding problems than poor families and to successfully lobby industrial policy, this model suggests a new channel from more inequality to less growth and a potentially new channel from wealth redistribution to growth.

Our results are very robust, as they hold under different behavioural and technological scenarios. Therefore we first obtain them in Section 2, by assuming constant shares of the labour force employed in manufacturing and R&D: the reason is the simplicity of exposition. In Sections 3 and 4 all the results previously obtained in the Solow framework of Section 2 are confirmed in a fully dynamic Ramsey-Cass-Koopmans setting. Section 5 concludes with some observations. The Appendix contains some of the most cumbersome proofs.

## 2. An Elementary Golden Rule Analysis

Let us assume[2] a unit mass of infinitely lived families whose members grow at constant rate $n > 0$. Let $c_{ht}$ be the real consumption

[2] I am indebted to a Referee for suggesting the brilliant expositional style that follows, including the reference to the Golden Rule.

flow of a member of family $h \in [0, 1]$ at time $t$. In this stylized economy each individual is endowed with a unit density of flow labor that generates no disutility. Therefore, the labor supply at each date $t \geq 0$ is:

$$L_t = L_0 e^{nt},\ L_0 > 0.$$

A share $0 \leq s_{Yt} \leq 1$ of the labor force is assumed to be employed in the manufacturing production, yielding a final perishable consumption output of

$$Y_t = A_t L_t s_{Yt} = A_t L_t \left(1 - s_{At}\right)$$

consumption goods per unit time. $A_t$ denotes the stock of productive knowledge cumulated as of time $t$, and $s_{At} = 1 - s_{Yt}$ is the share of labor employed in the research and development (R&D) sector. Being our analysis highly aggregate, it is not necessary to think of only one kind of innovative activity: standard methods of Schumpeterian growth theory (shown by Grossman and Helpman 1991, Jones 2005, Aghion and Howitt 1998, Barro and Sala-I-Martin 2004, Acemoglu 2008) could be adopted to render our equations consistent with economies with improving product quality, improving production methods or extending product variety.

Consistently with most R&D-driven growth models, in this section I will assume that labor appropriates a constant fraction[3] $0 < \alpha < 1$ of its marginal productivity in the final consumption production. Hence real wages are equal to $w_t = \alpha A_t$.

Aggregate profits are given by:

$$\left(1 - \alpha\right) Y_t = \left(1 - \alpha\right) A_t L_t \left(1 - s_{At}\right).$$

It therefore follows that gross of R&D aggregate profits decrease with the R&D share of labor. In fact, more R&D employment implies less manufacturing employment, and hence less consumption good revenues and profits.

[3]Usually pinned down by the inverse of the monopolistic mark-up.

In this section, we will focus on steady states, i.e. on allocations in which the shares of labor employed in the manufacturing and in the R&D are constant: hence we will assume $s_{At} = s_A$ for all $t \geq 0$. In order to guarantee non-negative profits[4]:

$$(1-\alpha)\, Y_t - L_t s_A \alpha A_t = (1-\alpha)\, A_t L_t (1-s_A) - L_t s_A \alpha A_t \geq 0,$$

that is:

$$s_A \leq 1-\alpha. \tag{1}$$

Let us assume the following law of motion of technology:

$$\dot{A}_t = v A_t^{\phi} L_t s_{At}, \tag{2}$$

where $v > 0$ and $\phi < 1$. As suggested by Jones (1995 and 2005), this law of motion of technology departs from the early generation endogenous growth models, by implying that as technology evolves it becomes increasingly more complex for innovators to induce equiproportional increases in the percapita final output[5]. Following Jones (1995), the long run[6] growth rate of per-capita output is:

$$g_{\frac{Y}{L}} = g_{At} \equiv \frac{\dot{A}_t}{A_t} \rightarrow \frac{n}{1-\phi}.$$

As well known, Jones' solution to the strong scale effect implies that R&D policy, by affecting $s_{At}$, only affects the absolute (not the relative or percentage) long-run growth of productivity, in fact (assuming constant shares $s_{At} = s_A$):

[4] Alternatively, we could allow negative profits in so far as the difference can be financed by a tax on labour income.

[5] In fact, the growth rate of $\frac{Y}{L}$ is equal to $\frac{\dot{A}}{A} = \frac{vLs_A}{A^{\phi}}$. As knowledge cumulates $A$ increases and more and more labour is required to invent enough new ideas compared to their existing stock.

[6] For a general analysis of the convergence of Jones' (1995) economy to a balanced growth path, see Arnold (2006).

$$A_t \to \text{constant} \cdot \left(s_A e^{nt}\right)^{\frac{1}{1-\phi}}.$$

Long-run per-capita consumption is:

$$\frac{Y_t}{L_t} \to (1 - s_A) \left(\frac{\upsilon s_A (1-\phi)}{n}\right)^{\frac{1}{1-\phi}} L_0^{\frac{1}{1-\phi}} e^{\frac{n}{1-\phi}t}.$$

In this economy different families have different profit incomes. This can derive from different initial wealth levels, that are perpetuated by perfectly diversified portfolios of R&D firm shares. In fact, the usual models with vertical and/or horizontal innovation usually assume that perfectly efficient capital markets allow individual wealth to evolve deterministically, due to a law of large numbers operating on a continuum of independent stochastic processes. Therefore, despite perpetual leapfrogging at the industry level[7], the personal wealth distribution evolves deterministic.

Since perfectly diversified portfolios in bonds or in equities are actuarially equivalent in this stylized economy, I will work under the assumption that a scalar $\theta_h$ denotes the level of uniform stockholding of the existing profit generating firms possessed by each family $h$. A family with average wealth will have $\theta_h = 1$, a "rich" will have $\theta_h > 1$, while a family with below average stock holding $\theta_h < 1$ will be deemed "poor".

Family $h$'s per-member consumption is given by her income:

$$\begin{aligned} c_{ht} &= \underbrace{\alpha A_t}_{\text{Labour Income}} + \left[\underbrace{(1-\alpha) A_t (1 - s_A)}_{\text{Gross Profits}} - \underbrace{s_A \alpha A_t}_{\text{R\&D Investment}}\right] \cdot \underbrace{\theta_h}_{\text{Ownership Share}} \qquad (3) \\ &= \text{constant} \cdot \left(s_A e^{nt}\right)^{\frac{1}{1-\phi}} \left[(1 - \alpha - s_A)\,\theta_h + \alpha\right]. \end{aligned}$$

[7] In fact, our aggregate analysis is consistent with an explicit market for patents, with R&D being accomplished by separate firms that derive revenue from selling patents. The market then allocates labour between R&D and manufacturing and, in the steady state, our results hold.

Notice that eq. (3) decomposes individual income into a labor component, $\alpha A_t$, common to everybody, and a profit component, $(1-\alpha-s_A)\,A_t\theta_h$, that increases in the dividend share of the family, that is in its relative shareholding. It is important to note that the higher the profit component the stronger the negative effect of $s_A$.

Maximizing steady state consumption with respect to $s_A$ gives the following the ideal R&D share of labor:

$$s_A^*(\theta_h) = \min\left[\frac{1}{2-\phi}\left(\frac{\alpha}{\theta_h}+1-\alpha\right), 1-\alpha\right]. \tag{4}$$

It follows from eq. (4) that all wealth groups want a positive R&D investment, but the richer the individual the lower her desired R&D investment. The higher the labor share of one's income the higher her ideal R&D investment, because the benefits in terms of the higher marginal productivity of labor offset the negative effects on the profit level. Also notice that the upper constraint $s_A \leq 1-\alpha$ implies that eq. (4) gives an inverted relationship between shareholding and desired growth only for rich enough wealth groups, i.e. for individuals with

$$\theta_h \geq \frac{\alpha}{(1-\alpha)\,(1-\phi)} \equiv \widetilde{\theta}.$$

It is useful to remark that, in so far as the GDP share of labor is not lower than the share of profits (which is always the case in industrialized countries), $\widetilde{\theta} > 1$, that is only the relatively rich want slower growth.

### *2.1. Subsidies to R&D*

After reading the previous section, the reader might have thought that our main result is due to the fact that the poorer the individual the lower the R&D investment she has to pay, by indirectly possessing the firms in the economy. However, this is not the case: the main motive for the capitalist households' opposing too high R&D employment is their perception of the aggregate negative effect of R&D employment on profits, and this survives any amount of alleviation of individual direct R&D expenditures. In fact, let us assume that R&D employment is subsidized at rate $\sigma \in [0,1[$. Government budget is balanced: lump sum taxes are equally imposed on all individuals to fund R&D subsidies. Family $h$'s per-member consumption would now be given by her income:

$$c_h = [(1-\alpha)\,A(1-s_A) - (1-\sigma)s_A\alpha A]\,\theta_h + \alpha A - \sigma s_A\alpha. \qquad (5)$$

$$\begin{aligned} c_{ht} &= \underbrace{\alpha A_t}_{\text{Labour Income}} + \left[\underbrace{(1-\alpha)\,A_t(1-s_A)}_{\text{Gross Profits}} - \underbrace{(1-\sigma)s_A\alpha A_t}_{\text{R\&D Investment}}\right]\cdot \underbrace{\theta_h}_{\text{Ownership Share}} \\ - \underbrace{\sigma s_A\alpha A_t}_{\text{Lump Sum Tax}} &= \\ &\text{constant}\cdot\left(s_A e^{nt}\right)^{\frac{1}{1-\phi}}\{[(1-\alpha)\,A_t(1-s_A)-(1-\sigma)s_A\alpha A_t]\,\theta_h + \\ &(1-\sigma s_A)\alpha A_t\}. \qquad (6) \end{aligned}$$

By repeating the same calculations as in the previous section, we easily get to:

$$s_A^*(\theta_h;\sigma) = \min\left[\frac{(1-\alpha)\,\theta_h+\alpha}{(2-\phi)\,[(1-\sigma\alpha)\theta_h+\sigma\alpha]}, \frac{1-\alpha}{1-\alpha+\sigma\alpha}\right]. \qquad (7)$$

The derivative with respect to $\theta_h$ of the first term in the square bracket[8] is negative everywhere for all $\sigma < 1$. This proves that no matter how high - though never equal to 100% - the R&D subsidy rate, the richer families always want less R&D than the poorer families.

[8] Equal to $\frac{-\alpha(1-\sigma)}{(2-\phi)[(1-\sigma\alpha)\theta_h+\sigma\alpha]^2}$.

### *2.2. Capital Income Tax and Ideal R&D*

Let us now assume that capital incomes - profits - are taxed at rate $\tau \in [0, 1[$. How would this affect desired R&D employment? In order to isolate the effect of marginal tax rates on desired growth, in this section I will assume that the tax proceeds are just wasted. That is, let us assume that individual consumption is given by:

$$\begin{aligned} c_{ht} &= [(1-\alpha) A_t(1-s_A) - s_A \alpha A_t] \theta_h (1-\tau) + \alpha A_t = \\ &\quad A_t [(1-\alpha-s_A) \theta_h (1-\tau) + \alpha] . \end{aligned}$$

By repeating the same computations as in Section 2, it is straightforward to prove that:

$$s_A^*(\theta_h; \tau) = \min \left[ \frac{1}{2-\phi} \left( \frac{\alpha}{\theta_h (1-\tau)} + 1 - \alpha \right), 1-\alpha \right] . \quad (8)$$

Eq. (8) shows that the desired R&D employment is an increasing function of the marginal income tax rate. The reason why a more heavily taxed dividend earner wants more growth is that her after tax income is more similar to that of the poor. This implies that her desired R&D employment takes into account her labor incomes relatively more.

### *2.3. Ownership versus Skill*

If individuals differ in their labour skill endowments, as opposed to asset ownership, we obtain interesting implications. To view this in a particularly simple way, I will postulate perfectly persistent ability shocks (as for example in Alesina and Angeletos, 2005). Let us assume that each member of a household $h$ supplies $\omega_h$ units of labour, where $P(h)$ is the constant society skill distribution, and $\int \omega_h dP(h) = 1$.

Here a "high skill" household has $\omega_h > 1$ and a "low skill" household has $\omega_h < 1$.

If all members of dynasty $h$ are endowed with $\omega_h$ units of labour, the family $h$'s per-member consumption is now given by

$$c_{ht} = [(1-\alpha) A_t(1-s_A) - s_A \alpha A_t] \theta_h + \omega_h \alpha A_t = A_t [(1-\alpha-s_A) \theta_h + \alpha \omega_h], \quad (9)$$

which replaces eq. (3). As before we can multiply by the $A_t$ =constant$\cdot \left(s_A e^{nt}\right)^{\frac{1}{1-\phi}}$ and maximize it with respect to $s_A$, obtaining:

$$s_A^*(\theta_h) = \min \left[ \frac{1}{2-\phi} \left( \frac{\omega_h}{\theta_h} \alpha + 1 - \alpha \right), 1-\alpha \right]. \quad (10)$$

Now the link between income and the preferred level of R&D labour allocation is sharper and depends on the relationship between the asset distribution and the skill distribution. Parameter $\frac{\omega_h}{\theta_h}$ plays a major role, as it allows us to determine the political preference of an individual toward R&D by simply considering the ratio of labour to ownership incomes. The suggested empirical prediction is that the higher the labour share of someone's income relative to her ownership share the higher her preference for R&D.

An obvious qualification is that, while it is reasonable to assume that non-human wealth is inherited, the skill distribution of a family's offspring is likely more akin to that in society. If this is the case, eq. (10) has to be integrated over family members according to $P(h)$, making us re-obtain eq. (4)'s predictions despite skill heterogeneity.

An important point is the following: if wealth brings about higher labour productivity - for several reasons such as better nutrition/health/location, more human capital, more social capital (family connections), richer internal knowledge of the workings of the firms, etc. - it is possible that $\omega_h$ be positively correlated with $\theta_h$. Then depending of the kind of correlation our results could be either strengthened or weakened. For example, assume we had estimated a linear model characterized by $\omega_h = a + b\theta_h$, with $a > 0$ being the minimum labour skill of the poorest and $b > 0$ measuring the increase in skill associated with firm ownership. Then we would have to rewrite eq. (10) as follows:

$$\begin{aligned} s_A^*(\theta_h) &= \min\left[\frac{1}{2-\phi}\left(\frac{a+b\theta_h}{\theta_h}\alpha+1-\alpha\right),1-\alpha\right]= \\ & \min\left\{\frac{1}{2-\phi}\left[\left(\frac{a}{\theta_h}+b\right)\alpha+1-\alpha\right],1-\alpha\right\}. \end{aligned} \quad (11)$$

Clearly eq. (11) generates a negative correlation between firm ownership and desired R&D employment, the more so the higher the labour endowment of the poorest.

## 3. A Fully Dynamic Political Equilibrium

In this section, I will show that the main qualitative result of the previous sections' simple Golden Rule analysis carries through to the fully dynamic version *a la* Ramsey-Cass-Koopmans, in which the R&D policy - $s_A$ - is not restricted to be constant from the beginning. We will instead assume that each wealth group $h$ decides its most preferred R&D policy as the time trajectory $s_A(\cdot)$ that maximizes its welfare from the beginning of time on. Individual preferences are represented by the following utility functional:

$$\int_0^\infty \ln c_{ht} e^{-(\rho-n)t} dt,$$

where $\rho > 0$ is the subjective rate of time preferences, satisfying $0 < n < \rho$. Therefore let us consider the following optimization problem:

$$\max_{c_{ht}, s_{At}, \theta_{ht}} \int_0^\infty \ln c_{ht} e^{-(\rho-n)t} dt,$$

subject to:

$$c_{ht} = A_t\left[(1-\alpha-s_{At})\,\theta_{ht}+\alpha\omega_h\right], \quad (12)$$

$$\dot{A}_t = vA_t^\phi L_t s_{At},$$

$$\dot{L}_t = nL_t, \quad (13)$$

where $A_0$, and $L_0$ are given and positive, and $\theta_{h0}$ are non-negative numbers. The following holds:

The following hold:

**Lemma 1** *The dynamic equilibrium family shares of aggregate profits are constant:* $\theta_{ht} = \theta_{h0} \equiv \theta_h$ *for all* $t \geq 0$ *and for all* $h$.

Focussing - without loss of generality - on wealth classes such that inequality (1) holds strict, we obtain:

**Proposition 2** *The dynamic equilibrium chosen by each household is determinate and the long run level of* $s_A$ *preferred by household* $h$ *is:*

$$s_A^*(\theta_h) = \frac{1}{\frac{\rho(1-\phi)}{n} + 1} \left( \frac{\alpha}{\theta_h} + 1 - \alpha \right). \tag{14}$$

**Proof.** See the Appendix.

Eq. (14) shows that a household's preferred long-run R&D share of the labour force is a decreasing function of its profit share. The value added by the Ramsey-Cass-Koopmans logic followed in this section is in the presence of the subjective rate of time preference $\rho$ in the denominator: the more patient the households (the lower $\rho$) the higher their desired growth enhancing policy. Comparing eq.s (14) and (4) we see how the conclusions of our Golden Rule exercise do not differ qualitatively from the more sophisticated analysis of this section. In particular, the ratio of the most prefererred policies of two households $h$ and $h'$ is

$$\frac{s_A^*(\theta_h)}{s_A^*(\theta_{h'})} = \left( \frac{\alpha}{\theta_h} + 1 - \alpha \right) / \left( \frac{\alpha}{\theta_{h'}} + 1 - \alpha \right)$$

in both cases.

By re-defining inequality in terms of heterogeneous labour skill as well as initial wealth, as in eq. (9), and by repeating the same derivations[9] we easily get to the following counterpart of eq. (10):

[9] Notice that eq. (25) holds anyway.

**Corollary 3** *If the individuals differ in their labour skills, as assumed in section 2.3, the long-run preferred policy becomes:*

$$s_A^*(\theta_h) = \frac{1}{\frac{\rho(1-\phi)}{n} + 1} \left( \frac{\alpha\omega_h}{\theta_h} + 1 - \alpha \right). \tag{15}$$

Like eq. (10), eq. (15) shows that the larger the unskill related source of wealth $\frac{\theta_h}{\omega_h}$ the weaker the desire for innovation-enhancing policies. In case skill and ownership be correlated, similar considerations could be made here as were made in the discussion of eq. (11).

It is important to remark that our conclusions about R&D policy preferences and inequality are not restricted to the specific utility function adopted. In fact the following holds:

**Corollary 4** *All our qualitative conclusions are valid for a general CRRA utility function with finite intertemporal elasticity of substitution.*

**Proof.** See the Appendix.

## 4. Alternative Laws of Motion of Technology in Fully Dynamic Equilibrium

### *4.1. Schumpeterian Knowledge Production Function*

We can easily test our results under an equally important and alternative law of motion of technology suggested by a number of authors (Smulders and Van de Klundert, 1995, Young, 1998, Peretto, 1998, Dinopoulous and Thompson, 1998, and Howitt, 1999) and supported by recent empirical analysis (Madsen, 2008, Ha and Howitt, 2007). This is a way out of the "strong scale effect" prediction of early generation endogenous growth models, that maintains endogenous growth by assuming that the larger population the more diluted aggregate R&D. According to this alternative, we replace eq. (2) with the following:

$$\dot{A}_t = vA_t \frac{L_{At}}{L_t} = vA_t s_{At}, \tag{16}$$

where $v > 0$. Hence we shall now solve:

$$\max_{c_{ht}, s_{At}, \theta_{ht}} \int_0^\infty \ln c_{ht} e^{-(\rho-n)t} dt,$$

subject to:

$$\begin{aligned} c_{ht} &= A_t \left[ (1 - \alpha - s_{At}) \theta_{ht} + \alpha\omega_h \right], & (17) \\ \dot{A}_t &= vA_t s_{At}, \\ \dot{L}_t &= nL_t, & (18) \end{aligned}$$

where $A_0$, and $L_0$ are given and positive, and $\theta_{h0}$ are non-negative numbers.

The following hold:

**Proposition 5** *The dynamic equilibrium level of $s_A$ preferred by household $h$ is constant for all $t \geq 0$ and equal to:*

$$s_A^*(\theta_h) = \left( \frac{\alpha\omega_h}{\theta_h} + 1 - \alpha \right) - \frac{\rho - n}{v}. \tag{19}$$

**Proof.** See Appendix.

As in Jones' (1995) "semi-endogenous" solution to the strong scale effect, also in this case the higher the profit share of someone's income the less growth promoting her ideal policy.

### *4.2. Updating Costs*

As a further test of robustness, we can check what happens if we account for positive research updating time costs, as modelled by Cozzi (2003) and Cozzi and Spinesi (2004) to be decreasing in productivity, due to increasingly better information and communication technology. In fact, in order for a researcher to target innovation correctly, he or she must first make sure to get informed and to understand the most recent new developments in his or her particular fields. Amending the previous models by assuming that the labour cost of updating each researcher's knowledge is $\frac{c}{A_t} > 0$, we can write:

$$\dot{A}_t = vA_t s_{At} \left(1 - c\frac{\dot{A}_t}{A_t}\right). \tag{20}$$

This implies that the growth rate is always equal to

$$\frac{\dot{A}_t}{A_t} = \frac{vs_{At}}{1 + cvs_{At}}. \tag{21}$$

Focussing on wealth levels such that inequality (1) holds strict, we obtain:

**Proposition 6** *The fully dynamic equilibrium with updating costs is characterized by* $\frac{\partial s_A^*}{\partial \theta_h} < 0$.

**Proof.** See Appendix.

## 5. Conclusions

This paper has analytically proved that - in the generality of the scale effect free R&D-driven growth technological assumptions - the R&D share of aggregate employment desired by every wealth group differs in a regular way: the more profit-rich the household lower its ideal aggregate R&D employment.

This paper has also showed that the higher profit tax rates the higher the aggregate R&D employment desired by each wealth group, thereby suggesting a positive relationship between marginal income tax rates and pro-innovation social attitude. The reason is that, though profits and wages gain in the same proportion from total factor productivity growth, the larger the relative weight of profits in someone's income the stronger the negative effect of diverting people from profitable manufacturing firms to R&D laboratories.

Future research may develop this point further. For example, if in the real world the rich shareholders are able to lobby the private sector at least to some extent, this model recreates the negative relationship between inequality and growth and the positive relationship between marginal tax rates and growth found in some empirical studies such as Perotti (1996).

# Appendix

**Proof of Lemma 1.** The problem can be analyzed in two stages. First characterize the rational expectations equilibrium for each R&D policy, and then maximize the household welfare with respect to that policy. At any time $t \geq 0$ each household knows that everyone in the economy is solving the following:

$$\max_{c_h(\cdot), s_A(\cdot), \theta_h(\cdot)} \int_t^\infty \ln c_{hs} e^{-(\rho-n)(s-t)} ds,$$

subject to:

$$\int_t^\infty c_{hs} e^{-\int_t^s (r_\tau - n) d\tau} ds \leq a_{ht}, \tag{22}$$

where $r(t)$ denotes time $t$'s real rate of return, common to all assets (consumer loans, firm stocks and bonds) viewed as perfect substitutes by the consumer; and $a_{ht}$ denotes household wealth. Clearly:

$$\dot{a}_{ht} = a_{ht} r_t + w_{ht} - c_{ht}. \tag{23}$$

Standard methods imply the Euler equation:

$$\dot{c}_{ht} = c_{ht} (r_t - \rho)$$

and hence - after solving it - $c_{hs} = c_{ht} e^{\int_t^s (r_\tau - \rho) d\tau}$. Plugging this into (22) gives:

$$\int_t^\infty c_{hs} e^{-\int_t^s (r_\tau - n) d\tau} ds = c_{ht} \int_t^\infty e^{-\int_t^s (\rho - n) d\tau} ds \leq a_{ht}, \tag{24}$$

which - due to non-satiatiated preferences - implies: $c_{ht} = (\rho - n) a_{ht}$. Hence, considering two families indexed by $h$ and $h'$, the following must hold:

$$\frac{\dot{c}_{ht}}{c_{ht}} = \frac{\dot{c}_{h't}}{c_{h't}} = \frac{\dot{a}_{ht}}{a_{ht}} = \frac{\dot{a}_{h't}}{a_{h't}},$$

which implies that $\frac{a_{ht}}{a_{h't}}$ stays constant and equal to $\frac{a_{h0}}{a_{h'0}}$. Therefore $\frac{a_{ht}r_t}{a_{h't}r_t}$ stays constant, which implies zero equilibrium consumer loans (otherwise they would last forever). Only firm assets will be held in household portfolios, hence

$$\frac{a_{ht}}{a_{h't}} = \frac{a_{ht}r_t}{a_{h't}r_t} = \frac{A_t\left(1-\alpha-s_{At}\right)\theta_{ht}}{A_t\left(1-\alpha-s_{At}\right)\theta_{h't}} = \frac{\theta_{ht}}{\theta_{h't}}. \tag{25}$$

From $\int \theta_{ht} dP(h) = 1$, it follows that $\theta_{ht} = \theta_{h0}$ for all $t \geq 0$.

**Proof of Proposition 2.** Based on the previous Lemma 1, we can assume constant shares from the beginning of time. The first order conditions for an optimum are:

$$\begin{aligned} \frac{\theta_h}{\left(1-\alpha-s_{At}\right)\theta_h+\alpha\omega_h} &= \lambda_t v A_t^{\phi} L_t, \\ \dot{\lambda}_t &= \lambda_t \left(\rho - n - \phi v A_t^{\phi-1} L_t s_{At}\right) - \frac{1}{A_t}, \\ \dot{A}_t &= v A_t^{\phi} L_t s_{At} \\ \dot{L}_t &= n L_t \end{aligned} \tag{26}$$

where $\lambda_t$ is the current value costate variable and the transversality condition becomes $\lim_{t\to\infty} \lambda_t A_t = 0$. Defining $x_t \equiv A_t^{\phi-1} L_t$ and $y_t \equiv \lambda_t A_t$ we can reduce the dimensionality of this system -thereby stationarizing it - obtaining:

$$\frac{\theta_h}{\left(1-\alpha-s_{At}\right)\theta_h+\alpha\omega_h} = y_t v x_t, \tag{27}$$

$$\dot{x}_t = x_t\left[n-(1-\phi)v s_{At} x_t\right], \tag{28}$$

$$\dot{y}_t = y_t\left[\rho-n+(1-\phi)v s_{At} x_t\right]-1. \tag{29}$$

Solving (27) for $s_A$ and plugging it into system (28)-(29) gives:

$$\begin{aligned} \dot{x}_t &= x_t\left[n-(1-\phi)\left(\frac{\alpha}{\theta_h}+1-\alpha\right)vx_t-\frac{1-\phi}{y_t}\right], & (30)\\ \dot{y}_t &= y_t\left[\rho-n+(1-\phi)v\left(\frac{\alpha}{\theta_h}+1-\alpha\right)x_t\right]-2+\phi. & (31)\end{aligned}$$

Therefore the unique economically meaningful steady state is:

$$\begin{aligned} s_A &= \frac{1}{\frac{\rho(1-\phi)}{n}+1}\left(\frac{\alpha}{\theta_h}+1-\alpha\right), & (32)\\ x &= \frac{\frac{n}{(1-\phi)}+\rho}{\left(\frac{\alpha}{\theta_h}+1-\alpha\right)v}, \\ y &= \frac{1}{\rho}. \end{aligned}$$

From the definition of $x$, we have:

$$A=\left[\frac{\left(\frac{\alpha}{\theta_h}+1-\alpha\right)vL}{\frac{n}{(1-\phi)}+\rho}\right]^{\frac{1}{(1-\phi)}}. \tag{33}$$

The Jacobian of system (30)-(31) at this steady state is:

$$\begin{pmatrix} -(1-\phi)\left(\frac{\alpha}{\theta_h}+1-\alpha\right)v, & \frac{1-\phi}{y^2}x \\ y(1-\phi)v\left(\frac{\alpha}{\theta_h}+1-\alpha\right), & \frac{2-\phi}{\rho} \end{pmatrix},$$

and its determinant is clearly negative. Therefore there are two real eigenvalues of opposite sign: hence the equilibrium is determinate.

**Proof of Corollary 4.** Simply repeat all the steps we have made, after assuming utility functional:

$$\int_0^\infty \frac{c_{ht}^{1-\eta}-1}{1-\eta}e^{-(\rho-n)t}dt,$$

where $\eta > 0$ is a parameter. The crucial part is the following: instead of first order conditions (26), we now have

$$\begin{aligned}
\lambda_t v A_t^{\phi} L_t &= A_t^{1-\eta}\theta_h \left[\left(1-\alpha-s_{At}\right)\theta_h + \alpha\omega_h\right]^{-\eta}, \\
\dot{\lambda}_t &= \lambda_t\left(\rho - n - \phi v A_t^{\phi-1} L_t s_{A_t}\right) - A_t^{-\eta}\theta_h\left[\left(1-\alpha-s_{A_t}\right)\theta_h + \alpha\omega_h\right]^{1-\eta}, \\
\dot{A}_t &= v A_t^{\phi} L_t s_{A_t} \\
\dot{L}_t &= n L_t
\end{aligned}$$

and the transversality condition. Defining $x_t \equiv A_t^{\phi-1}L_t$ and $y_t \equiv \lambda_t A_t^{\eta}$ we can reduce the dimensionality of this system - thereby stationarizing it - obtaining:

$$v s_{A_t} x_t = \frac{s_{A_t}\theta_h}{\left(1-\alpha-s_{A_t}\right)\theta_h + \alpha\omega_h}\frac{\left[\left(1-\alpha-s_{A_t}\right)\theta_h + \alpha\omega_h\right]^{1-\eta}}{y_t}, \tag{34}$$

$$\frac{\dot{x}_t}{x_t} = n - (1-\phi) v s_{A_t} x_t, \tag{35}$$

$$\frac{\dot{y}_t}{y_t} = \rho - n + (\eta - \phi) v s_{A_t} x_t - \frac{\left[\left(1-\alpha-s_{A_t}\right)\theta_h + \alpha\omega_h\right]^{1-\eta}}{y_t} \tag{36}$$

In a steady state $\frac{\dot{x}_t}{x_t} = 0$, and hence eq. (35) implies that $s_{A_t}x$ does not change with $\theta_h$. Consequently, $\frac{\dot{y}_t}{y_t} = 0$, and hence eq. (36) implies that $\frac{\left[\left(1-\alpha-s_{A_t}\right)\theta_h+\alpha\omega_h\right]^{1-\eta}}{y}$ does not change with $\theta_h$. These results and eq. (34) imply that $\frac{s_{A_t}\theta_h}{\left(1-\alpha-s_{A_t}\right)\theta_h+\alpha\omega_h}$ does not change with $\theta_h$. Hence, since $\frac{s_{A_t}\theta_h}{\left(1-\alpha-s_{A_t}\right)\theta_h+\alpha\omega_h}$ is an increasing function of both $s_{A_t}$ and $\theta_h$, the implicit function theorem implies that $\frac{\partial s_{A_t}}{\partial\theta_h} < 0$. Therefore the optimal steady state share of R&D employment is a decreasing function of the profit share of the family, at least locally.

**Proof of Proposition 5.** Based on the previous Lemma 1, we can assume constant shares from the beginning of time. The first order conditions for an optimum are:

$$\begin{aligned}
\frac{\theta_h}{(1-\alpha-s_{A_t})\,\theta_h+\alpha\omega_h} &= \lambda_t v A_t, \\
\dot{\lambda}_t &= \lambda_t\left(\rho-n-vs_{A_t}\right)-\frac{1}{A_t}, \\
\dot{A}_t &= vA_t s_{A_t} \\
\dot{L}_t &= nL_t
\end{aligned}$$

and the transversality condition. Defining $y_t \equiv \lambda_t A_t$ we can reduce the dimensionality of this system -thereby stationarizing it - obtaining:

$$\frac{\theta_h}{(1-\alpha-s_{A_t})\,\theta_h+\alpha\omega_h} = vy_t, \tag{37}$$

$$\dot{y}_t = y_t\left(\rho-n\right)-1. \tag{38}$$

Notice that (38) is unstable, hence there is no transitional dynamics[10] for $y_t$ and hence for $s_{A_t}$. Hence $y_t$ jumps immediately to its steady state value $\frac{1}{(\rho-n)}$, which (plugged into 37) allows us to easily solve for the unique constant value of $s_{A_t}$ stated in the proposition.

**Proof of Proposition 6.** Based on the previous Lemma 1, we can assume constant shares from the beginning of time. The first order conditions for an optimum are:

$$\begin{aligned}
\frac{\theta_h}{(1-\alpha-s_{A_t})\,\theta_h+\alpha\omega_h} &= \lambda_t\frac{v}{\left(1+cvs_{A_t}\right)^2}A_t, \\
\dot{\lambda}_t &= \lambda_t\left(\rho-n-\frac{vs_{A_t}}{1+cvs_{A_t}}\right)-\frac{1}{A_t}, \\
\dot{A}_t &= A_t\frac{vs_{A_t}}{1+cvs_{A_t}} \\
\dot{L}_t &= nL_t
\end{aligned}$$

[10] As is generally the case with this "endogenous growth" solution, which, once reduced in dimensionality, behaves exactly as in the original Grossm and Helpman's (1991a and b) case.

and the transversality condition. Defining $y_t \equiv \lambda_t A_t$, as before, we reduce the dimensionality of this system, obtaining:

$$\frac{\theta_h}{(1-\alpha-s_{A_t})\,\theta_h+\alpha\omega_h} = \frac{v}{(1+cvs_{A_t})^2}y_t, \tag{39}$$

$$\dot{y_t} = y_t\,(\rho-n)-1. \tag{40}$$

Since eq. (40) is unstable, there is no transitional dynamics for $y_t$, and hence for $s_{A_t}$. Hence $y_t$ jumps immediately to its steady state value $\frac{1}{(\rho-n)}$, which (plugged into 39) allows us to implicitly differentiate with respect to $s_{A_t}$, thereby obtaining $\frac{\partial s_{A_t}}{\partial \theta_h} < 0$.

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