Welfare Implications of Public Education Spending Rules

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Abstract

In this paper, we quantitatively assess the welfare implications of alternative public education spending rules. To this end, we employ a dynamic stochastic general equilibrium model in which human capital externalities and public education expenditures, financed by distorting taxes, enhance the productivity of private education choices. We allow public education spending, as share of output, to respond to various aggregate indicators in an attempt to minimize the market imperfection due to human capital externalities. We also expose the economy to varying degrees of uncertainty via changes in the variance of total factor productivity shocks. Our results indicate that, in the face of increasing aggregate uncertainty, active policy can significantly outperform passive policy (i.e. maintaining a constant public education to output ratio) but only when the policy instrument is successful in smoothing the growth rate of human capital.

1 Introduction

Public education spending on, for example, schools, libraries and state universities, is an economic and political reality in all countries. The overall public education shares of GDP and of total government spending can vary quite considerably. For example, in 2005, the World Bank reported public education spending shares of GDP and of total public spending respectively of (5.9, 15.2) for the U.S.; (4.7, 9.7) for Germany; (3.7, 10.7) for Japan; and (5.5, 11.9) for the U.K.. One of the most important economic justifications for public education spending, irrespective of its relative size, is that human capital creates externalities. In other words, one agent's return to human capital is positively affected by the human capital of other agents in the society. Hence, decentralized decision-making leads to a growth rate of human capital that is inefficiently low (see e.g. Romer (1986), Lucas (1988), Azariadis and Drazen (1990) and Tamura (1991)). The importance of externalities in this context is emphasized by Lucas (2002) who states that "if ideas are the engine of growth and if an excess of social over private returns is an essential feature of the production of ideas, then we want to go out of our way to introduce external effects into growth theory, not to try to do without them" and "the existence of important external effects of investment in human capital – in knowledge – has long been viewed as an evident and important aspect of reality".[1]

The quantitative role of public education expenditure in generating endogenous growth and improving social welfare remains an open issue. Although there is a growing theoretical literature (see e.g. Glomm and Ravikumar (1992), Zhang (1996), Blankenau and Simpson (2004), Su (2004), and Blankenau (2005)), we are not aware of any estimation or calibration research which explores the empirical link between public education expenditure and social welfare in a general equilibrium micro-founded setup.[2]

In this context, a number of relevant macro-oriented policy questions can be addressed. For example, should the output share of public education expenditure be held constant over time, or should policymakers follow a state-dependent rule relating this share to the degree of market imperfection resulting from the presence of externalities? If the latter, what indicator(s) should the state-dependent policy rule respond to? What is the optimal reaction of the public education share to the observed value of the indicator(s)? With the answers to these in hand, it is then possible to establish the quantitative welfare gains/losses of moving from a constant to a state-dependent

[1] For further discussion and examples of externalities, see e.g. McMahon (2007) and van der Ploeg and Veugelers (2008, pp. 111-2).

[2] An exception here is our recent research on this topic (see Angelopoulos, *et al.* (2008)).

education spending share under various assumptions regarding the degree of uncertainty present in the economy. In light of the current global economics crisis, being able to address issues pertaining to uncertainty is particularly useful.

In an attempt to provide answers to the above questions, we calibrate, solve and conduct policy analysis using a simple dynamic stochastic general equilibrium (DSGE) model whose engine of long-term growth is human capital accumulation. Human capital externalities and public education expenditures can both enhance the productivity of private education choices. The way we model human capital externalities follows Azariadis and Drazen (1990) and Tamura (1991), while the way we model public education expenditure is based on e.g. Blankenau and Simpson (2004), Su (2004) and Blankenau (2005). To be in a position to realistically assess the effects of public education expenditure, we assume that it is financed by a distorting tax on income.

In contrast to the rest of the literature however, when modelling public education spending, we allow its share in output to respond endogenously to the gap between one of the model's state variables and its corresponding social planner's value, where the latter serves as a benchmark or target. This reflects the idea that, in the presence of positive externalities from aggregate human capital, market outcomes are inefficient, and that such a gap serves as an indicator of the extent of the underlying market inefficiency. Note that this is similar in approach to the monetary and fiscal stabilization policy literature (see, e.g. Schmitt-Grohé and Uribe (2007)). However, in contrast, here the fiscal stance reacts to a measure of resource misallocation, rather than to a measure of cyclical conditions. As far as we know, there have not been any other studies adopting this approach.

We experiment with a number of different simple feedback policy rules/ gaps. In each case, the feedback policy coefficient on the gap is chosen optimally so as to maximize household's welfare. When policy makers do not react to the state-dependent part of the rule, they follow a simple passive policy by keeping the public education share constant over time. To solve the model and welfare evaluate different policies under uncertainty, we work as in Schmitt-Grohé and Uribe (2004, 2007) by approximating both the equilibrium solution and welfare (defined as household's expected lifetime utility) to second-order.

Our empirical base of departure for our model calibration is the post-war U.S. economy. Our calibration profits significantly from having access to a dataset which includes consistent measures for human and physical capital (see e.g. Jorgenson and Fraumeni (1989)). In contrast to the relevant empirical studies referred to above, this data allows us to correctly distinguish between inputs to and output from the human capital production function.[3]

Our main findings are as follows. First, our model economy for the post-war U.S. is consistent with positive externalities from economy-wide private human capital and public education expenditure. Second, for low levels of uncertainty, welfare gains can be realized for active versus passive policy irrespective of the rule. However, for relatively high levels of uncertainty, some forms of active, state-dependent policy become counter-productive vis-a-vis passive policy. Thus, fiscal action comes at a cost. Changes in public education spending trigger changes in income taxes that disrupt private decisions and can destabilize the economy at high levels of uncertainty. Third, reaction to the gap between the growth rate of human capital and its social planner value (what we call the γ *gap* rule) welfare dominates any other simple state-dependent rule, as well as passive policy. Fourth, when we use the γ *gap* rule, the attractiveness of active versus passive policy is increasing in the degree of macroeconomic uncertainty; also, counter-cyclical spending on public education is welfare improving in this case, even if it requires higher distorting tax rates in bad times. Fifth, when we calculate welfare for the U.S., over the whole period 1981-2004 as well as over selected sub-periods, we find that substantial welfare gains could have been realized under the γ *gap* rule.

The rest of the paper is organized as follows. Section 2 presents the theoretical model. Section 3 discusses the data, calibration and long-run solution. Section 4 contains the results and Section 5 the conclusions. Finally, the Appendix presents the derivation of welfare, the social planner's solution and Figures containing the model's impulse response functions.

2 Theoretical model

We consider the implications of public education expenditure in a $DSGE$ model in which the engine of endogenous, long-term growth is human capital accumulation. We allow for positive externalities generated by the average stock of human capital in the society. These externalities can in turn justify public expenditure on education.

[3] Given the lack of comparable cross-country human capital data, other empirical work generally resorts to the use of measures of school enrolment ratios or years of schooling as general proxies of labor quality or human capital.

2.1 Households

The economy is populated by a large number of identical households indexed by the superscript h and an equal number of identical firms indexed by the superscript f, where $h, f = 1, 2, ..., N_t$. The population size, N_t, evolves at a constant rate $n \geq 1$, so that $N_{t+1} = nN_t$ where N_0 is given. Each household's preferences are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^h, l_t^h) \tag{1}$$

where E_0 is the conditional expectations operator, C_t^h is consumption of household h at time t, l_t^h is h's leisure at t, and $0 < \beta < 1$ is the subjective rate of time preference. The instantaneous utility function, $U_t^h = U(C_t^h, l_t^h)$, is increasing in all its arguments, concave and satisfies the Inada conditions. Specifically, we use a Cobb-Douglas form:

$$U_t^h = \frac{[(C_t^h)^{\mu}(l_t^h)^{1-\mu}]^{1-\sigma}}{1-\sigma} \tag{2}$$

where, $1/\sigma$ ($\sigma > 1$) is the intertemporal elasticity of substitution between consumption in adjacent periods and $0 < \mu < 1$ is the weight given to consumption relative to leisure.

Each household h consumes C_t^h, invests X_t^h in the production of its own human capital and saves I_t^h in physical capital. It receives interest income, $r_t K_t^h$, where r_t is the return to capital and K_t^h is the beginning-of-period physical capital stock. Each household also has one unit of time in each period t, which it allocates to leisure, l_t^h, work, u_t^h and education, e_t^h, so that $l_t^h + u_t^h + e_t^h = 1$. A household with a stock of human capital, H_t^h, receives labour income, $w_t u_t^h H_t^h$, where w_t is the wage rate and $u_t^h H_t^h$ is effective labour. Finally, each h receives dividends paid by firms, Π_t^h, and an average lump-sum transfer/tax, $\overline{G}_t^o$. Thus, the budget constraint of each household is:

$$C_t^h + X_t^h + I_t^h = (1 - \tau_t)\left(r_t K_t^h + w_t u_t^h H_t^h + \Pi_t^h\right) + \overline{G}_t^o \tag{3}$$

where $0 < \tau_t < 1$ is the distortionary income tax rate.

Each household's physical and human evolve according to:

$$K_{t+1}^h = (1 - \delta^p) K_t^h + I_t^h \tag{4}$$

and

$$H_{t+1}^h = (1 - \delta^h) H_t^h + \left(e_t^h H_t^h\right)^{\theta_1} (X_t^h)^{\theta_2} \left(\overline{H}_t\right)^{1-\theta_1-\theta_2} \widetilde{B}_t \tag{5}$$

where, $0 \leq \delta^p, \delta^h \leq 1$ are depreciation rates on physical and human capital respectively. The second term on the r.h.s. of (5), can be interpreted as the quantity of "new" human capital created at time t. This expression is comprised of the following arguments: (i) $e_t^h H_t^h$, which is $h's$ effective human capital; (ii) X_t^h, which is $h's$ private expenditure on education; (iii) $\overline{H}_t$, which is the average human capital stock in the economy; and (iv) $\widetilde{B}_t \equiv B\left(g_t^e\right)^{\theta_3}$, where $B > 0$ is a constant scale parameter and g_t^e is average public education expenditure expressed in efficiency units (see below for further detail). Finally, $0 < \theta_1 < 1$, $0 < 0 < \theta_2 < 1$ and $0 \leq \theta_3 < 1$ are constant parameters.[4]

The assumption that individual human capital accumulation is an increasing function of the per capita level of economy-wide human capital, $\overline{H}_t$, encapsulates the idea that the existing know-how of the economy provides an external positive effect. Equivalently, it can be thought of as a learning-by-doing effect, as discussed in Romer (1986). Or, it can be interpreted more generally, e.g. Hanushek (2002, p. 2064) argues that "external benefits involve citizen involvement in the community and government, crime reduction, family decision making and child upbringing, and economic growth. There is evidence that more schooling does have positive impact in each of these areas." Examples of other papers which use the per capita level of aggregate human capital in either the goods or human capital production functions include Lucas (1988), Azariadis and Drazen, (1990), Tamura (1991) and Glomm and Ravikumar (1992).

The assumption that individual human capital accumulation is an increasing function of both private and public expenditure on education can nest several models in the literature and reflects the idea that public spending applies more to primary and secondary education, while private spending applies more to college education and on-the-job training (see e.g. the discussion in Blankenau and Simpson, 2004, p. 586). Jones et al. (1997) and Jones et al. (2005) use private spending only, while the inclusion of public education, g_t^e, is consistent with the goal of public education policy in practice, as well as with previous theoretical work (see e.g. Glomm and Ravikumar (1992), Blankenau and Simpson (2004), Su (2004) and Blankenau (2005)).

Given the above setup, households act competitively by taking prices, policy and aggregate outcomes as given. Thus, each h chooses $\{C_t^h, X_t^h, I_t^h, l_t^h, u_t^h, e_t^h, K_{t+1}^h, H_{t+1}^h\}_{t=0}^{\infty}$ to maximize (1) subject to $(3)-(5)$, the time constraint

[4] Note that the parameter restrictions employed in (5) imply increasing returns to scale (IRS) at social level. Lucas (1988) and Benhabib and Perli (1994) are examples of other studies which employ the IRS assumption in either or both the physical and human capital production functions. In general, some restrictions on returns to scale are unavoidable to obtain a stationary equilibrium (see below).

$l_t^h + u_t^h + e_t^h = 1$ and initial conditions for K_0^h and H_0^h.

2.2 Firms

To produce its homogenous final product, Y_t^f, each firm, f, uses physical capital, K_t^f, and effective labour, $u_t^f H_t^f$. Its production function is:

$$Y_t^f = A_t \left(K_t^f\right)^\alpha \left(u_t^f H_t^f\right)^{1-\alpha} \tag{6}$$

where A_t is the level of Hick neutral technology available to firms, and $0 < \alpha < 1$ is a parameter.

Firms act competitively by taking prices, policy and aggregate outcomes as given. Accordingly, each f chooses K_t^f and $u_t^f H_t^f$ to maximize a series of static profit functions:

$$\Pi_t^f = Y_t^f - r_t K_t^f - w_t u_t^f H_t^f. \tag{7}$$

2.3 Government budget constraint

Total expenditure on public education, G_t^e and lump-sum transfers/taxes, G_t^o, are financed by total income tax revenues. Thus,

$$G_t^e + G_t^o = \tau_t \sum_{h=1}^{N_t} \left(r_t^k K_t^h + w_t u_t^h H_t^h + \Pi_t^h\right) \tag{8}$$

where only two of the three policy instruments, (i.e. G_t^e, G_t^o, τ_t) can be exogenously set in each t.[5] We choose the income tax rate, τ_t, to be the residually determined policy instrument. Note that, when we calibrate the model, the inclusion of G_t^o will make the residually determined value of the income tax rate correspond to the rate which exists in the data. This will allow for a realistic assessment of the trade-offs between increased spending on public goods versus increased distortions due to higher tax rates.

2.4 Stationary decentralized competitive equilibrium

Given the paths of two policy instruments, $\{G_t^e, G_t^o\}_{t=0}^\infty$, and initial conditions for the state variables, $\left(K_0^h, H_0^h\right)$, a decentralized competitive equilibrium

[5] To focus on public education, we abstract from other common types of government spending like public investment in infrastructure and utility-enhancing public consumption. Also note that equation (8) is as in e.g. Baxter and King (1993) in the sense that we use a balanced budget. Ignoring public debt is not important here because lump-sum taxes/transfers are equivalent to debt financing in this class of models.

(DCE) is defined to be a sequence of allocations $\{C_t, X_t, I_t, l_t, u_t, e_t, K_{t+1}, H_{t+1}\}_{t=0}^{\infty}$, prices $\{r_t, w_t\}_{t=0}^{\infty}$ and the tax rate $\{\tau_t\}_{t=0}^{\infty}$ such that (i) households maximize utility; (ii) firms maximize profits; (iii) markets clear; and (iv) the government budget constraint is satisfied in each period. Note that market clearing values will be denoted without the superscripts h, f.

Since human capital is the engine of long-run endogenous growth, we transform quantities to make them stationary. We first define *per capita* quantities for any variable Z as $\overline{Z}_t \equiv Z_t/N_t$, where $Z_t \equiv (Y_t, C_t, X_t, I_t, K_t, H_t, G_t^e, G_t^o)$. We next express these quantities as shares of per capita human capital, e.g. $z_t \equiv \overline{Z}_t/\overline{H}_t$. Also, the gross human capital growth rate is defined as $\gamma_t \equiv \overline{H}_{t+1}/\overline{H}_t$.

Using this notation and substituting out prices $\{r_t, w_t\}_{t=0}^{\infty}$,we obtain the following per capita stationary DCE:[6]

$$y_t = A_t (k_t)^{\alpha} (u_t)^{1-\alpha} \tag{9}$$

$$n\gamma_t k_{t+1} - (1-\delta^p) k_t + c_t + x_t + g_t^e = y_t \tag{10}$$

$$n\gamma_t = 1 - \delta^h + B_t (e_t)^{\theta_1} (x_t)^{\theta_2} (g_t^e)^{\theta_3} \tag{11}$$

$$\lambda_t = \mu (c_t)^{\mu(1-\sigma)-1} (1 - e_t - u_t)^{(1-\mu)(1-\sigma)} \tag{12}$$

$$\psi_t = \frac{(1-\mu)(c_t)^{\mu(1-\sigma)} (1-e_t-u_t)^{(1-\mu)(1-\sigma)-1}}{B_t\theta_1 (e_t)^{\theta_1-1} (x_t)^{\theta_2} (g_t^e)^{\theta_3}} \tag{13}$$

$$\frac{(1-\mu) c_t}{(1-e_t-u_t)} = \frac{\mu (1-\alpha)(1-\tau_t) y_t}{u_t} \tag{14}$$

$$\frac{(1-\mu) e_t}{(1-e_t-u_t)} = \frac{\mu\theta_1 x_t}{\theta_2 c_t} \tag{15}$$

$$\lambda_t = \beta (\gamma_t)^{\mu(1-\sigma)-1} \lambda_{t+1} \left[1 - \delta^p + \frac{(1-\tau_{t+1}) \alpha y_{t+1}}{k_{t+1}}\right] \tag{16}$$

$$\begin{aligned} \psi_t \;&=\; \beta (\gamma_t)^{\mu(1-\sigma)-1} \{\lambda_{t+1} (1-\alpha)(1-\tau_{t+1}) y_{t+1} + \\ &\quad +\psi_{t+1} \left[1 - \delta^h + \theta_1 B_{t+1} (e_{t+1})^{\theta_1} (x_{t+1})^{\theta_2} \left(g_{t+1}^e\right)^{\theta_3}\right]\} \end{aligned} \tag{17}$$

$$g_t^e + g_t^o = \tau_t y_t \tag{18}$$

where λ_t and ψ_t are the transformed shadow prices associated with (3) and (5) respectively in the household's problem.[7]

[6] Note that the model's non-stationary optimality conditions are presented in the Appendix.

[7] In particular, $\lambda_t \equiv \Lambda_t/H_t^{\mu(1-\sigma)-1}$ and $\psi_t \equiv \Psi_t/H_t^{\mu(1-\sigma)-1}$ where h-superscripts are omitted.

Therefore, the stationary DCE is summarized by the above system of ten equations in the paths of $\{\gamma_t, y_t, c_t, x_t, u_t, e_t, k_{t+1}, \lambda_t, \psi_t, \tau_t\}_{t=0}^{\infty}$ given the paths of the exogenous policy instruments, $\{g_t^e, g_t^o\}_{t=0}^{\infty}$, and productivity, $\{A_t\}_{t=0}^{\infty}$, whose motion is defined in the next subsection.

2.5 Fiscal instruments and technology

We next specify the processes governing the evolution of exogenous fiscal policy instruments and technology. We assume that public education spending as share of output follows a feedback policy rule. This rule consists of an exogenous component, as well as an endogenous or state-contingent part designed to improve resource allocation, e.g.

$$\frac{g_t^e}{y_t} = g_0^e \left(z^* - z_t\right)^{\zeta} \tag{19}$$

where, $0 \leq g_0^e \leq 1$ is the exogenous constant public education to output share, z_t is an indicator of the current state of the economy, z^* is the value of this indicator in the associated social planner's long-run solution[8], and $0 \leq \zeta \leq 1$ is a feedback policy coefficient. Obviously, there are many possible candidates for z_t. For instance, we could choose z_t to be any endogenous variable in the DCE. Given that it is impossible to know *a-priori* which variable will lead to the largest efficiency gains, we report below (see Section 4) the rationale for considering a subset of the model's endogenous variables and the quantitative welfare implications associated with each.

Consistent with the public economics literature, the motivation for the state-contingent part of the fiscal rule is that governmental intervention in education is primarily justified by the presence of positive externalities from the economy's aggregate human capital. In the presence of such externalities, human capital accumulation is inefficiently low, i.e. $z_t < z^*$, and so the society steps in to allocate scarce public resources to education to improve efficiency. If the market and social planner solutions coincide i.e. $z^* = z_t$, then no public education spending is required and $\frac{g_t^e}{y_t} = 0$. Finally, a passive fiscal rule can be depicted as the state when $\zeta = 0$. In this special case, the public education spending share is constant and equal to g_0^e.

By relating the policy instrument to the observed value of some endogenous variable, we implicitly exploit the information contained in observations

[8]The social planner's or policy benchmark solution (hereafter SP) is defined to be the case in which a planner solves the same problem by internalizing externalities. Note that our quantitative results reported below change very little (i.e. at the fourth decimal place) when the target value is not the long-run social planner solution, z^*, but the current one, z_t^*. Hence we use z^* for simplicity.

of the endogenous variable in question. In principle, of course, welfare and the welfare-maximizing value of the feedback policy coefficient are not expected to be independent of which endogenous variable is chosen as an indicator or intermediate target, z_t (see below). Also, there is no reason to restrict policy to respond only to one endogenous variable, and not to a combination of two, or more, or all, endogenous variables, unless observations of that one variable contain all of the available information relevant to achieving the policy objective (see the discussion by Friedman (1990) pp. 1210-1212, and Walsh (2003a), ch.9). Here, we experiment with reacting to one endogenous variable at a time and compare results across different endogenous variables.[9] The question of what further role additional endogenous variables can play is an empirical issue.

Concerning lump-sum taxes/transfers, we simply assume:

$$\frac{g_t^o}{y_t} = g_0^o \tag{20}$$

where, $0 \leq g_0^o \leq 1$ is fixed at a constant share.

Finally, following the RBC literature we assume that technology, A_t, follows an AR(1) process:

$$A_t = A^{(1-\rho^a)} A_{t-1}^{\rho^a} e^{\varepsilon_t^a} \tag{21}$$

where $A > 0$ is a constant, $0 < \rho^a < 1$ is the autoregressive parameter and $\varepsilon_t^a \sim iid(0, \sigma_a^2)$ are the random shocks to productivity.

3 Data and Calibration

The model's structural parameters relating to preferences, production and physical and human capital accumulation are next calibrated using annual post-war data for the U.S.. As our aim is to use the model to evaluate welfare as approximated around the steady-state, it is important that the calibrated parameters imply a sensible long-run solution. This provides the criterion for choosing those parameters we cannot retrieve from the data or previous empirical studies, especially the exponents in the production function for human capital. Our base calibration reported below starts by incorporating a passive policy rule for government education spending. We then follow this, in the next section, with an exercise which searches for the values of the reaction coefficients in (19) which maximize welfare across a variety of active fiscal rules.

[9] Note that we also experiment with one period lags of z rather than the current one but find that our results are basically unaffected. As above, the differences show up at the fourth decimal place.

3.1 Data

To calibrate the model, we require data for the endogenous variables as shares of human capital. Thus it is important to obtain a measure of human capital that is comparable to monetary valued quantities such as consumption, income, physical capital and government spending. To obtain this, we use data from Jorgenson and Fraumeni (1989, 1992a,b) on human and physical capital.[10] These measures are reported in billions of constant 1982 dollars for 1949-1984. The additional (annual) data required for calibration include output (GDP), consumption, government spending on education, private spending on education, compensation of employees, GDP deflator, long term nominal interest rates, labor force, effective average tax rates on labor and capital income, hours worked and years of education in the labor force. These are obtained from the following sources: (i) Bureau of Economic Analysis (NIPA accounts); (ii) OECD (Economic Outlook database); (iii) U.S. Department of Labor, Bureau of Labor Statistics (BLS); (iv) ECFIN Effective Average Tax Base (see Martinez-Mongay, 2000) and (v) Ho and Jorgenson (2001).

3.2 Calibration

The numeric values for the model's parameters are reported in Table 1 and the long-run solution they imply is presented in Table 2. To calibrate the model, we work as follows.

3.2.1 Labour's share, discount rate, leisure and population growth

We set the value of $(1-\alpha)$ equal to labor's share in income (i.e. 0.578) using compensation of employees data from the OECD Economic Outlook. This figure is similar to others used in the literature, see e.g. Lansing (1998). Given labour's share, capital's share, α, is then determined residually.

The discount rate, $1/\beta$ is equal to 1 plus the *ex-post* real interest rate, where the interest rate data is from the OECD Economic Outlook. This implies a value 0.964 for β. Again this figure is similar to other U.S. studies, see e.g. King and Rebelo (1999), Lansing (1998) and Perli and Sakellaris (1998). Following Kydland (1995, ch. 5, p. 134), we set $(1-\mu)$, the weight given to leisure relative to consumption in the utility function, equal to the average value of leisure versus work time for the working population, which is

[10] As said above, generally empirical studies use measures of school enrolment ratios or years of schooling as general proxies of labor quality or human capital. However, in our setup, these proxies are measures of the input to the production function of human capital (time spent on education) and not of the output of this activity, new human capital.

obtained using data on hours worked from Ho and Jorgenson (2001).[11] This implies $\mu = 0.36$. The population gross growth rate n is set equal to the post war labor force growth rate, 1.016, obtained by using data from Bureau of Labor Statistics.

Table 1: Parameters

parameter	value	definition
$A > 0$	0.124	productivity in goods production
$B > 0$	0.507	productivity in human capital production
$0 < \alpha < 1$	0.422	productivity of private capital
$0 < 1 - \alpha < 1$	0.578	productivity of effective labor
$0 < \beta < 1$	0.964	rate of time preference
$n \geq 1$	1.016	population growth
$0 \leq \delta^p \leq 1$	0.049	depreciation rate on physical capital
$0 \leq \delta^h \leq 1$	0.018	depreciation rate on human capital
$0 < g_0^e < 1$	0.053	public education spending share of output
$0 < g_0^o < 1$	0.157	other public investment spending share of output
$\sigma > 1$	2.000	$1/\sigma$ is the intertemporal elasticity of consumption
$0 < \mu < 1$	0.360	consumption weight in utility
$0 \leq \theta_1 \leq 1$	0.550	productivity of household human capital
$0 \leq \theta_2 \leq 1$	0.050	productivity of private education spending
$0 \leq 1 - \theta_1 - \theta_2 \leq 1$	0.400	productivity of aggregate human capital
$0 \leq \theta_3 \leq 1$	0.100	productivity of public education spending
$0 < \tau < 1$	0.210	effective direct tax rate
$0 < \rho^a < 1$	0.933	AR(1) parameter technology
$\sigma_a > 0$	0.010	std. dev. of technology innovations

3.2.2 Depreciation rates, technology and public spending

The depreciation rates for physical, δ^p, and human capital, δ^h, are calculated by Jorgenson and Fraumeni to be, on average, 0.049 and 0.0178, respectively. We also use a value for the intertemporal elasticity of consumption ($1/\sigma$) that is common in the DSGE literature (i.e. $\sigma = 2$). Using a production function and time period similar to ours, Lansing (1998) provides estimates for TFP. Hence we use his parameters for the stationary TFP process in (20), e.g. $\rho = 0.933$ and $\sigma_a = 0.01$.

We also require constants for government education and other government spending as shares of output. The constant part of education spending ratio

[11] To obtain this we divide total hours worked by total hours available for work or leisure, following e.g. Ho and Jorgenson (2001). For example, they assume that there are 14 hours available for work or leisure on a daily basis with the remaining 10 hours accounted for by physiological needs.

is set at the data average using NIPA data, i.e. $g_0^e = 0.053$. We set other government spending, g_0^o, in the government budget equation (8) so that the long-run solution for the tax rate gives 0.21. This value corresponds to the effective income tax reported in the ECFIN dataset.[12] This implies $g_0^o = 0.157$ for the output share of other government spending.

3.2.3 Human capital

We next move on to the parameters θ_1, θ_2, θ_3 and B in the production function of human capital and A in the production function of goods. Note that the expression $\left(e_t^h H_t^h\right)^{\theta_1} (X_t^h)^{\theta_2} \left(\overline{H}_t\right)^{1-\theta_1-\theta_2} B\left(g_t^e\right)^{\theta_3}$ in equation (5) is essentially the production function for the creation of new human capital, or what Jorgenson and Fraumeni (1992a, b) call investment in human capital. Model consistent values for the scale parameters A and B are obtained by solving equations (5) and (6) using data averages and long-run values for the variables y, k, x, u, e, γ and g^e, as well as the calibrated parameters α, θ_1, θ_2, θ_3,n, and δ^h.[13] Data on y, k, x and γ are obtained using the NIPA accounts and the human capital data by Jorgenson and Fraumeni discussed above.

We also acquire a proxy for e and u to calibrate A and B by employing data on the allocation of time in the labor force in education, work and leisure. Using data on hours worked from Ho and Jorgenson (2001) (see above), we calculate the share of leisure time to be 0.64. We assume that, on average, agents have the same leisure versus non-leisure time allocation independent of whether non-leisure time is used to create human capital or to work. This implies that in the steady state, non-leisure time (i.e. $e + u$) should be equal to 0.36. Agents spend time to educate both in formal education and in on-the job learning. Private agents allocate approximately 30% of their non-leisure time to formal education as opposed to work (assuming an average of 14 years spent on education and 35 years in work). In addition, they spend time to improve their human capital while at work, in the form of on-the job-training. According to the 1976 *SRC Time Use Data* (see e.g. Kim and Lee, 2007), this amounts to 20% of their work time. Taken together, these imply that on average, 35% of the labor force spends its non-leisure time in human capital creation. Given the clear upward trend in the education data, we might expect this figure to increase in the future and treat this as a lower bound for our steady state results. In any case, for the purpose of calibrating A and B we set e and u to 0.126 and 0.234 respectively.

[12] We calculate this as the weighted average of the effective tax rates on (gross) capital income and labor income, where the weights are capital's and labor's shares in income.

[13] For this exercise, we obtain model consistent y, from equation (3), using NIPA data.

Given the functions for the calibration of A and B, we calibrate θ_1, θ_2 and θ_3 so that we obtain an economically meaningful and data-consistent long-run solution. In particular, we choose the three parameters so that the long-run solution for education time, e, growth, γ and private spending on education as a share of GDP, x/y, are close to the data. This can be obtained by setting values of $\theta_1 = 0.55$, $\theta_2 = 0.05$ and $\theta_3 = 0.1$.

It is also important to note the following regarding the calibrated values of θ_1, θ_2 and θ_3. For higher (lower) values of θ_2, the steady state x/y solution becomes higher (lower). The value $\theta_2 = 0.05$ implies that private education spending as a share of GDP is close to the data average (about 2%). Moreover, for higher externalities, the growth rate becomes too low, irrespective of the size of θ_3. This happens because, with very high externalities, there are free riding problems in the creation of human capital. On the other hand, for low externalities, the implied share of time allocated to education (e) in the long-run becomes unrealistically large. By contrast, our calibrated values $\theta_1 = 0.55$ and $\theta_3 = 0.1$. guarantee a growth rate consistent with the data average (2%)[14] and, at the same time, imply that agents in the long-run will spend about 50% of their non-leisure time in acquiring human capital.

3.2.4 Long-run solution

The steady-state solution implied by this calibration is reported in Table 2. As can be seen, the long-run solution is close to the data average for the U.S., (see e.g. King and Rebello, 1999). Concerning the allocation of time, the model's long-run solution implies that total leisure time in the labour force is 62.9% (see also above). In addition, the model solution indicates that education effort in the long run is about 50% of the labor force non-leisure time. As discussed above, this is consistent with increased education and training activities in the U.S. labor force.

To sum up, this model economy for the post-war U.S. is consistent with externalities in private human accumulation and productive public education expenditure. Lucas (1988) supports a value of human capital externality of 0.4, but (since his externality is modeled as a direct argument in the goods production function) its effect on output produced is much higher relative to our calibrated externality. The associated value of the productivity of public education expenditure, $\theta_3 = 0.1$, is also well within the range assumed in the

[14] Note that a value of the gross growth rate of 1.02 is the US per labour input growth rate for 1949-1984 using GDP data from the NIPA accounts and labour data from Bureau of Labor Statistics.

related literature (see e.g. Blankenau (2005) p. 501).

Table 2: Steady-state solution

variable	model solution
γ	1.020
l	0.629
e	0.132
u	0.239
c/y	0.676
x/y	0.023
k^p/y	2.903

4 Results

Recall that the *raison d'etre* for government in our simple setup is that externalities in human capital accumulation result in suboptimally low human capital accumulation in DCE. Hence, we find it natural to start by choosing the growth rate of human capital, γ_t, as the indicator variable z_t in our fiscal policy reaction function (19). Thus, the authorities react to the gap between the market value of γ_t and its social planner value, γ^* (we call this the γ gap rule). Other natural candidates for z_t include the returns to education as perceived by the household in DCE, as these returns determine the resources that individuals allocate to human capital. Thus, we will also explore the quantitative implications of reacting to the gap between the shadow price of human capital (ψ_t) and its social planner value, as well as the gap between the wage rate (w_t) and its social planner value. Finally, for reasons of comparability with the stabilization literature that looks at output, we will also consider reaction to the output growth, $\frac{y_t}{y_{t-1}}$, gap (see, e.g. Walsh (2003b)).[15]

We will compute welfare under each of the alternative policy rules (e.g. alternative indicator variables, z_t) in (19)). Welfare is defined as the conditional expectation of the discounted sum of lifetime utility. We will approximate both the equilibrium solution and welfare to second-order around the non-stochastic steady state (see e.g. Schmitt-Grohé and Uribe (2004, 2007)). Note that, in contrast to solutions which impose certainty equivalence, the solution of the second-order system allows us to take account of the uncertainty agents face when making decisions. Also note, as pointed out by e.g. Rotemberg and Woodford (1997), Woodford (2003) and Schmitt-Grohé and

[15] Note that in the steady state, output growth is equal to the human capital growth, so this target is fully compatible with the above reasoning for human capital growth. Obviously, along the transition path the two targets will have different welfare implications.

Uribe (2004), the second-order approximation to the model's policy function helps to avoid potential spurious welfare rankings of different regimes that may arise under certainty equivalence.[16] Further, the welfare gains/losses, associated with alternative policy rules, denoted as ξ, will be obtained by computing the percentage compensation in private consumption that the individual would require so as to be equally well off between two policy rules.[17]

4.1 Optimal resource allocation and net welfare

In this subsection, we start our analysis by computing welfare, and the associated welfare-maximizing value of the feedback policy coefficient, ζ, for alternative indicators used as z_t in the policy rule (19) (see 4.1.1.). We then study the differences between active and passive policy as TFP uncertainty rises and we do so for each policy rule (see 4.1.2). Finally, we present further results regarding, according to our analysis, the best available policy rule (see 4.1.3).

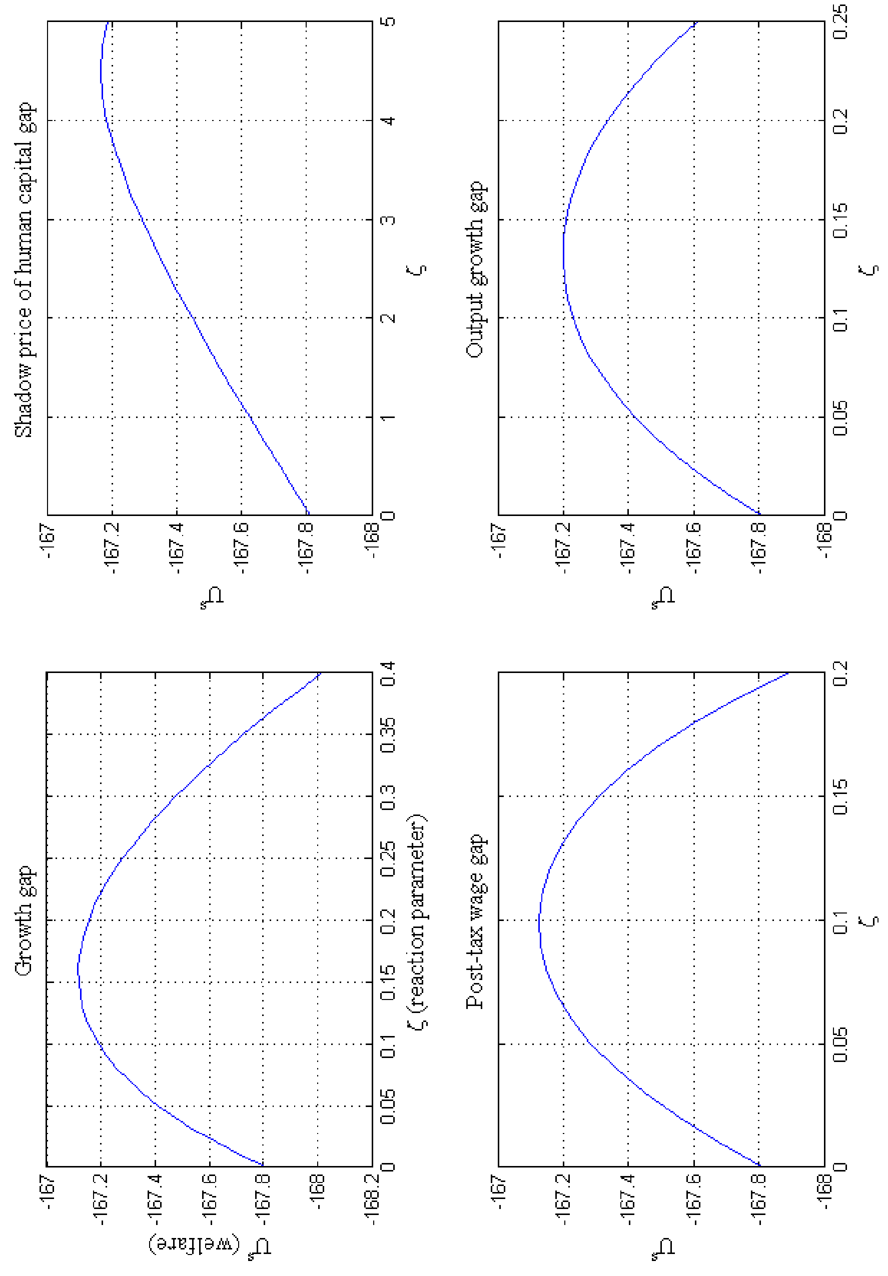
4.1.1 Welfare under different indicators

Using the baseline parameters in Table 1 above, we compute welfare for a wide range of feedback policy coefficients, ζ, under each feedback rule. This allows us to find the welfare-maximizing value of ζ under each rule. The four diagrams in Figure 1 plot the welfare associated with different values of ζ when the government reacts to the gaps of γ_t, ψ_t, w_t and $\frac{y_t}{y_{t-1}}$ from their respective social planner values. The figure shows that the optimal ζ values are 0.16, 4.5, 0.1 and 0.13 respectively. It also shows that, at the optimum, the level of welfare associated with each active rule is approximately the same (however see below). It finally shows that this value is clearly higher than the corresponding value under the passive rule (which happens when we set $\zeta = 0$ in each case) at least at the baseline calibrated level of uncertainty (however see below).

[16] See the Appendix for further details on the derivation of welfare. Also note that we evaluate the conditional expectation in the welfare function using Monte-Carlo integration. To this end we conduct 1000 simulations and approximate an agent's infinite life-time by 300 years since $\beta^{300} \simeq 0$, where β =0.933.

[17] Our derivation of ξ in the Appendix follows Lucas (1990), Cooley and Hansen (1992) and Schmitt-Grohé and Uribe (2007).

Figure 1: Optimal feedback policy



4.1.2 Passive versus active policy and uncertainty

We now compute the welfare gains/losses associated with active relative to passive policy, for each of the four indicators (γ_t, ψ_t, w_t and $\frac{y_t}{y_{t-1}}$) discussed above, across a varying degree of macroeconomic uncertainty. By welfare under active policy, we mean the maximum welfare resulting from the optimally chosen ζ in (19). By passive, we mean the case where the government's public education share in output is held constant at its data average (i.e. when we set $\zeta = 0$ in equation (19)).

Table 3 summarizes the welfare gains/losses (i.e. the value of ξ) of active relative to passive policy across different values of the standard deviation of TFP, σ_a. In particular, we present results for the deterministic case (i.e. $\sigma_a = 0$), the base calibration case where $\sigma_a = 0.01$ (which was also the case in Figure 1), and for two more scenarios representing higher levels of uncertainty which might reasonably be observable in practice, i.e. $\sigma_a = \{0.03, 0.06\}$.

Table 3: Optimal rules and welfare gains/losses from active policy

	γ *gap*	ψ *gap*	w *gap*	γ_y *gap*
$\sigma_a = 0$	0.011	0.011	0.011	0.011
$\sigma_a = 0.01$	0.012	0.011	0.011	0.010
$\sigma_a = 0.03$	0.012	0.006	0.010	-0.001
$\sigma_a = 0.06$	0.013	-0.013	0.007	-0.079

To interpret the findings in Table 3, it is important to note that welfare gains/losses from active policy can be attributed to both resource reallocation and stabilization effects. The latter are obviously absent in a deterministic environment where the economy is at its steady state.

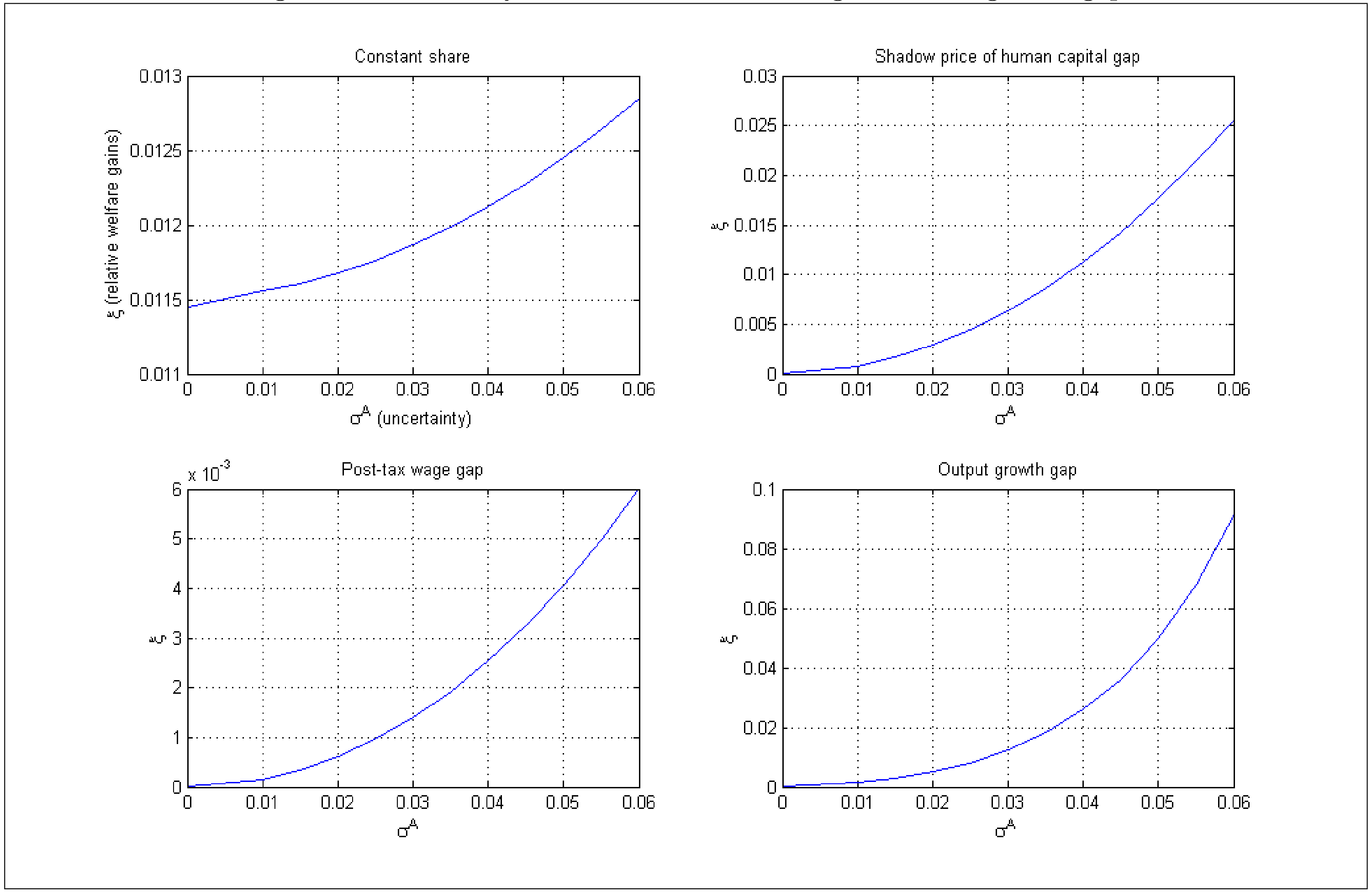
In a deterministic environment, $\sigma_a = 0$, active policy is used for resource allocation only. Since all endogenous variables, used as policy indicators in (19), are obtained from the solution of the same problem (i.e. the DCE for z and the SP for z^*), the particular choice of z makes no difference to the extent that active policies (namely, the feedback policy coefficients, ζ) are chosen optimally. Hence, all indicators imply the same welfare gain relative to passive policy (as reported in row 2 of Table 3, when $\sigma_a = 0$, applying any of the active rules leads to welfare gains of 1.1%). Therefore, to the extent that fiscal reaction is optimally chosen, and there is no uncertainty, the choice of the endogenous variable serving as a policy indicator, that the authorities react to, does not matter to welfare. In other words, in a deterministic environment, all endogenous variables are equally good as indicators of the market inefficiency.

By contrast, in a stochastic environment, $\sigma_a > 0$, active rules might also yield benefits/costs from a stabilization perspective. A higher value of ξ in rows 3-5 in Table 3, relative to the value reported in row 1, indicates the extra benefits associated with a particular optimal active rule's ability to smooth the economy along the transition path. A negative value of ξ indicates that active policy is destabilizing relative to passive policy. The results in Table 3 suggest that the potential gains and losses associated with active versus passive policy rules contain a strong degree of asymmetry. For example, only when we use the γ *gap* rule, the higher the uncertainty, the stronger the scope for using active policy (although the extra gains are small). In contrast, when we use the remaining gaps, as uncertainty increases, the desirability of active policy decreases, especially when we use the ψ *gap* and the $\frac{y_t}{y_{t-1}}$ *gap* rules.

Therefore, in a stochastic world, the choice of the endogenous variable used as an indicator, that policymakers react to, does matter to the macroeconomy. The same results suggest that, for relatively high uncertainty, some forms of active policy can even become counter-productive vis-a-vis passive policy ($\xi < 0$). This happens because action always comes at a cost (here, changes in public education spending trigger changes in distorting income taxes). Hence, excessive action destabilizes the economy.[18] Only, when we react to the gap between the market value of the growth rate of human capital and its social planner value, the attractiveness of active versus passive policy is increasing in the degree of overall uncertainty (at least in the empirically plausible range of standard deviations reported in Table 3). Therefore, reaction to the γ *gap* is superior to reaction to all other gaps in terms of welfare. This happens because the growth rate of human capital is the engine of perpetual growth and hence the key determinant of welfare. To put it differently, the growth rate of human capital is at the heart of the market imperfection, or - strictly speaking - closer to that than any other indicator, so that reaction to this gap is the most efficient way of policy action.

[18] In general, the macro effects of fiscal policy and government size are not monotonic to the magnitude and type of shocks. For a rich survey, see the European Commission's edition on Public Finances in the EU, 2008.

Figure 2: Uncertainty and the relative welfare gains of the growth gap



4.1.3 Gains from the γ gap rule

To obtain a fuller picture of these relationships, we next plot in Figure 2 (above) the welfare gains associated with reacting to the γ *gap* rule relative to passive policy as well as the other active rules. Again this is carried out for different levels of uncertainty. The results suggest that the relative welfare benefits of following the human capital growth rate rule are highest when the alternative is the output growth rate rule. In this case, the relative gains can be nearly 10% at high levels of uncertainty. The rank-ordering from highest to lowest for the remaining rules is that reacting to the shadow price of human capital gap is second; and this is followed by being passive and by reacting to the wage gap respectively. For example, at high levels of uncertainty, these rules suggest relative welfare gains of 2.5%, 1.28% and 0.006%. Thus, it appears that the γ *gap* rule outperforms the competitors considered when the focus is varying degrees of uncertainty.

To further explore what is driving the above results, we next examine the impulse response (IR) functions of the three first-moment arguments in the welfare function, i.e. the growth rate (γ), leisure (l) and consumption (c), for all policy rules, in response to a 1% standard deviation shock to TFP. The IRs are presented in the Appendix. As can be seen in these Figures, the advantage of the γ *gap* rule off steady-state is that the path of human capital growth (γ) is relatively smoother than the competing rules.[19] The other rules involve trade-offs, as they result in making some series smoother, while increasing the volatility in others. Hence, their overall second-order result on welfare is not clear and, as we saw above, depends on the level of uncertainty. Regarding the output growth rule, in addition to making consumption more volatile, notice that the effect of this rule is that human capital growth (γ) becomes countercyclical for the first periods after the shock.

Therefore, the γ *gap* rule implies additional benefits over the business cycle, when compared to the other active rules, as it makes the growth rate in the economy more stable. Although the other active rules improve the resource allocation in the deterministic long run, they effectively de-stabilize the economy along the transition path. The general message is that the human capital growth rule is preferred.

An interesting implication of the γ *gap* rule is that it suggests that gov-

[19] Note from the second order approximation to the welfare function (see the Appendix) that, in addition to the steady state values of γ,l,c and the deviations of γ_t,l_t,c_t from their steady state values, what also matters for welfare is the squared deviations and cross products of γ_t,l_t,c_t from their steady state values. Hence the variance of the series and their covariances are also important in determining the level of welfare.

ernments should increase the output share of public education spending in bad times (and decrease it in good times). This is opposite to the policy implication obtained from the passive rule case. In the passive rule case, when a negative shock hits the economy, so that output falls, the government should also decrease the level of public spending on education to keep its output share (and thus the tax rate) constant. On the contrary, according to the γ *gap* rule, when a negative shock hits the economy, the government should *increase* education spending as a share in GDP. The reason is that after a negative shock, the growth rate of human capital falls further below the target, so that the government should spend more on education to support human capital accumulation, which is the engine of growth. As we have seen, such countercyclical policy implies welfare gains because it smooths human capital growth. Notice also that these gains are realized here even if the government has to raise the income tax to finance the extra spending. In other words, in this model, countercyclical spending on public education is welfare improving, even if it requires increases in distorting tax rates in bad times.

4.2 Welfare gains over time

We next examine the welfare gains over time, when following different policy rules, by evaluating welfare in varying time periods after a temporary TFP shock for a low level of uncertainty, i.e. $\sigma_a = 0.01$.[20] In Table 4, we employ time horizons of 5, 10 and 20 years after the temporary shock hits the economy and report welfare in each case. In addition, we report the welfare gains from using an active rule versus a passive rule for all types of active policy and all time horizons. Finally, we report the welfare gains from using the γ *gap* rule as opposed to the other (active and passive) rules for all these time horizons.

There are a number of interesting results to be observed in Table 4. The first pattern observed is that all active rules imply welfare gains over the passive rule for all time horizons. Hence, for low levels of uncertainty, the active rules are preferable, not only by a lifetime utility criterion, but also when the interest is in shorter term benefits. Notice that, for all rules, the welfare gains versus passive policy settle down to 1% to 1.2% in the long run, as expected given our previous evaluations in Table 3. Another useful observation, consistent with the findings in Table 3, is that the human capital growth rule outperforms the other rules, with implied welfare gains in the

[20] This is in effect the average in the US data and is consistent with the value used in most studies in the literature.

range 0.1% to 0.8%. Notice that, in the long run, when the effects of the shock have faded away, all active rules give rise to the same level of welfare. Again, given the evaluations in Table 3, this is as expected since all active rules imply the same resource allocation in the steady state.

Table 4: Effects of a temporary TFP shock

			Welfare		
Years	passive	γ *gap*	ψ *gap*	w *gap*	γ_y *gap*
5	-26.92	-26.30	-26.36	-26.31	-26.35
10	-54.54	-53.40	-53.52	-53.42	-53.55
20	-94.55	-92.90	-93.06	-92.93	-93.12
∞	-167.34	-166.66	-166.62	-166.67	-166.63
		Gains relative to passive			
Years	passive	γ *gap*	ψ *gap*	w *gap*	γ_y *gap*
5	0	0.065	0.058	0.064	0.059
10	0	0.059	0.052	0.058	0.051
20	0	0.049	0.044	0.048	0.042
∞	0	0.011	0.012	0.011	0.012
		Gains relative to γ *gap*			
Years	passive	γ *gap*	ψ *gap*	w *gap*	γ_y *gap*
5	0.065	0	0.006	0.001	0.005
10	0.059	0	0.006	0.001	0.008
20	0.049	0	0.005	0.001	0.007
∞	0.011	0	-0.001	0.000	-0.001

4.3 Welfare gains for the U.S. (1981-2004)

Since the γ *gap* rule appears to perform the best over the criteria set out in Tables 3 and 4, we next illustrate the potential welfare gains that can be obtained by following this rule when we use actual data. For example, we feed the model with actual TFP growth shocks[21] and calculate welfare under the passive rule and the γ *gap* rule for the U.S., over the whole period 1981-2004 as well as over selected sub-periods.

The results in Table 5 suggest that quite substantial welfare gains could have been realized over the various periods considered. Considering that the model only has one (non-internalised) market imperfection, these gains

[21] Note that the TFP growth shocks have been calculated using annual data from the Gronigen project (see www.ggdc.net/index-dseries.html).

appear to be quite significant.

Table 5: Welfare under actual TFP shocks (1981-2004)			
Years	passive	γ *gap*	welfare gain
1981-1985	-26.94	-26.32	0.065
1981-1990	-54.39	-53.26	0.058
1991-2004	-72.26	-70.88	0.054
1981-2004	-105.07	-103.39	0.045

However, in our present work, (i) the government reacts to a measure of resource misallocation rather than a measure of cyclical conditions as in the stabilization literature (ii) we use an endogenous growth model where the engine of long-term growth is human capital accumulation (iii) fiscal reaction is most beneficial when the policymakers relate their fiscal stance to the human capital growth indicator, which is the indicator closest to the underlying misallocation problem. In other words, our (second-best) policy message is: do react to a market imperfection, but do so as directly as possible by relating the policy instrument to an indicator close to that imperfection.[22]

5 Conclusions

In this paper, we have quantitatively assessed the welfare implications of alternative public education spending rules. To achieve this, we employed a DSGE model in which human capital externalities and public education expenditures, financed by distorting taxes, enhance the productivity of private education choices. We allowed public education spending, as share of output, to respond to various aggregate indicators in an attempt to minimize the market imperfection due to human capital externalities. We also exposed the economy to varying degrees of uncertainty via changes in the variance of total factor productivity shocks. Our results indicated that, in the face of increasing aggregate uncertainty, active policy can significantly outperform passive policy but only when the policy instrument is successful in smoothing the growth rate of human capital.

Despite establishing that it is possible to design simple fiscal resource allocation rules which can significantly improve welfare, much more research

[22] Contrary to traditional *ad hoc* macroeconomic models, recent micro-founded general equilibrium studies imply that the potential gains from active stabilization policy may be limited (see, e.g. Beetsma (2008) for a review of this literature, Malley *et al.* (2008), and Schmitt-Grohé and Uribe (2007)). In contrast, in this paper the government's goal is to improve resource allocation not to smooth fluctuations so that any stabilizing effects occur as a by-product of more efficient resource allocation.

is needed to make these implementable in practice. This will require continual improvement in the measurement of human capital but also most probably the econometric estimation of models and fiscal rules along the lines of the ones developed in this paper. We leave these extensions for future research.

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6 Appendix

6.1 Non-stationary optimality conditions

On the households' side, the first-order conditions for $C_t^h, X_t^h, u_t^h, e_t^h, K_{t+1}^h, H_{t+1}^h$ are respectively:

$$\Lambda_t^h = \mu \left(C_t^h\right)^{\mu(1-\sigma)-1} (1 - u_t^h - e_t^h)^{(1-\mu)(1-\sigma)} \tag{22}$$

$$\Psi_t^h = \frac{\Lambda_t^h}{\widetilde{B}_t \theta_2 (e_t^h H_t^h)^{\theta_1} (X_t^h)^{\theta_2 - 1} \left(\overline{H}_t\right)^{1-\theta_1-\theta_2}} \tag{23}$$

$$\Lambda_t^h (1-\tau_t) w_t H_t^h = (1-\mu) \left(C_t^h\right)^{\mu(1-\sigma)} (1 - u_t^h - e_t^h)^{(1-\mu)(1-\sigma)-1} \tag{24}$$

$$\Psi_t^h = \frac{(1-\mu) \left(C_t^h\right)^{\mu(1-\sigma)} (1 - u_t^h - e_t^h)^{(1-\mu)(1-\sigma)-1}}{\widetilde{B}_t \theta_1 \left(e_t^h\right)^{\theta_1 - 1} \left(H_t^h\right)^{\theta_1} (X_t^h)^{\theta_2} \left(\overline{H}_t\right)^{1-\theta_1-\theta_2}} \tag{25}$$

$$\Lambda_t^h = \beta E_t \Lambda_{t+1}^h \left[1 - \delta^p + (1-\tau_{t+1}) r_{t+1}\right] \tag{26}$$

$$\begin{aligned} \Psi_t^h &= \beta E_t \Lambda_{t+1}^h (1-\tau_{t+1}) w_{t+1} u_{t+1}^h + \\ &\quad \beta E_t \Psi_{t+1}^h [1 - \delta^h + \widetilde{B}_{t+1} \theta_1 \left(e_{t+1}^h\right)^{\theta_1} \left(H_{t+1}^h\right)^{\theta_1 - 1} (X_{t+1}^h)^{\theta_2} \left(\overline{H}_{t+1}\right)^{1-\theta_1-\theta_2}] \end{aligned} \tag{27}$$

where Λ_t^h and Ψ_t^h are the multipliers associated with (3) and (5) respectively.

On the firms' side, the first-order conditions for u_t^f and K_t^f are:

$$\frac{(1-\alpha) Y_t^f}{u_t^f H_t^f} = w_t \tag{28}$$

$$\frac{\alpha Y_t^f}{K_t^f} = r_t. \tag{29}$$

6.2 Welfare analysis

6.2.1 Instantaneous utility

Using the notation set out in the paper, first consider the per capita representation of the instantaneous utility function given by (2) :

$$\overline{U}_t = \frac{[(\overline{C}_t)^{\mu}(l_t)^{1-\mu}]^{1-\sigma}}{1-\sigma} \tag{30}$$

or using our notation for stationary variables:

$$\overline{U}_t = \frac{[(c_t\overline{H}_t)^{\mu}(l_t)^{1-\mu}]^{1-\sigma}}{1-\sigma} \tag{31}$$

where $\overline{H}_t$ is the beginning-of-period human capital stock. Since $\gamma_t \equiv \overline{H}_{t+1}/\overline{H}_t$, we have for $t \geq 1$:

$$\overline{H}_t = \overline{H}_0 \left(\prod_{s=0}^{t-1} \gamma_s\right) \tag{32}$$

and $\overline{H}_0$ is given from initial conditions.

Substituting (32) into (31) gives

$$\overline{U}_t = \frac{\left\{\left[\overline{H}_0 c_t \left(\prod_{s=0}^{t-1} \gamma_s\right)\right]^{\mu} (l_t)^{1-\mu}\right\}^{1-\sigma}}{1-\sigma} \quad \text{for } t \geq 1 \tag{33}$$

$$\overline{U}_0 = \frac{\left[\left(\overline{H}_0 c_0\right)^{\mu} (l_0)^{1-\mu}\right]^{1-\sigma}}{1-\sigma} \quad \text{for } t = 0. \tag{34}$$

6.2.2 Steady-state utility

We define the long-run as the state without stochastic shocks and constant stationary variables. Using (33) – (34), steady state utility is written as

$$\overline{U}_t^* = \frac{[(\overline{H}_0 c\gamma^t)^{\mu}(l)^{1-\mu}]^{1-\sigma}}{1-\sigma} \tag{35}$$

where the * superscript denotes steady-state per capita utility. In the steady-state, non-stationary $\overline{C}_t$ grows at the constant rate γ, which in turn implies for $\sigma, \gamma > 1$ that the growth of $\overline{U}_t^*$ is constant and less than unity.

6.2.3 Second-order approximation of within period utility

Define for simplicity a variable $z_t \equiv c_t \left(\prod_{s=0}^{t-1} \gamma_s \right)$, so that the second-order approximation of the within-period utility function in (33) around its long-run is:

$$\overline{U}_t^s \simeq \overline{U}_t^* + [U_z z]\widehat{z}_t + [U_l l]\widehat{l}_t + \frac{1}{2}[U_z z + U_{zz} z^2]\widehat{z}_t^2 + \frac{1}{2}[U_l l + U_{ll} l^2]\widehat{l}_t^2 + [U_{zl} z l]\widehat{z}_t \widehat{l}_t \quad (36)$$

where

$$\begin{aligned}
&\widehat{z}_t = \widehat{c}_t + \textstyle\sum_{s=0}^{t-1} \widehat{\gamma}_s; \\
&\widehat{l}_t \simeq -\left(\tfrac{u}{1-u-e}\right)\widehat{u}_t - \left(\tfrac{e}{1-u-e}\right)\widehat{e}_t - \tfrac{1}{2}\left[\tfrac{u}{1-u-e} + \left(\tfrac{u}{1-u-e}\right)^2\right](\widehat{u}_t)^2 \\
&-\tfrac{1}{2}\left[\tfrac{e}{1-u-e} + \left(\tfrac{e}{1-u-e}\right)^2\right](\widehat{e}_t)^2 - \left(\tfrac{ue}{(1-u-e)^2}\right)\widehat{u}_t\widehat{e}_t; \\
&(\widehat{z}_t)^2 = (\widehat{c}_t)^2 + \left(\textstyle\sum_{s=0}^{t-1} \widehat{\gamma}_s\right)^2 + 2\widehat{c}_t \textstyle\sum_{s=0}^{t-1} \widehat{\gamma}_s; \\
&(\widehat{l}_t)^2 \simeq \left(\tfrac{u}{1-u-e}\right)^2 (\widehat{u}_t)^2 + \left(\tfrac{e}{1-u-e}\right)^2 (\widehat{e}_t)^2 + 2\left(\tfrac{ue}{(1-u-e)^2}\right)\widehat{u}_t\widehat{e}_t;
\end{aligned}$$

and the partial derivatives in (36), evaluated at the steady-state, are:

$$\begin{aligned}
U_z &= \tfrac{\mu[\left(\overline{H}_0 z\right)^\mu (l)^{1-\mu}]^{1-\sigma}}{z} \\
U_l &= \tfrac{(1-\mu)[\left(\overline{H}_0 z\right)^\mu (l)^{1-\mu}]^{1-\sigma}}{l} \\
U_{zz} &= \tfrac{\mu[\mu(1-\sigma)-1][\left(\overline{H}_0 z\right)^\mu (l)^{1-\mu}]^{1-\sigma}}{z^2} \\
U_{ll} &= \tfrac{(1-\mu)[(1-\mu)(1-\sigma)-1][\left(\overline{H}_0 z\right)^\mu (l)^{1-\mu}]^{1-\sigma}}{l^2} \\
U_{zl} &= \tfrac{\mu(1-\mu)(1-\sigma)[\left(\overline{H}_0 z\right)^\mu (l)^{1-\mu}]^{1-\sigma}}{zl};
\end{aligned}$$

The expression in (36) gives the second-order approximation, $\overline{U}_t^s$, for any $t \geq 1$. Therefore at $t = 0$, the expression for $\overline{U}_t^s$ is the same except that $z_0 \equiv c_0$.

6.2.4 Second-order approximation of lifetime utility

Finally, expected lifetime utility, V_t, is given by the expected discounted sum of $\overline{U}_0^s$ and $\overline{U}_t^s$, i.e.:

$$V_t \simeq \overline{U}_0^s + E_0 \sum_{t=1}^{\infty} \beta^t \overline{U}_t^s \quad (37)$$

In the simulations, lifetime is approximated by $T = 300$ years and the sample average for V is calculated using 1000 simulations.

6.2.5 Welfare comparison of two regimes

Say there are two regimes denoted by the superscripts A and B. Then, following e.g. Lucas (1990), we define ξ as the constant fraction of regime B's consumption supplement that the household would be willing to give up to be as well off under A as under B. Hence, we write:

$$V_t^A = (1-\xi)^{\mu(1-\sigma)} V_t^B \tag{38}$$

Solving for ξ, we obtain:

$$\begin{aligned} \ln(1-\xi) &= \frac{1}{\mu(1-\sigma)} \times \ln\left(\frac{V_t^A}{V_t^B}\right) \\ &\Rightarrow \ \xi \simeq \frac{1}{\mu(1-\sigma)} \times \ln\left(\frac{V_t^A}{V_t^B}\right) \end{aligned} \tag{39}$$

where, V_t^B and V_t^A are calculated by using the second-order approximation of welfare as defined in (37) above and averaged over 1000 simulations.

6.3 Social planner's solution

This is defined to be the case in which human capital externalities are internalized. Since, in this model, taxes are used to finance public education spending, where the latter is not required in the absence of human capital externalities, we drop public education spending and set taxes at zero.

Hence the planner maximizes (we omit h and f superscripts denoting households and firms and express all quantities in per capita terms):

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{[(C_t)^{\mu}(l_t)^{1-\mu}]^{1-\sigma}}{1-\sigma} \right\} \tag{40}$$

subject to:

$$C_t + X_t + I_t = A_t\,(K_t)^{\alpha}\,(u_t H_t)^{1-\alpha} \tag{41}$$

$$K_{t+1} = (1-\delta^p)K_t + I_t \tag{42}$$

$$H_{t+1} = (1-\delta^h)H_t + (e_t)^{\theta_1}\,(X_t)^{\theta_2}\,(H_t)^{1-\theta_2}\,B \tag{43}$$

as well as, the time constraint $l_t + u_t + e_t = 1$ and initial conditions for K_0 and H_0.

Working as in the DCE, we obtain a new per capita stationary equilibrium in nine equations and nine variables $\{\gamma_t, y_t, c_t, x_t, u_t, e_t, k_{t+1}, \lambda_t, \psi_t\}_{t=0}^{\infty}$.

$$y_t = A_t\,(k_t)^{\alpha}\,(u_t)^{1-\alpha} \tag{44}$$

$$n\gamma_t k_{t+1} - (1-\delta^p)\, k_t + c_t + x_t = y_t \tag{45}$$

$$n\gamma_t = 1 - \delta^h + B\,(e_t)^{\theta_1}\,(x_t)^{\theta_2} \tag{46}$$

$$\lambda_t = \mu\,(c_t)^{\mu(1-\sigma)-1}\,(1-e_t-u_t)^{(1-\mu)(1-\sigma)} \tag{47}$$

$$\psi_t = \frac{(1-\mu)\,(c_t)^{\mu(1-\sigma)}\,(1-e_t-u_t)^{(1-\mu)(1-\sigma)-1}}{B\theta_1\,(e_t)^{\theta_1-1}\,(x_t)^{\theta_2}} \tag{48}$$

$$\frac{(1-\mu)\,c_t}{(1-e_t-u_t)} = \frac{\mu\,(1-\alpha)\,y_t}{u_t} \tag{49}$$

$$\frac{(1-\mu)\,e_t}{(1-e_t-u_t)} = \frac{\mu\theta_1 x_t}{\theta_2 c_t} \tag{50}$$

$$\lambda_t = \beta\,(\gamma_t)^{\mu(1-\sigma)-1}\,\lambda_{t+1}\left[1-\delta^p+\frac{\alpha y_{t+1}}{k_{t+1}}\right] \tag{51}$$

$$\begin{aligned}\psi_t \;&=\; \beta\,(\gamma_t)^{\mu(1-\sigma)-1}\,\{\lambda_{t+1}\,(1-\alpha)\,y_{t+1}+\\ &\quad +\psi_{t+1}\left[1-\delta^h+(1-\theta_2)B\,(e_{t+1})^{\theta_1}\,(x_{t+1})^{\theta_2}\right]\}\end{aligned} \tag{52}$$

Figure 3: Responses of $\widehat{g}$, $\widehat{l}$ and $\widehat{c}$ to a temporary TFP shock relative to automatic stabilisation

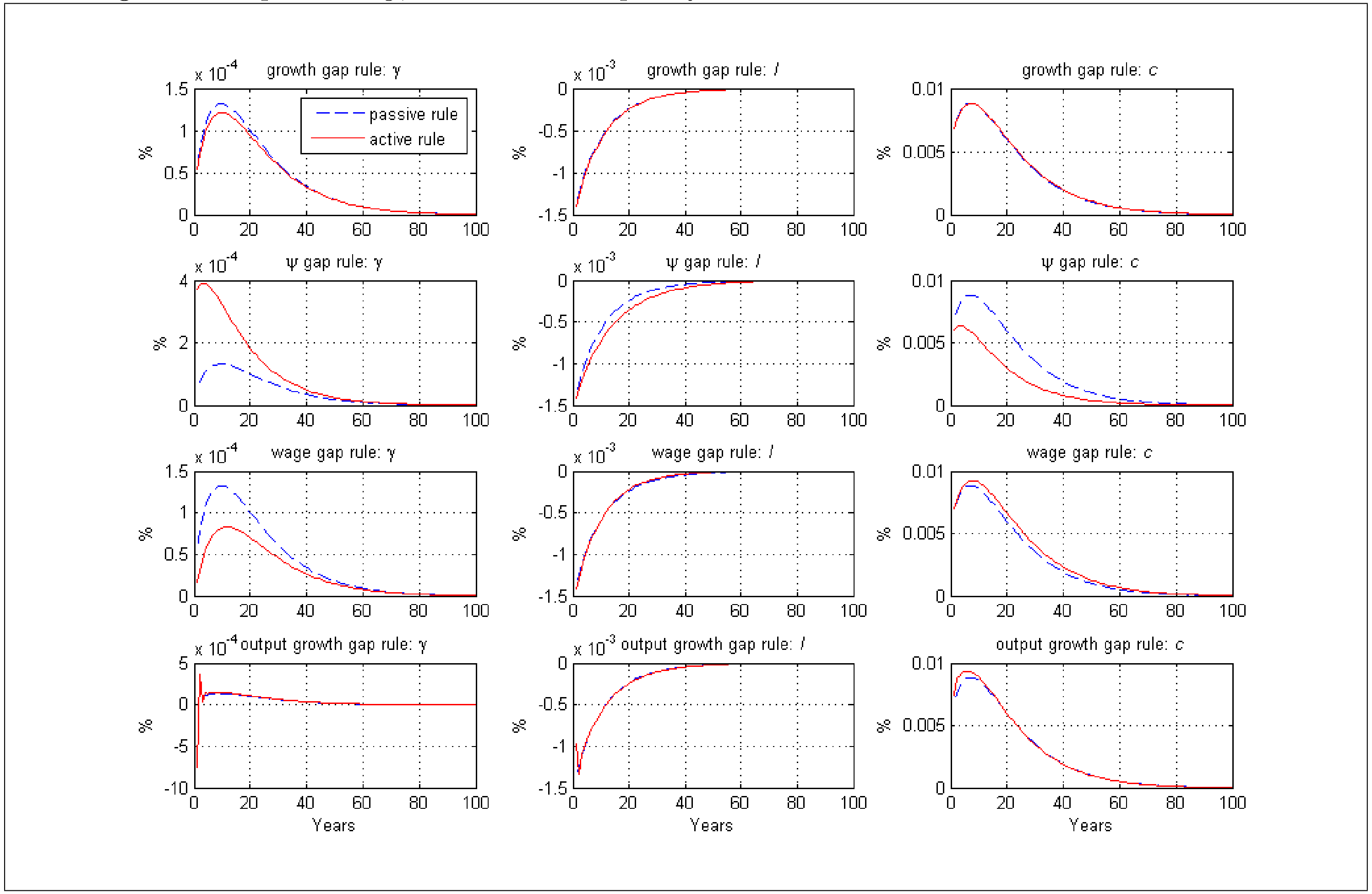

Figure 4: Responses of $\widehat{g}$, $\widehat{l}$ and $\widehat{c}$ to a temporary TFP shock relative to the growth gap rule